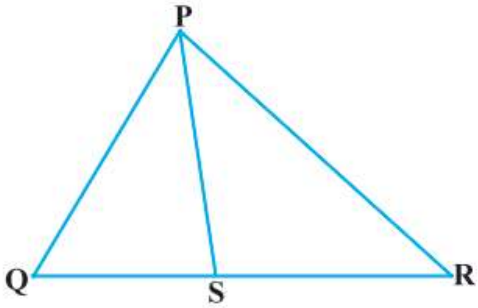
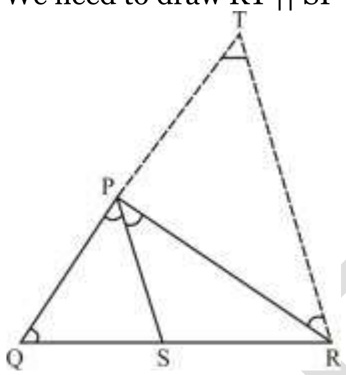
Triangles: Exercise - 6.6

Q.1 In Figure, PS is the bisector of ∠QPR of ΔPQR. Prove that QS/SR = PQ/PR



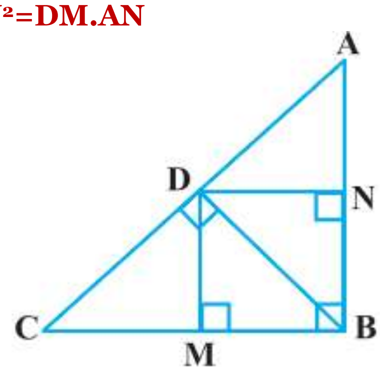
Sol.
Given: In **Δ**PQR, PS is the internal bisector of ∠QPR meeting QR at S.
So, ∠QPS = ∠SPR
To prove: QS/SR = PQ/PR
Construction: We need to draw RT || SP to cut QP produced at T as shown in figure.



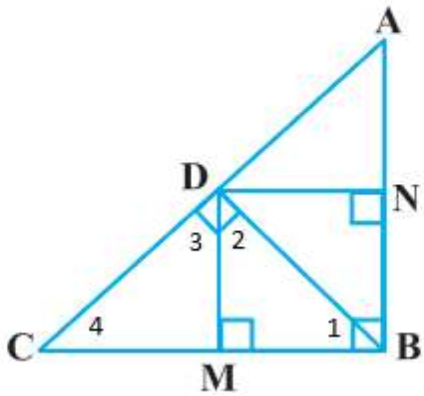
Proof: Since PS || TR and PR cuts them,

∠SPR=∠PRT .... (i) (Alternate angles)
and, ∠QPS=∠PTR .... (ii) (Corresponding angles)
But, ∠QPS=∠SPR (given)
So, ∠PRT=∠PTR (From (i) and (ii))
⇒ PT = PR .... (iii) (Since, sides opposite to equal angles are equal.]
Now, in ΔQRT,
RT || SP (From construction)
So, QS/SR = PQ/PT (From Basic Proportionality Theorem)
From (iii),
⇒ QS/SR = PQ/PR ....Hence Proved.

Q.2 In figure, D is a point on hypotenuse AC of ΔABC, BD⊥AC, DM⊥BC and DN⊥AB. Prove that
(i) DM^2=DN.MC
(ii) DN^2=DM.AN



Sol. Given: AB ⊥ BC and DM ⊥ BC
Since both lines are parallel. So,
⇒ AB || DM………………..(i)
Similarly, BC⊥AB and DN⊥AB
⇒ CB || DN…………………..(ii)
From (i) & (ii), quadrilateral BMDN is a rectangle.
So, BM = ND …………………………(iii)



(i) Now, in ΔBMD,
$\angle 1 + \angle BMD + \angle 2 = 180^o$
$\Rightarrow \angle 1 + 90^o + \angle 2 = 180^o$
$\Rightarrow \angle 1 + \angle 2 = 90^o$ …………….(iv)
Similarly, in ΔDMC,
$\angle 3 + \angle 4 = 90^o$ …………….(v)
Since BD⊥AC.
So, $\angle 2 + \angle 3 = 90^o$ …………….(vi)
From (iv) & (vi)
$\Rightarrow \angle 1 + \angle 2 = \angle 2 + \angle 3$
$\Rightarrow \angle 1 = \angle 3$……………………..(vii)
Also, From (v) & (vi)
$\Rightarrow \angle 3 + \angle 4 = \angle 2 + \angle 3$
$\Rightarrow \angle 2 = \angle 4$…………….(viii)
Thus, in ΔBMD and ΔDMC,
$\angle 1 = \angle 3$ and $\angle 2 = \angle 4$ (From (vii) & (viii))
So, from AA-criterion of similarity,
ΔBMD~ΔDMC
⇒ BM/DM = MD/MC
⇒ DN/DM = DM/MC (Since, BM = ND)
⇒ $DM^2 = DN \times MC$…………………Hence Proved.

