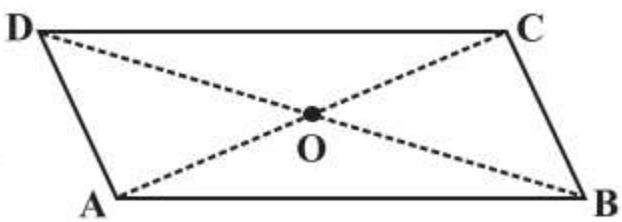
Understanding Quadrilaterals: Exercise 3.3

Q.1 Given a parallelogram ABCD. Complete each statement along with the definition or property used.



(i) AD =……
(ii) ∠ DCB =……
(iii) OC =……
(iv) m ∠DAB + m ∠CDA = ……

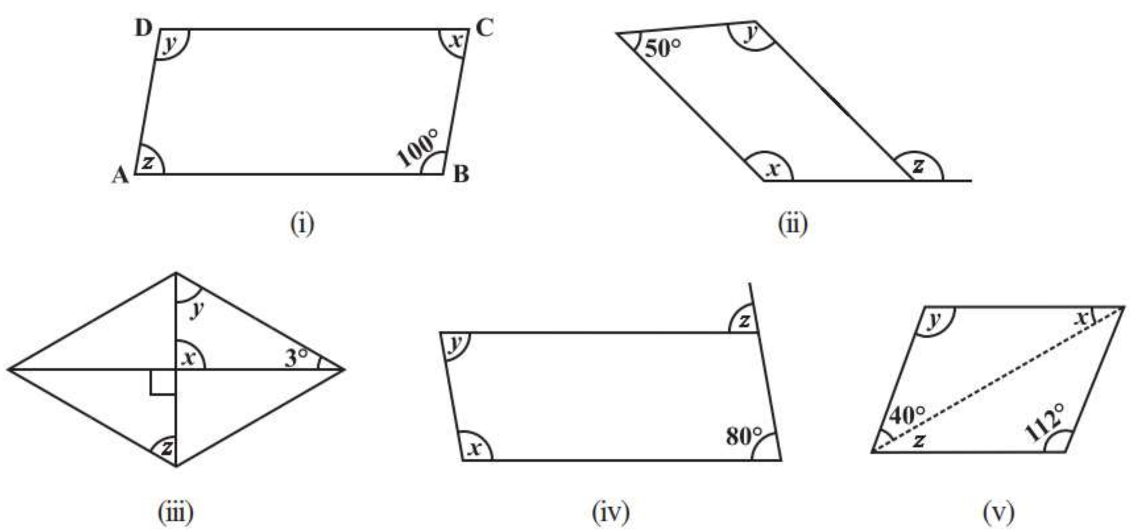
Sol. (i) AD = **BC**
Since, opposite sides are equal of a parallelogram.

(ii) ∠ DCB = **∠ DAB**
Since, opposite angles are equal of a parallelogram.

(iii) OC = **OA**
Since, diagonals of a parallelogram bisect each other.

(iv) m ∠DAB + m ∠CDA = **180^o**
Since, adjacent angles are supplementary to each other of a parallelogram.

Q.2 Consider the following parallelograms. Find the values of the unknowns *x*, *y*, *z*.



Sol. (i) As we know that adjacent angles of a parallelogram are supplementary to each other.
So, $\angle A + \angle B = 180^o$
$z + 100^o = 180^o$
$z = 180^o - 100^o$
$z = 80^o$
Now, $z = x$ or $\angle C = 80^o$ (Since in a parallelogram, opposite angles are equal)
And , y or $\angle D = \angle B = 100^o$ (Since, in a parallelogram, opposite angles are equal.)

(ii) As we know that, adjacent angles of a parallelogram are supplementary to each other.

So, $x + 50^\circ = 180^\circ$

$x = 180^\circ - 50^\circ$

$x = 130^\circ$

Now, $y = x = 130^\circ$ (Since, in a parallelogram, opposite angles are equal.)

So, $z = x = 130^\circ$ (Since, in a parallelogram, corresponding angles are equal.)

(iii) As we know that in a parallelogram, diagonals intersect each other at 90° (perpendicularly) and also vertically opposite angles are equal.

