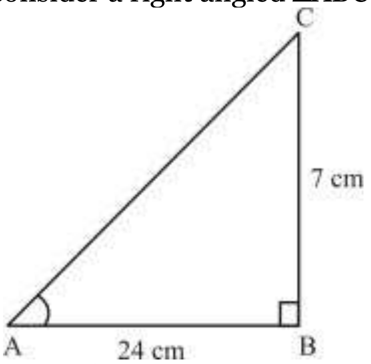
# Introduction to Trigonometry: Exercise - 8.1

**Q.1 In ΔABC, right angled at B, AB = 24 cm, BC = 7 cm. Determine:**
**(i) sin A, cos A (ii) sin C, cos C**

***Sol.*** Let us consider a right angled ΔABC.




From the Pythagoras theorem,

$AC^2 = AB^2 + BC^2 = (24)^2 + (7)^2$

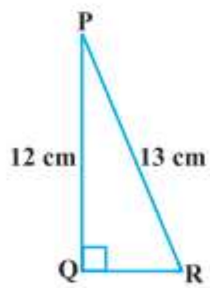
$= 576 + 49 = 625$

$\Rightarrow AC = \sqrt{625} = 25cm$

**(i)** For ∠A, Base=AB, Perpendicular = BC and Hypotenuse = AC
sinA = P/H = BC/AC = 7/25,
cosA = B/H = AB/AC = 24/25

**(ii)** For ∠C, Base = BC, Perpendicular = AB and Hypotenuse = AC
sinC = P/H = AB/AC = 24/25,
cosC = B/H = BC?AC = 7/25.

**Q.2 In adjoining figure, find tan P – cot R.**




***Sol.*** From the Pythagoras theorem,

$PR^2 = PQ^2 + QR^2$

$\Rightarrow 13^2 = 12^2 + QR^2$

$\Rightarrow QR^2 = 13^2 - 12^2$

$= 169 - 144 = 25$

$\Rightarrow QR = \sqrt{25} = 5$

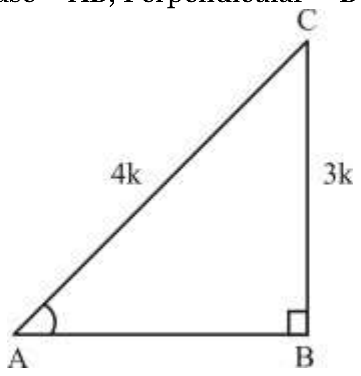
So, tanP = P/B = QR/PQ
= 5/12
and cotR = B/P = QR/PQ = 5/12

Thus, $\tan P - \cot R = \frac{5}{12} - \frac{5}{12}$

$= 0$

**Q.3 If $\sin A = \frac{3}{4}$, calculate cos A and tan A.**

***Sol.*** Consider a $\Delta ABC$ right angled at $\angle B = 90^\circ$.
For $\angle A$, Base = AB, Perpendicular = BC and Hypotenuse = AC




So, $\sin A = P/H$,
$= BC/AC = 3/4$

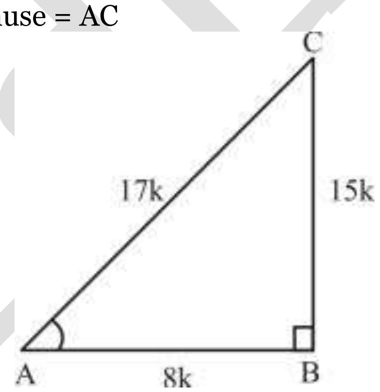
Suppose, BC = 3k and AC = 4k.
From the Pythagoras theorem,
Then, $AB^2 = AC^2 - BC^2$
$= (4k)^2 - (3k)^2$
$= 16k^2 - 9k^2$
$= 7k^2$
$AB = \sqrt{7}\,k$

Now, $\cos A = B/H = AB/AC = \sqrt{7}\,k / 4k$
$= \sqrt{7} / 4$

and $\tan A = P/B = BC/AB = 3k/\sqrt{7}\,k = 3/\sqrt{7}$

**Q.4 Given 15 cot A = 8, find sin A and sec A.**

***Sol.*** Consider a $\Delta ABC$ right angled at $\angle B = 90^\circ$.
For $\angle A$, Base = AB, Perpendicular = BC
and Hypotenuse = AC




Since, 15 cot A = 8
$\Rightarrow \cot A = 8/15$
Suppose, AB = 8k and BC = 15k.
From the Pythagoras theorem,
Then, $AC^2 = AB^2 + BC^2$
$= (8k)^2 + (15k)^2$
$= 64k^2 + 225k^2$
$AC = \sqrt{289}\,k$
$= 17k$
Now, $\sin A = P/H = BC/AC = 15k/17k = 15/17$
and, $\sec A = H/B = AC/AB = 17k/8k = 17/8$

**Q.5 Given $\sec\theta = 13/12$, calculate all other trigonometric ratios.**

***Sol.*** Lets consider a $\Delta ABC$ right angled at $\angle B = 90^o$ and $\angle A = \theta^o$.

For $\angle A$, Base = AB, Perpendicular = BC and Hypotenuse = AC

So, $\sec\theta = H/B = AC/AB = 13/12$

Let AC = 13k and AB = 12k.

