Introduction to Trigonometry: Exercise - 8.2

Q.1 Evaluate:

(i) sin 60° cos 30° + sin 30° cos 60°

(ii) $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

(iii) $\dfrac{\cos 45^0}{\sec 30^0 + \operatorname{cosec} 30^0}$

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(iv) $\dfrac{\sin 30^0 + \tan 45^0 - \operatorname{cosec} 60^0}{\sec 30^0 + \cos 60^0 + \cot 45^0}$

(v) $\dfrac{5\cos^2 60^0 + 4\sec^2 30^0 - \tan^2 45^0}{\sin^2 30^0 + \cos^2 30^0}$

Sol. **(i)** sin 60° cos 30° + sin 30° cos 60°

Since, sin 60° = $\frac{\sqrt{3}}{2}$; cos30° = $\frac{\sqrt{3}}{2}$; sin 30° = $\frac{1}{2}$; cos 60° = $\frac{1}{2}$

Now, putting the values,

$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}$

$= \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$

(ii) $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

Since, tan45° = 1; cos30° = $\frac{\sqrt{3}}{2}$; sin60° = $\frac{\sqrt{3}}{2}$

Now, putting the values,

$= 2(1)^2 + (\frac{\sqrt{3}}{2})^2 - (\frac{\sqrt{3}}{2})^2$

$= 2 + \frac{3}{4} - \frac{3}{4} = 2$

(iii) $\dfrac{\cos 45^0}{\sec 30^0 + \operatorname{cosec} 30^0}$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + 2} = \frac{\frac{1}{\sqrt{2}}}{\frac{2 + 2\sqrt{3}}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}}{\sqrt{2} \text{ x } 2(\sqrt{3}+1)} \text{ x } \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{\sqrt{3}(\sqrt{3}-1)}{\sqrt{2} \text{ x } 2(3-1)}$$

$$= \frac{3\sqrt{2}-\sqrt{6}}{8}$$

(iv) $\dfrac{\sin 30^0 + \tan 45^0 - \operatorname{cosec} 60^0}{\sec 30^0 + \cos 60^0 + \cot 45^0}$

Since, sin 30º = $\frac{1}{2}$; tan 45º = 1; cosec 60º = $\frac{2}{\sqrt{3}}$; cos 60º = $\frac{1}{\sqrt{2}}$; cot 45º = 1; sec 30º = $\frac{2}{\sqrt{3}}$

Now the value in given expression:

$$= \frac{\frac{1}{2}+1-\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}+\frac{1}{2}+1}$$

$$= \frac{\frac{\sqrt{3}+2\sqrt{3}-4}{2\sqrt{3}}}{\frac{4+\sqrt{3}+2\sqrt{3}}{2\sqrt{3}}} = \frac{3\sqrt{3}-4}{3\sqrt{3}+4}$$

Now rationalize,

$$= \frac{3\sqrt{3}-4}{3\sqrt{3}+4} \text{ x } \frac{3\sqrt{3}-4}{3\sqrt{3}-4}$$

$$= \frac{27-12\sqrt{3}-12\sqrt{3}+16}{27-12\sqrt{3}+12\sqrt{3}+16}$$

$$= \frac{43-24\sqrt{3}}{11}$$

(v) $\dfrac{5\cos^2 60^0 + 4\sec^2 30^0 - \tan^2 45^0}{\sin^2 30^0 + \cos^2 30^0}$

Sol. Since, Cos60º; sec30º = $\frac{2}{\sqrt{3}}$; tan45º=1; sin30º = ½; cos30º = $\frac{\sqrt{3}}{2}$

Now, putting the value in given expression:

$$= \frac{5(\frac{1}{2})^2 + 4(\frac{2}{\sqrt{3}})^2 - 1}{(\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2}$$

$$= \frac{5\text{x}\frac{1}{4} + 4\text{x}\frac{4}{3} - 1}{\frac{1}{4}+\frac{3}{4}} = \frac{5}{4}+\frac{16}{3}-1 = \frac{\frac{1}{12}(15+64-12)}{\frac{1+3}{4}} = \frac{67}{12}$$

Q.2 Choose the correct option and justify :