So, $x = 90^\circ$

Since, sum of interior angles of triangle = 180°

$$y + x + 30^\circ = 180^\circ$$
$$y + 90^\circ + 30^\circ = 180^\circ$$
$$y + 120^\circ = 180^\circ$$
$$y = 180^\circ - 120^\circ$$
$$y = 60^\circ$$

Now, $z = y = 60^\circ$ (Since, alternate angles of a parallelogram are equal.)

(iv) Since, corresponding angles of a parallelogram are equal.

So, $z = 80^\circ$

And adjacent angles of a parallelogram are supplementary to each other.

So, $x + 80^\circ = 180^\circ$

$x = 180^\circ - 80^\circ$

$x = 100^\circ$

Since, opposite angles are equal.

So, $y = 80^\circ$

(v) As we know that opposite angles of a parallelogram are equal.

So, $y = 112^\circ$

And sum of interior angles of triangle = 180°

$$40^\circ + y + x = 180^\circ$$
$$40^\circ + 112^\circ + x = 180^\circ$$
$$152^\circ + x = 180^\circ$$
$$x = 180^\circ - 152^\circ$$
$$x = 28^\circ$$

Now, $z = x = 28^\circ$ (Since, alternate angles are equal.)

Q.3 Can a quadrilateral ABCD be a parallelogram if:
(i) ∠D + ∠B = 180°?
(ii) AB = DC = 8 cm, AD = 4 cm and BC = 4.4 cm?
(iii) ∠A = 70° and ∠C = 65°?

Sol. **(i) Given:** ∠D + ∠B = 180°,

Quadrilateral ABCD can be a parallelogram, if it follows the given condition:

(a) Sum of the adjacent angles = 180° (b) Its opposite angles are equal.

(ii) Given: AB = DC = 8 cm, AD = 4 cm and BC = 4.4 cm,

Given quadrilateral ABCD cannot be a parallelogram.

Since opposite sides, AD ≠ BC.

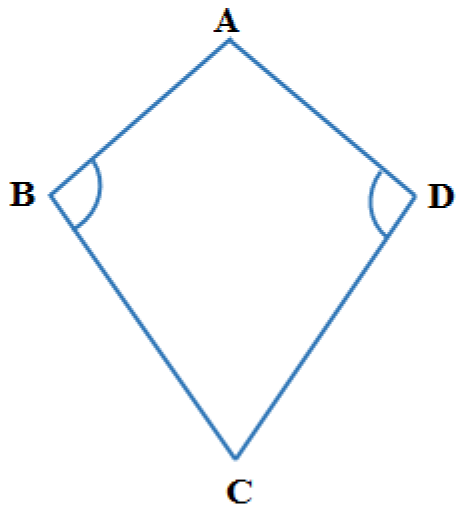
(iii) Given: ∠A = 70° and ∠C = 65°,

Given quadrilateral ABCD cannot be a parallelogram.

Since, opposite angles ∠A ≠ ∠C

Q.4 Draw a rough figure of a quadrilateral that is not a parallelogram but has exactly two opposite angles of equal measure.

Sol. Kite like quadrilateral ABCD as shown in figure. In this quadrilateral, two interior opposite angles ∠A and ∠C of same measure. But, still quadrilateral ABCD is not a parallelogram because the measure of the other pair of opposite angles ∠D and ∠B are not equal.



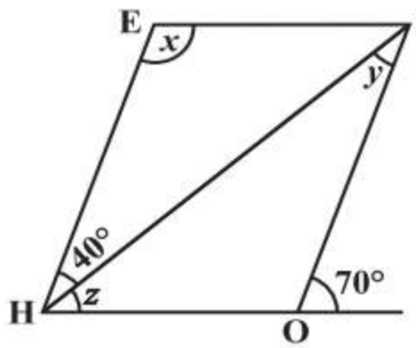
Q.5 The measures of two adjacent angles of a parallelogram are in the ratio 3:2. Find the measure of each of the angles of the parallelogram.

Sol. Let 3x and 2x be the two adjacent angles of a parallelogram.
Since in a parallelogram, adjacent angles are supplementary to each other.
So, $3x + 2x = 180^\circ$
$5x = 180^\circ$
$x = 180^\circ/5 = 36^\circ$
So, one angle, $3x = 3 \times 36^\circ = 108^\circ$
And other angle, $2x = 2 \times 36^\circ = 72^\circ$

Q.6 Two adjacent angles of a parallelogram have equal measure. Find the measure of each of the angles of the parallelogram.