(ii) Since, we have proved.
ΔBND~ΔDNA
⇒ BN/DN = ND/NA
⇒ DM/DN = DN/AN (Since BN = DM)
⇒ $DN^2 = DM \times AN$………………………Hence Proved.

Q.3 In figure, ΔABC is a triangle in which $\angle ABC > 90^o$ and AD⊥CB produced. Prove that $AC^2=AB^2+BC^2+2BC.BD$

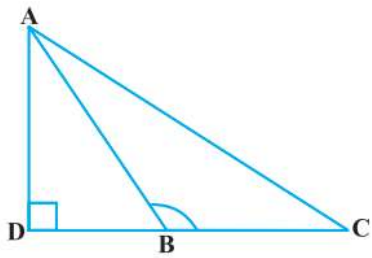
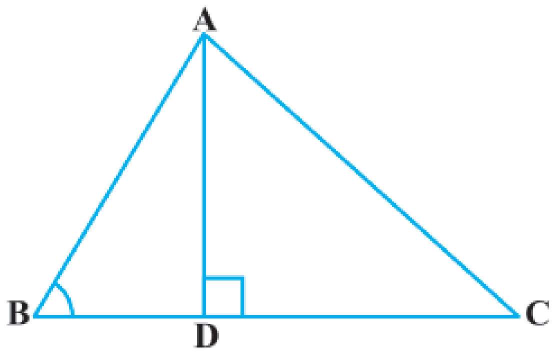


Fig. 6.58

Sol. Given: ΔABC, in which ∠ABC > 90° and AD⊥CB
To Prove: $AC^2 = AB^2 + BC^2 + 2BC.BD$
Proof: Since, ΔADB is a right triangle, right-angled at D.
So, from Pythagoras theorem,
$AB^2 = AD^2 + DB^2$ .... (i)

Again, ΔADC is a right triangle, right-angled at D.
So, from Pythagoras theorem,
$AC^2 = AD^2 + DC^2$
$\Rightarrow AC^2 = AD^2 + (DB + BC)^2$
$\Rightarrow AC^2 = AD^2 + DB^2 + BC^2 + 2DB.BC$
$\Rightarrow AC^2 = (AD^2 + DB^2) + BC^2 + 2BC.BD$
$\Rightarrow AC^2 = AB^2 + BC^2 + 2BC.BD$ (from (i))
Hence Proved.

Q.4 In figures, ABC is a triangle in which ∠ABC < 90° and AD⊥BC. Prove that $AC^2=AB^2+BC^2-2BC.BD$



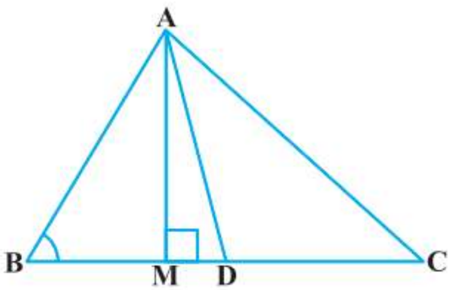
Sol. Given: **Δ**ABC, in which ∠ABC < 90° and AD⊥BC.
To prove: $AC^2 = AB^2 + BC^2 - 2BC.BD$
Proof: Since, ΔADB is a right triangle, right-angled at D.
So, from Pythagoras theorem,
$AB^2=AD^2+BD^2$ .... (i)
Again, in right angle triangle ΔADC, right-angled at D.
So, from Pythagoras theorem,
$AC^2 = AD^2 + DC^2$
$\Rightarrow AC^2 = AD^2 + (BC - BD)^2$
$\Rightarrow AC^2 = AD^2 + (BC^2 + BD^2 - 2BC.BD)$
$\Rightarrow AC^2 = (AD^2 + BD^2) + BC^2 - 2BC.BD$
$\Rightarrow AC^2 = AB^2 + BC^2 - 2BC.BD$ (From (i))
Hence Proved.

Q.5 In figure, AD is a median of a triangle ABC and AM⊥BC. Prove that

(i) $AC^2 = AD^2 + BC.DM + (\frac{BC}{2})^2$

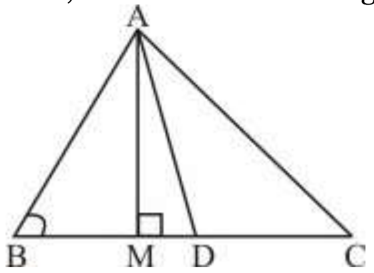
(ii) $AB^2 = AD^2 - BC.DM + (\frac{BC}{2})^2$

(iii) $AC^2 = AB^2 = 2AD^2 + \frac{1}{2}BC^2$



Sol. In given figure, $\angle AMD = 90^\circ$, so, $\angle ADM < 90^\circ$ and $\angle ADC > 90^\circ$.
Therefore, $\angle ADM$ is acute and $\angle ADC$ is obtuse.

