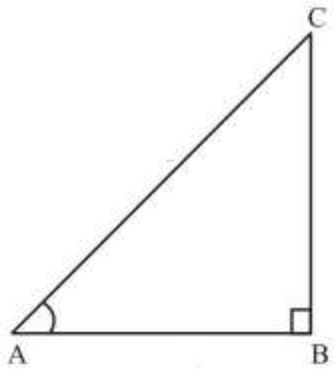
Introduction to Trigonometry: Exercise - 8.4

Q.1 Express the trigonometric ratios sin A, sec A and tan A in terms of cot A.

Sol. Let us consider a ΔABC, right angled at ∠B=90°.

For ∠A, Base = AB, Perpendicular = BC and Hypotenuse = AC

So, cot A = B/P = AB/BC



⇒AB/BC = cotA = cotA / 1

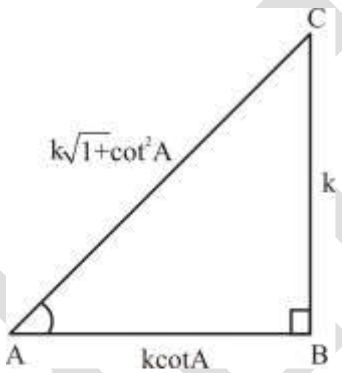
Let AB = k cotA and BC = k.

So, from Pythagoras Theorem,

$AC^2 = AB^2 + BC^2$

$AC^2 = k^2 \cot^2 A + k^2$

$AC = k\sqrt{1+cot^2 A}$



(i) So, sinA = P/B = BC/AC = $\frac{k}{k\sqrt{1+cot^2 A}}$

$= \frac{1}{\sqrt{1+cot^2 A}}$

(ii) secA = H/B = AC/AB = $\frac{k\sqrt{1+cot^2 A}}{k \cot A}$

$= \frac{\sqrt{1+cot^2 A}}{\cot A}$

(iii) and, tanA = P/B = BC/AB = $\frac{k}{k \cot A} = \frac{1}{\cot A}$

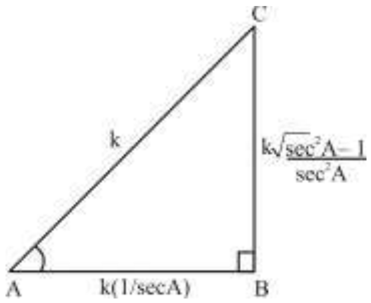
Q.2 Write the other trigonometric ratios of A in terms of secA.

Sol. Let us consider a ΔABC, right angled at ∠B = 90°.

For ∠A, Base = AB, Perpendicular = BC and Hypotenuse = AC

So, secA = H/B =AC/AB

And AC/AB = secA = secA/1



$\Rightarrow$AB/AC = 1/secA = $\dfrac{\frac{1}{\sec A}}{1}$

Suppose, AB = k (1/secA), AC = k

So, from Pythagoras theorem

$BC^2 = AC^2 - AB^2$

$BC^2 = k^2 - k^2 (1/\sec^2 A)$

BC = $k\sqrt{\dfrac{\sec^2 A - 1}{\sec^2 A}} = \dfrac{k\sqrt{\sec^2 A - 1}}{\sec A}$

So, sinA = BC/AC = $\dfrac{\frac{k\sqrt{\sec^2 A - 1}}{\sec A}}{k} = \dfrac{\sqrt{\sec^2 A - 1}}{\sec A}$

cosA = AB/AC = $\dfrac{k\left(\frac{1}{\sec A}\right)}{k} = \dfrac{1}{\sec A}$

tanA = BC/AB = $\dfrac{\frac{k\sqrt{\sec^2 A - 1}}{\sec A}}{k\left(\frac{1}{\sec A}\right)} = \sqrt{\sec^2 A - 1}$

cotA = 1/tanA = $\dfrac{1}{\sqrt{\sec^2 A - 1}}$

cosecA = 1/sinA = $\dfrac{\sec A}{\sqrt{\sec^2 A - 1}}$

So, we have found all the trigonometry ratios in term of secA.