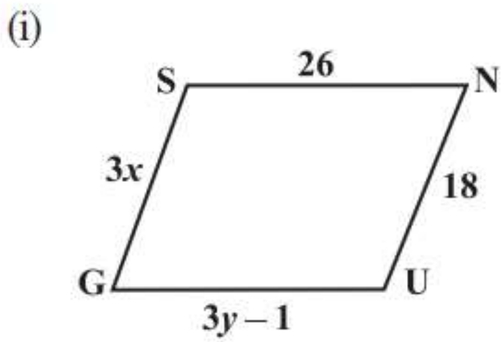
Sol. Let x be the each adjacent angle of a parallelogram ABCD.
Since in a parallelogram, adjacent angles are supplementary to each other.
So, $\angle A + \angle B = 180^\circ$
$x + x = 180^\circ$
$2x = 180^\circ$
$x = 90^\circ$
Therefore, each adjacent angle = $\angle A = \angle B = 90^\circ$
Now, $\angle A = \angle C = 90^\circ$ and $\angle B = \angle B = 90^\circ$ (Opposite angles of a parallelogram are equal)
Therefore, the measure of each angle of parallelogram is 90°, 90°, 90° and 90°.

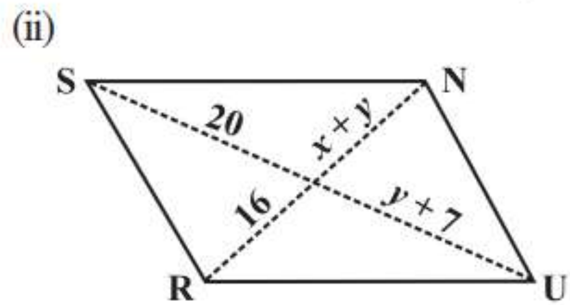
Q.7 The adjacent figure HOPE is a parallelogram. Find the angle measures x, y and z. State the properties you use to find them.



Sol. Since, in figure ∠EHP and ∠y are alternate interior angles.
So, $\angle y = 40^\circ$
Now, in a parallelogram, corresponding angles are equal.
So, $70^\circ = z + 40^\circ$
$70^\circ - 40^\circ = z$
$z = 30^\circ$
Since, adjacent pair of angles of a parallelogram are equal.
So, $x + (z + 40^\circ) = 180^\circ$
$x + 70^\circ = 180^\circ$
$x = 180^\circ - 70^\circ = 110^\circ$

Q.8 The following figures GUNS and RUNS are parallelograms. Find x and y. (Lengths are in cm).

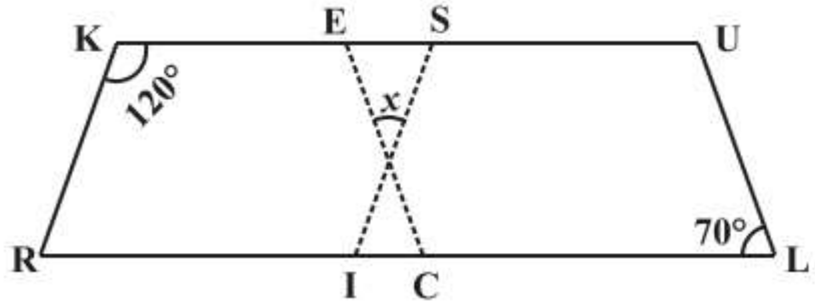




Sol. (i) Given: A parallelogram GUNS,
Since, in a parallelogram, opposite sides are equal.
So, GS = UN
$3x = 18$
$x = 6$ cm
In the same way, GU = SN
$3y - 1 = 26$
$3y = 26 + 1$
$3y = 27$
$y = 27/3 = 9$ cm
Thus, the measures of x and y are 6 cm and 9 cm respectively.