(i) Since, in ΔADC, $\angle ADC$ is an obtuse angle.



So, $AC^2 = AD^2 + DC^2 + 2DC.DM$

$\Rightarrow AC^2 = AD^2 + (\frac{BC}{2})^2 + 2.\frac{BC}{2}.DM$ (Since, $DC = \frac{BC}{2}$, AD is median.)

$\Rightarrow AC^2 = AD^2 + (\frac{BC}{2})^2 + BC.DM$

$\Rightarrow AC^2 = AD^2 + BC.DM + (\frac{BC}{2})^2$ .... Hence Proved.

(ii) Now, in ΔABD, $\angle ADM$ is an acute angle.
So, $AB^2 = AD^2 + BD^2 - 2BD.DM$

$\Rightarrow AB^2 = AD^2 + (\frac{BC}{2})^2 - 2.\frac{BC}{2}.DM$ (Since, $BD = \frac{BC}{2}$, AD is median.)

$\Rightarrow AB^2 = AD^2 - BC.DM + (\frac{BC}{2})^2$ .... Hence Proved.

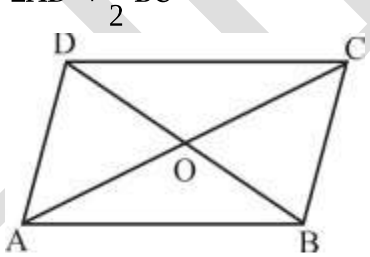
(iii) By adding result (i) and (ii),

$AB^2 + AC^2 = 2AD^2 + \frac{1}{2}BC^2$ ....Hence Proved.

Q.6 Prove that the sum of the squares of the diagonals of parallelogram is equal to the sum of the squares of its sides.

Sol. Since we have proved that if AD is a median of ΔABC,

then $AB^2 + AC^2 = 2AD^2 + \frac{1}{2}BC^2$



Given that the diagonals of a parallelogram bisect each other, therefore, BO is the median of ΔABC and DO is medians of Δ ADC.

So, $AB^2 + BC^2 = 2BO^2 + \frac{1}{2}AC^2$........... (i)

and, $AD^2 + CD^2 = 2DO^2 + \frac{1}{2}AC^2$.......... (ii)

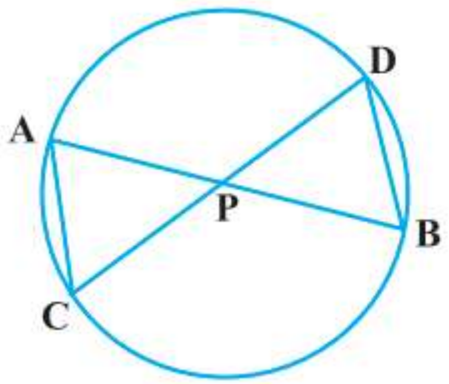
Now add (i) and (ii),

$AB^2 + BC^2 + AD^2 + CD^2 = 2BO^2 + \frac{1}{2}AC^2 + 2DO^2 + \frac{1}{2}AC^2$

$\Rightarrow AB^2 + BC^2 + AD^2 + CD^2 = 2(\frac{1}{4}BD^2 + \frac{1}{4}BD^2) + AC^2$ (Since, $DO = \frac{1}{2}BD$)

$\Rightarrow AB^2 + BC^2 + AD^2 + CD^2 = AC^2 + BD^2$ ....Hence Proved.

Q.7 In figure, two chords AB and CD intersect each other at the point P. Prove that
(i) ΔAPC~ΔDPB
(ii) AP.PB = CP.DP

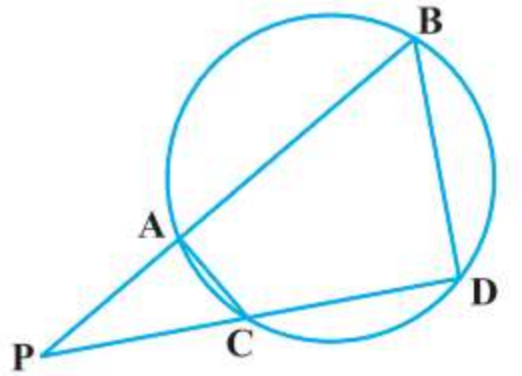


Sol. Given: Two chords AB and CD intersect each other at the point P.