Q.3 Evaluate:

(i) $\dfrac{\sin^2 63^0 + \sin^2 27^0}{\cos^2 17^0 + \cos^2 73^0}$

(ii) sin 25° cos 65° + cos25° sin65° [Since, sin (90°–θ) = cosθ]

Sol. **(i) Given:** $\dfrac{\sin^2 63^0 + \sin^2 27^0}{\cos^2 17^0 + \cos^2 73^0}$

Since, sin63° = sin (90° – 27°) = cos27°.............(i)

and cos17° = cos(90° – 73°) = sin 73° ....(ii)

Therefore, from (i) & (ii), we can write the expression:

$$\frac{\sin^2 63^0 + \sin^2 27^0}{\cos^2 17^0 + \cos^2 73^0} = \frac{\cos^2 27^0 + \sin^2 27^0}{\sin^2 73^0 + \cos^2 73^0}$$

Since, $\cos^2 A + \sin^2 A = 1$

=1/1 = 1

(ii) sin25° cos65° + cos25° sin65°

$= \sin(90° - 65°) . \cos 65° + \cos(90° - 65°) \sin 65°$

Since, $\sin(90° - \theta) = \cos\theta$ and $\cos(90° - \theta) = \sin\theta$

$= \cos 65° \cos 65° + \sin 65° \sin 65°$

$= \cos^2 65° + \sin^2 65°$

Since, $\sin^2\theta + \cos^2\theta = 1$

$= 1$

Q.4 Choose the correct option. Justify your choice:

(i) $9\sec^2 A - 9\tan^2 A =$

(A) 1 **(B) 9**

(C) 8 **(D) 0**

(ii) $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta) =$

(A) 0 **(B) 1**

(C) 2 **(D) None of these**

(iii) $(\sec A + \tan A)(1 - \sin A) =$

(A) secA **(B) sinA**

(C) cosecA **(D) cosA**

(iv) $\dfrac{1 + \tan^2 A}{1 + \cot^2 A} =$

(A) $\sec^2 A$ **(B) -1**

(C) $\cot^2 A$ **(D) none of these**

***Sol.* (i)** Given: $9\sec^2 A - 9\tan^2 A$

$= 9(\sec^2 A - \tan^2 A)$

Since, $1 + \tan^2 A = \sec^2 A$, so $\sec^2 A - \tan^2 A = 1$

$= 9 \times (1) = 9$

Thus, correct option: (B)

(ii) Given: $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta)$

Since, $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$; $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$; $\sec\theta = \dfrac{1}{\cos\theta}$; $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$

Put it in above expression:

$= \left(1 + \dfrac{\sin\theta}{\cos\theta} + \dfrac{1}{\cos\theta}\right)\left(1 + \dfrac{\cos\theta}{\sin\theta} - \dfrac{1}{\sin\theta}\right)$

$= \left(\dfrac{\cos\theta + \sin\theta + 1}{\cos\theta}\right)\left(\dfrac{\sin\theta + \cos\theta - 1}{\sin\theta}\right)$

Since, $(a + b)(a - b) = a^2 - b^2$

$= \dfrac{(\cos\theta + \sin\theta)^2 - 1}{\sin\theta \cos\theta}$

$= \dfrac{(\cos^2\theta + \sin^2\theta) + 2\cos\theta\sin\theta - 1}{\sin\theta\cos\theta}$

Since, $\sin^2\theta + \cos^2\theta = 1$

$= \dfrac{1 + 2\cos\theta\sin\theta - 1}{\sin\theta\cos\theta} = \dfrac{2\cos\theta\sin\theta}{\sin\theta\cos\theta} = 2$

So, correct option: (C)

(iii) Given: $(\sec A + \tan A)(1 - \sin A)$

Since, $\sec A = 1/\cos A$; $\tan A = \sin A/\cos A$

$= \left(\dfrac{1}{\cos A} + \dfrac{\sin A}{\cos A}\right)(1 - \sin A)$

$= \left(\dfrac{1 + \sin A}{\cos A}\right)(1 - \sin A)$

Since, (a + b) (a−b) = a² − b²

$$= \frac{1-\sin^2 A}{\cos A}$$

Since, sin²A + cos²A = 1, so cos²A = 1- sin²A

$$= \frac{\cos^2 A}{\cos A} = \cos A$$

Thus, correct option: (D)