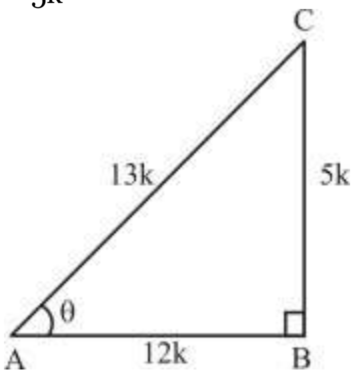
$BC^2 = AC^2 - AB^2$

$= (13k)^2 - (12k)^2$

$= 169k^2 - 144k^2$

$= 25k^2$

BC = 5k




Now, $\sin\theta = P/H = BC/AC$

$= 5k/13k$

$= 513$

$\cos\theta = B/H = AB/AC = 12k/13k = 12/13$

$\tan\theta = P/B = BC/AB = 5k/12k = 5/12$

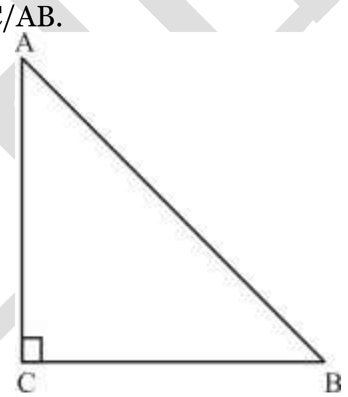
$\cot\theta = 1/\tan\theta = 12/5$

$\mathrm{cosec}\theta = 1/\sin\theta = 13/5$

**Q.6 If $\angle A$ and $\angle B$ are acute angles such that cos A = cos B, then show that $\angle A = \angle B$.**

***Sol.*** Consider $\Delta ABC$ right angled at $\angle C$ such that cosA = B/H = AC/AB.

and, cosB = BC/AB.




Since, given: cos A = cos B

$\Rightarrow$ AC/AB = BC/AB

$\Rightarrow$ AC = BC

If in $\Delta ABC$, AC = BC, then

$\Rightarrow \angle A = \angle B$ ....................Hence Proved

Since, angles opposite to equal sides are equal. So, It is isosceles triangle.

**Q.7 If cotθ = 7/8, evaluate:**

**(i)** $\dfrac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

**(ii)** $\cot^2\theta$

***Sol.*** Consider a ΔABC right angled at ∠B=90° and ∠A = θ°

From the Pythagoras theorem,

Then, $AC^2 = BC^2 + AB^2$

$= (8k)^2 + (7k)^2$

$= 64k^2 + 49k^2$

$= 113k^2$

$= \sqrt{113}\,k$

So, $\sin\theta = P/H = BC/AC = 8k/\sqrt{113}\ k = 8/\sqrt{113}\ k$..........(i)

and $\cos\theta = B/H = AB/AC = 7k/\sqrt{113}\ k = 7/\sqrt{113}\ k$..........(ii)

**(i)** Now, Putting the value from (i) & (ii)

$$\text{Therefore, } \frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1-\sin^2\theta}{1-\cos^2\theta} = \frac{1-\frac{64}{113}}{1-\frac{49}{113}}$$

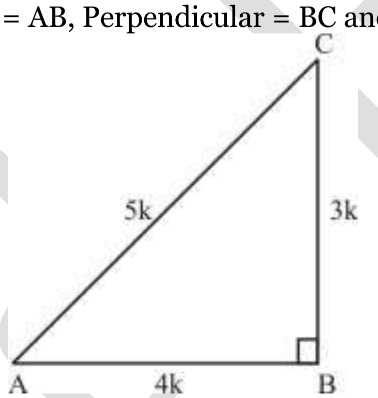
$$= \frac{113-64}{113-49} = 49/64$$

**(ii)** Now, $\cot^2\theta = (7/8)^2 = 49/64$

**Q.8 If 3 cot A = 4, check whether $\dfrac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.**

***Sol.*** Lets consider a ΔABC right angled at ∠B = 90°.

For ∠A, Base = AB, Perpendicular = BC and Hypotenuse = AC




Since, 3cot A = 4

⇒cot A = 4/3

So, cotA = B/P = AB/BC = 4/3

Suppose, AB = 4k and BC = 3k

Then, $AC^2 = AB^2 + BC^2$

$= (4k)^2 + (3k)^2$

$= 16k^2 + 9k^2$

$= 25k^2$

AC = 5k

So, sinA = P/H = BC/AC = 3k/5k = 3/5

cosA = B/H = AB/AC = 4k/5k = 4/5

Since, $\tan A = \dfrac{1}{\cot A} = 3/4$

Now taking L.H.S. $= \frac{1-tan^2 A}{1+tan^2 A} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}} = \frac{7}{25}$ ...............(i)

and R.H.S. = $\cos^2 A - \sin^2 A$

$= (4/5)^2 - (3/5)^2$

$= \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$ ..........................(ii)