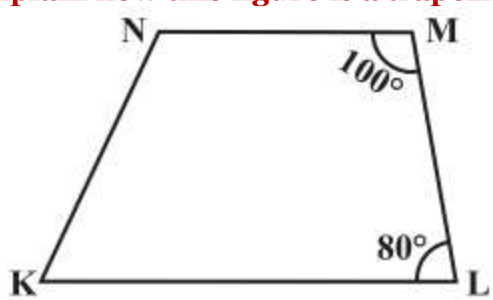
(ii)Since, diagonals bisect of a parallelogram each other.
So, $y + 7 = 20$
$y = 20 - 7 = 13$ cm
In the same way, $x + y = 16$
$x + 13 = 16$
$x = 16 - 3 = 3$ cm
Thus, the measures of x and y are 3 cm and 13 cm respectively.

Q.9In the above figure both RISK and CLUE are parallelograms. Find the value of x.



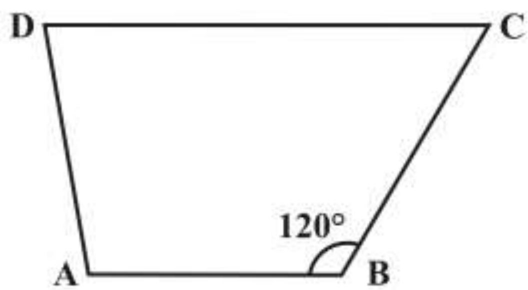
Sol. Since, adjacent angles of a parallelogram are supplementary.
So, from parallelogram RISK,
$\angle RKS + \angle ISK = 180^\circ$
$120^\circ + \angle ISK = 180^\circ$
$\angle ISK = 60^\circ$
And opposite angles of a parallelogram are equal.
So, from parallelogram CLUE,
$\angle ULC = \angle CEU = 70^\circ$
Since, the sum of measures of all the interior angles of a triangle = 180°
So, $x + 60^\circ + 70^\circ = 180^\circ$
$x = 50^\circ$

Q.10 Explain how this figure is a trapezium. Which of its two sides are parallel?



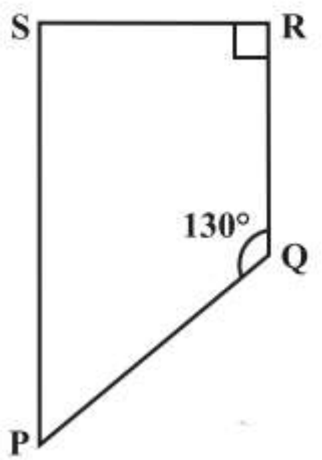
Sol. Since, if a transversal line intersects two given lines and sum of the measures of the angles on the same side of transversal line is 180°, then the given two lines will be parallel to each other.
For the given quadrilateral KLMN,
$\angle M + \angle L = 100^\circ + 80^\circ = 180^\circ$ (Since, sum of interior opposite angles = 180°)
So, lines NM and KL are parallel.
Since, Quadrilateral KLMN has one pair of parallel line.
So, this proves that KLMN is a trapezium.

Q.11 Find m ∠C in Fig if AB || DC.



***Sol.* Given:** $\overline{AB} \parallel \overline{CD}$
Since, line segment AB and DC are two parallel lines and one transversal line BC intersect them at points B and C Respectively.
So, ∠B + ∠C = 180° (Since, both angles on same side of transversal line)
120° + ∠C = 180°
∠C = 180° - 120°
∠C = 60°

Q.12 Find the measure of ∠P and ∠S if $\overline{SP} \parallel \overline{RQ}$ in Fig. (If you find m∠R, is there more than one method to find m∠P?)



***Sol.* Given:** $\overline{SP} \parallel \overline{RQ}$
Since, line segment SP and RQ are two parallel lines and one transversal line PQ intersect them at points P and Q Respectively.
From the figure,
∠P + ∠Q = 180° (Since, ∠P and ∠Q are angles on the same side of transversal)
∠P = 180° - 130° = 50°
Again, line segment SP and RQ are two parallel lines and one transversal line SR intersect them at points S and R Respectively.
From the figure,
∠R +∠S = 180° (Since, ∠R and ∠S are angles on the same side of transversal)
90° + ∠S = 180°
∠S = 180°- 90° = 90°

Alternative method to find ∠P:
By using the angle sum property of quadrilateral PQRS,
∠P + ∠Q + ∠R + ∠S = 360°
∠P + 130° + 90° + 90° = 360° (Since, ∠R = 90°. So, ∠ S =90 because adjacent angles of a parallelogram are supplementary to each other)
∠P = 360° - 310°
∠P = 50°