(iv) Given: $\mathbf{\frac{1+\tan^2 A}{1+\cot^2 A}}$

Since, cotA = 1/tanA

$$\frac{1+\tan^2 A}{1+\left(\frac{1}{\tan A}\right)^2} = \frac{1+\tan^2 A}{\frac{\tan^2 A+1}{\tan^2 A}} = \tan^2 A$$

Thus, correct option: (D)

Q.5 Prove the following identities, where the angles involved are acute angles for which the expressions are defined :

(i) (cosecθ − cotθ)² = $\frac{1-\cos\theta}{1+\cos\theta}$

(ii) $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A$

(iii) $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$ = 1+ secθ cosecθ

(iv) $\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$

(v) $\frac{\cos A-\sin A+1}{\cos A+\sin A-1}$ = cosecA + cotA, **using the identity** cosec²A = 1 + cot²A

(vi) $\sqrt{\frac{1+\sin A}{1-\sin A}}$ = secA + tanA

(vii) $\frac{\sin\theta-2\sin^3\theta}{2\cos^3\theta-\cos\theta} = \tan\theta$

(viii) (sinA + cosecA)² + (cosA + secA)² = 7 + tan²A + cot²A

(ix) (cosecA − sinA) (secA − cosA) = $\frac{1}{\tan A+\cot A}$

(x) $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\frac{1-\tan A}{1-\cot^2 A}\right)^2 = \tan^2 A$

Sol. **(i)** Given: (cosecθ − cotθ)² = $\frac{1-\cos\theta}{1+\cos\theta}$

L.H.S. = (cosecθ − cotθ)²

Since, cosecθ = $\frac{1}{\sin\theta}$; cotθ = $\frac{\cos\theta}{\sin\theta}$

$$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2 = \left(\frac{1-\cos\theta}{\sin\theta}\right)^2$$

$$= \frac{(1-\cos\theta)^2}{\sin^2\theta} = \frac{(1-\cos\theta)^2}{1-\cos^2\theta}$$ (Since, sin²θ = 1− cos²θ)

Since, $a^2 - b^2 = (a + b)(a - b)$

$$= \frac{(1-\cos\theta)^2}{(1+\cos\theta)(1-\cos\theta)}$$

$$= \frac{1-\cos\theta}{1+\cos\theta}$$

= R.H.S. ....Hence Proved.

(ii) Given: $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A$

L.H.S. = $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A}$

$$= \frac{\cos^2 A + (1+\sin A)^2}{\cos A(1+\sin A)}$$

$$= \frac{\cos^2 A + 1 + 2\sin A + \sin^2 A}{\cos A(1+\sin A)}$$

$$= \frac{(\cos^2 A + \sin^2 A) + 1 + 2\sin A}{\cos A(1+\sin A)}$$

Since, $\cos^2 A + \sin^2 A = 1$

$$= \frac{2(1+\sin A)}{\cos A(1+\sin A)}$$

= 2/cosA

= 2 secA (Since, secA = 1/cosA)

= R.H.S.................Hence Proved.

(iii) Given: $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta \operatorname{cosec}\theta$

L.H.S. = $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$

Since cot θ = $\frac{1}{\tan\theta}$ put it in expression

$$= \frac{\tan\theta}{1-\frac{1}{\tan\theta}} + \frac{\frac{1}{\tan\theta}}{1-\tan\theta}$$

$$= \frac{\tan\theta}{\frac{\tan\theta-1}{\tan\theta}} + \frac{1}{\tan\theta(1-\tan\theta)}$$