From (i) & (ii)

⇒ L.H.S = R.H.S

So, $\frac{1-tan^2 A}{1+tan^2 A} = \cos^2 A - \sin^2 A$ ................Hence Proved.

**Q.9 In ΔABC right angled at B, if tan A = $\frac{1}{\sqrt{3}}$, find the value of**

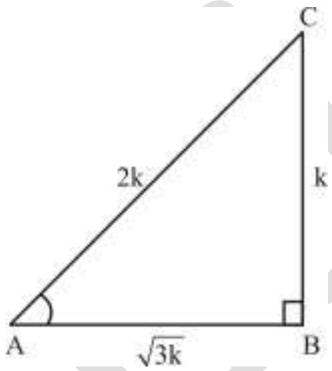
**(i) sin A cos C + cos A sin C**
**(ii) cos A cos C – sin A sin C**

***Sol.*** Lets consider a Δ ABC, right angled at ∠B=90°.

For ∠A, Base = AB, Perpendicular = BC and Hypotenuse = AC

tanA = P/H

$= BC/AB = \frac{1}{\sqrt{3}}$




Let BC = k and AB = $\sqrt{3}$ k.

From the Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$= 3k^2 + k^2$

$= 4k^2$

AC =2k

So, sinA = P/H = BC/AC = k/2k = 1/2

cosA = B/H = AB/AC = $\sqrt{3}$ k/2k = $\sqrt{3}$ / 2

Now, for ∠C, Base = BC, Perpendicular = AB and Hypotenuse = AC

So, sin C = P/H = AB/AC = $\sqrt{3}$ k / 2k = $\sqrt{3}$ / 2

and, cos C = B/H = BC/AC = k/2k = 1/2

**(i)** sinA cosC + cosA sinC $= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

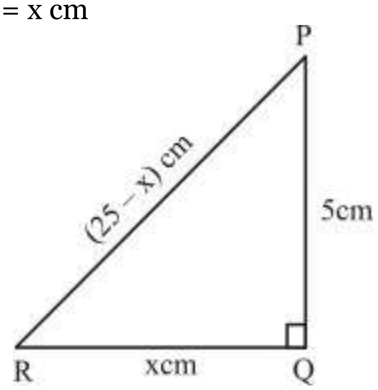
$= \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$

**(ii)** cos A cosC – sin A sin C $= \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2} = 0$

**Q.10 In ΔPQR, right angled at Q, PR + QR 25 cm and PQ = 5 cm. Determine the values of sin P cos P and tan P.**

***Sol.*** Given: In ΔPQR, right ∠d at Q and PR + QR = 25 cm and PQ = 5cm

Let QR = x cm




So, PR = (25 – x) cm

From Pythagoras theorem,

$RP^2 = RQ^2 + QP^2$

$\Rightarrow (25-x)^2 = x^2 + 5^2$

$\Rightarrow 625 - 50x + x^2 = x^2 + 25$

$\Rightarrow -50x = -600$

$\Rightarrow x = -600/-50 = 12$

So, RQ = 12cm

$\Rightarrow$ RP = (25 – 12)cm = 13cm

Now, sinP = RQ/RP = 12/13

cosP = PQ/RP = 5/13

and tanP = RQ/PQ = 12/5

**Q.11 State whether the following are true or false. Justify your answer.**

**(i) The value of tan A is always less than 1.**

**(ii) sec A=12/5 for some value of angle A .**

**(iii) cos A is the abbreviation used for the cosecant of angle A.**

**(iv) cot A is the product of cot and A.**

**(v) sinθ = 4/3 for some angle θ.**

***Sol.*** **(i)** This statement is false because sides of a right triangle may have any length, therefore tanA may have any value.

**(ii)** This statement is true because Sec A = H/B. Since Hypotenuse is numerator. So, sec A is always greater than 1.

**(iii)** This statement is false because cos A is the abbreviation used for cosine A.

**(iv)** This statement is false because cot A is not the product of 'cot' and A. 'cot' separated from A has no meaning.

**(v)** This statement is false because sinθ = P/H . so, it cannot be > 1.