(i) $\dfrac{2\tan 30^0}{1+\tan^2 30^0} =$

(A) sin 60° **(B) cos 60°**
(C) tan 60° **(D) sin 30°**

(ii) $\dfrac{1-\tan^2 45^0}{1+\tan^2 45^0} =$

(A) tan 90° **(B) 1**
(C) sin 45 ° **(D) 0**

(iii) sin 2A = 2 sinA is true when A =

(A) 0° **(B) 30°**

(C) 45° **(D) 60°**

(iv) $\frac{2tan^2 30^0}{1-tan^2 30^0} =$

(A) cos 60° **(B) sin 60°**

(C) tan 60 ° **(D) none of these**

Sol. **(i)** Since, tan30° = $\frac{1}{\sqrt{3}}$,

Now, put the value in given expression:

$$\frac{2tan30^0}{1+tan^2 30^0} = \frac{2x\frac{1}{\sqrt{3}}}{1+\left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{\frac{2}{\sqrt{3}}}{1+\left(\frac{1}{3}\right)} = \frac{2}{\sqrt{3}} x \frac{3}{3+1}$$

$$= \frac{\sqrt{3}}{2} = \sin 60^o$$

So, correct Option: (A)

(ii) Since, tan45° = 1

Now, put the value in the given expression:

$$\frac{1-tan^2 45^0}{1+tan^2 45^0} = \frac{1-1}{1+1} = \frac{0}{2} = 0$$

So, correct Option (D)

(iii) If we put the value of A = 0, then sin 2 A = sin 0

= 0

and, 2 sinA = 2 sin 0 = 2 × 0

= 0

⇒ So, sin 2A = 2sinA, when A = 0

Thus, correct option: (A)

(iv) Since, tan30° = $\frac{1}{\sqrt{3}}$

Now, put the value in the given expression:

$$\frac{2tan^2 30^0}{1-tan^2 30^0} = \frac{2x\frac{1}{\sqrt{3}}}{1-\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1-\left(\frac{1}{3}\right)} = \frac{2}{\sqrt{3}} x \frac{3}{2}$$

$$= \sqrt{3} = \tan 60^o$$

So, correct option: (C)

Q.3 If tan (A + B) = $\sqrt{3}$ and tan (A – B) = $\frac{1}{\sqrt{3}}$; 0° < A+B ≤ 90°; A > B, find A and B.

Sol. Given: tan (A + B) = $\sqrt{3}$

⇒tan (A+B) = tan60°

Then,

$\Rightarrow A + B = 60^o$ ....(i)

and $\tan(A-B) = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan(A-B) = \tan 30^o$

$\Rightarrow A - B = 30^\circ$...(ii)

On solving (i) and (ii),

$A = 45^\circ$ and $B = 15^\circ$

Thus, $A = 45^\circ$ and $B = 15^\circ$

Q.4 State whether the following are true of false. Justify your answer.

(i) sin (A + B) = sin A + sin B.

(ii) The value of sin θ increases as θ increases.

(iii) The value of casθ increases as θ increases.

(iv) sinθ = cosθ for all values of θ.

(v) cot A is not defined for A = 0°.

Sol. (i) This statement is false because if we put the value of $A = 60^\circ$ and $B = 30^\circ$. Then,

LHS: $\sin(A + B) = \sin(60^\circ + 30^\circ)$

$= \sin 90^\circ = 1$ ....(i)

and RHS, $\sin A + \sin B = \sin 60^\circ + \sin 30^\circ$

$= \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{\sqrt{3}+1}{2}$ ....(ii)

From (i) & (ii), LHS ≠ RHS

$\Rightarrow \sin(A + B) \neq \sin A + \sin B$

(ii) This statement is true, because from the table of sin function:

θ	0°	30°	45°	60°	90°
sinθ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1

Clearly we can see that the value of sinθ increases as θ increases.

(iii) This statement is false, because from the table of cos function:

θ	0°	30°	45°	60°	90°
cosθ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0

Clearly we can see that the value of cosθ decreases as θ increases.

(iv) From the table of sin and cos, this statement is false. This statement is only true for $\theta = 45^\circ$.

$(\sin 45^o = \frac{1}{\sqrt{2}} = \cos 45^o)$

(v) This statement is true.

Since, $\tan 0^\circ = 0$ and $\cot 0^o = \frac{1}{\tan 0^0} = \frac{1}{0}$; which is undefined.