$$= \frac{\tan^2\theta}{\tan\theta-1} + \frac{1}{\tan\theta(1-\tan\theta)}$$

$$= \frac{\tan^2\theta}{\tan\theta-1} - \frac{1}{\tan\theta(1-\tan\theta)}$$

$$= \frac{\tan^3\theta-1}{\tan\theta(1-\tan\theta)}$$ (since, $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$)

$$= \frac{(\tan\theta-1)(\tan^2\theta+\tan\theta+1)}{\tan\theta(1-\tan\theta)}$$

$$= \frac{\tan^2\theta+\tan\theta+1}{\tan\theta}$$

= tanθ + 1 + cotθ

Since, $\tan\theta = \frac{\sin\theta}{\cos\theta}$; $\cot\theta = \frac{\cos\theta}{\sin\theta}$

$= 1 + \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$= 1 + \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$ (Since, $\sin^2\theta + \cos^2\theta = 1$)

$= 1 + \frac{1}{\sin\theta\cos\theta}$

$= 1 + \text{cosec}\theta \sec\theta$

= R.H.S.........................Hence Proved.

(iv) Given: $\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$

R.H.S. = $\frac{\sin^2 A}{1-\cos A}$

Since, $\sin^2 A = 1 - \cos^2 A$

$= \frac{1-\cos^2 A}{1-\cos A}$

Since, $a^2 - b^2 = (a + b)(a - b)$

$= \frac{(1-\cos A)(1+\cos A)}{1-\cos A} = (1 + \cos A)$

$= 1 + \frac{1}{\sec A} = \frac{1+\sec A}{\sec A}$ = L.H.S.

(v) Given: $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$ = cosecA + cotA

L.H.S. = $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$

Dividing sinA in numerator and denominator

$= \frac{\frac{\cos A - \sin A + 1}{\sin A}}{\frac{\cos A + \sin A - 1}{\sin A}}$

$= \frac{\cot A - 1 + \text{cosec}A}{\cot A + 1 - \text{cosec}A}$

Since, $1 + \cot^2 A = \text{cosec}^2 A$, so $\text{cosec}^2 A - \cot^2 A = 1$

$= \frac{\cot A + \text{cosec}A - (\text{cosec}^2 A - \cot^2 A)}{\cot A - \text{cosec}A + 1}$

Since $a^2 - b^2 = (a + b)(a - b)$

$= \frac{\cot A + \text{cosec}A - (\text{cosec}A + \cot A)(\text{cosec}A - \cot A)}{\cot A - \text{cosec}A + 1}$

By taking common (cosecA + cotA)

$= \frac{(\text{cosec}A + \cot A)(1 - \text{cosec}A + \cot A)}{(\cot A - \text{cosec}A + 1)}$

= cosec A + cot A

= R.H.S.....................Hence Proved.

(vi) Given: $\sqrt{\frac{1+\sin A}{1-\sin A}}$ = secA + tanA

L.H.S. = $\sqrt{\frac{1+\sin A}{1-\sin A}}$

By multiplying and dividing by $\sqrt{1 + sinA}$

$$= \sqrt{\frac{1+sinA}{1-sinA} \text{ x } \frac{1+sinA}{1+sinA}}$$

Since, $\sin^2 A + \cos^2 A = 1$, so $1 - \sin^2 A = \cos^2 A$

$$= \sqrt{\frac{(1+sinA)^2}{\cos^2 A}}$$

$$= \frac{1+sinA}{\cos A}$$

$$= \frac{1}{\cos A} + \frac{sinA}{\cos A}$$

Since, tanA = sinA/cosA and secA =1/cosA

= secA + tanA

= R.H.S. ....Hence proved.

(vii) Given: $\frac{sin\theta - 2sin^3\theta}{2cos^3\theta - cos\theta} = \tan\theta$

$$\text{L.H.S.} = \frac{sin\theta - 2sin^3\theta}{2cos^3\theta - cos\theta} = \frac{sin\theta(1-2sin^2\theta)}{cos\theta(2cos^2\theta - 1)}$$

$$= \tan\theta\left[\frac{1-2sin^2\theta}{2(1-sin^2\theta)-1}\right]$$

$$= \tan\theta\left[\frac{1-2sin^2\theta}{2-2sin^2\theta-1}\right]$$

$$= \tan\theta\left[\frac{1-2sin^2\theta}{1-2sin^2\theta}\right]$$

= tanθ

= R.H.S...............................Hence Proved.

(viii) Given: $(\sin A + \text{cosec}A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

Taking L.H.S., $(\sin A + \text{cosec}A)^2 + (\cos A + \sec A)^2$

$= (\sin^2 A + \text{cosec}^2 A + 2\sin A\,\text{cosec}A) + (\cos^2 A + \sec^2 A + 2\cos A \sec A)$

$= (\sin^2 A + \text{cosec}^2 A + 2\sin A.\ \frac{1}{\sin A}) + (\cos^2 A + \sec^2 A + 2\cos A.\ \frac{1}{\cos A})$

$= (\sin^2 A + \text{cosec}^2 A + 2) + (\cos^2 A + \sec^2 A + 2)$

$= \sin^2 A + \cos^2 A + \text{cosec}^2 A + \sec^2 A + 4$

Since, $\sin^2 A + \cos^2 A = 1$

$= 1 + (1 + \cot^2 A) + (1 + \tan^2 A) + 4$

Since, $\text{cosec}^2 A = 1 + \cot^2 A$ and $\sec^2 A = 1 + \tan^2 A$

$= 7 + \tan^2 A + \cot^2 A$

= R.H.S.........................Hence Proved.

(ix) Given: (cosecA – sinA) (secA – cosA) = $\frac{1}{tanA + cotA}$

Taking L.H.S., (cosec A – sinA) (secA – cosA)

Since, Cosec A = $\frac{1}{\sin A}$ and SecA = $\frac{1}{\cos A}$

$$= (\frac{1}{\sin A} - \sin A)(\frac{1}{\cos A} - \cos A)$$

$$= (\frac{1-sin^2 A}{\sin A})(\frac{1-cos^2 A}{\cos A})$$

$$= \frac{cos^2 A}{\sin A} \times \frac{\sin^2 A}{\cos A}$$

= sinA cosA

Since, $\sin^2A + \cos^2A = 1$, Putting the value of 1 in denominator.

$$= \frac{sinA\ cosA}{sin^2 A + cos^2 A}$$

By dividing Numerator and Denominator by sinA cosA

$$= \frac{\frac{sinA\ cosA}{sinA\ cosA}}{\frac{sin^2 A + cos^2 A}{sinA\ cosA}}$$

$$= \frac{\frac{sinA\ cosA}{sinA\ cosA}}{\frac{sin^2 A}{sinA\ cosA} + \frac{cos^2 A}{sinA\ cosA}}$$

$$= \frac{1}{\frac{sinA}{cosA} + \frac{cosA}{sinA}}$$

$$= \frac{1}{\tan A + \text{Cot}A}$$

= R.H.S.....................Hence Proved.

(x) Given: $\left(\frac{1+tan^2 A}{1+cot^2 A}\right) = \left(\frac{1-tanA}{1-cot^2 A}\right)^2 = \tan^2 A$

By taking, L.H.S. = $\left(\frac{1+tan^2 A}{1+cot^2 A}\right)$

Since, $1 + \tan^2A = \sec^2A$

$$= \frac{sec^2 A}{cosec^2 A}$$

Since, secA = $\frac{1}{\cos A}$ and cosec A = $\frac{1}{\sin A}$

$$= \frac{1}{\cos^2 A} \times \frac{sin^2 A}{1} = \tan^2 A \text{...............(i)}$$

Now taking, R.H.S. = $\left(\frac{1-tanA}{1-cotA}\right)^2$

Since, cotA = $\frac{1}{\tan A}$

$$= \left(\frac{1-tanA}{1-\frac{1}{\tan A}}\right)^2$$

$$= \left(\frac{1-tanA}{\frac{\tan A - 1}{\tan A}}\right)^2$$

$= (-\tan A)^2$

$= \tan^2 A$ ....(ii)

From (i) & (ii), L.H.S. = R.H.S..............Hence Proved.