(i) Now, in ΔAPC and ΔDPB,
∠APC = ∠DPB (Vertically opposite angles)
∠CAP = ∠BDP (Since, angles of same segment of circle are equal.]
So, from AA-criterion of similarity,
ΔAPC~ΔDPB........................Hence Proved.

(ii) Since we have proved that ΔAPC~ΔDPB
So, AP/DP = CP/PB
⇒ AP×PB = CP×DP.....................Hence Proved.

Q.8 In figure, two chords AB and CD of a circle intersect each other at the point P(when produced) outside the circle, Prove that
(i) ΔPAC~ΔPDB
(ii) PA.PB = PC.PD

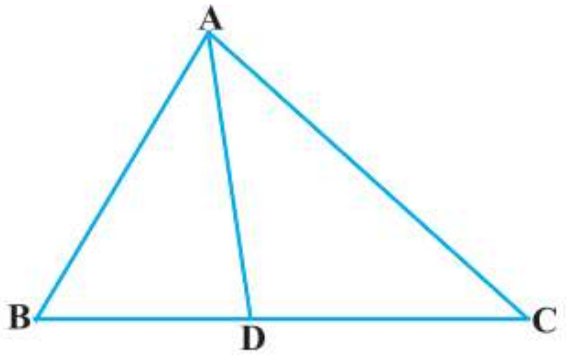


Sol.

(i) In ΔPAC and ΔPDB,
∠APC = ∠BPD (Common Angle)
∠PAC = ∠PDB (Since, exterior angle of a cyclic quadrilateral is ∠PAC and ∠PDB is opposite interior angle, which are both equal.)
So, from, AA-criterion of similarity,
ΔPAC~ΔDPB ....Hence Proved

(ii) Since, we have proved that ΔPAC~ΔDPB
PA/PD = PC/PB
⇒ PA.PB = PC.PD....................Hence Proved

Q.9 In figure, D is a point on side BC of ΔABC such that BD/CD = AB/AC . Prove that AD is the bisector of ∠BAC.

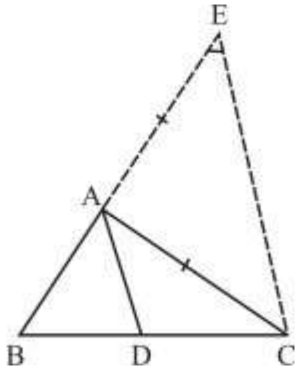


Sol. Given: In **Δ**ABC, D is a point on BC:

BD/CD = AB/AC

To prove: AD is the angle bisector of ∠BAC.

Construction: Produce BA to E so that AE = AC. Now join CE.



Proof: In ΔAEC,

AE = AC (Construction)

∠AEC = ∠ACE .... (i) (Since, angles opposite to equal sides)

And BD/CD = AB/AC (given)

⇒ BD/CD = AB/AE (From (i))

So, from converse of Basic Proportionality Theorem,

DA || CE

and CA is a transversal, So

∠BAD = ∠AEC .... (ii) (Corresponding angles)

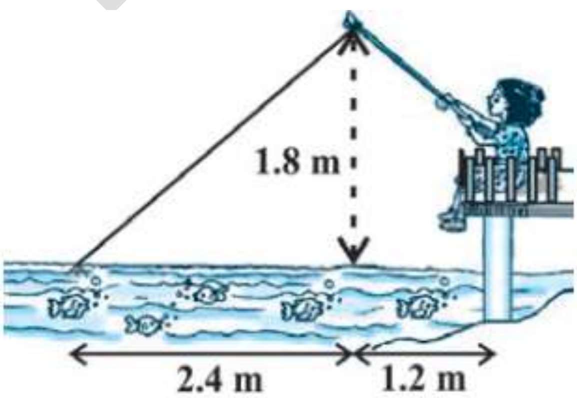
and, ∠DAC = ∠ACE .... (iii) (Alternate angles)

Also, ∠AEC = ∠ACE (From (i))

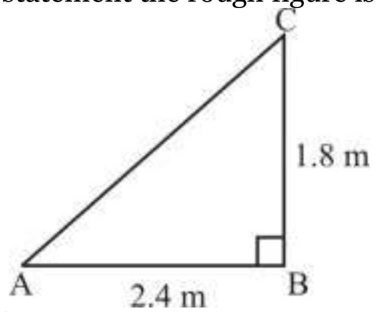
So, ∠BAD = ∠DAC (from (ii) and (iii))

Thus, we can say that AD bisects, ∠BAC ....Hence Proved

Q.10 Nazima is fly fishing in a stream. The tip of her fishing rod is 1.8 m above the surface of the water and the fly at the end of the string rests on the water 3.6 m away and 2.4 m from a point directly under the tip of the rod. Assuming that her string (from the tip of her rod to the fly) is taur, how much string does she have out (see figure)? If she pulls in the string at the rate of 5 cm per second, what will the horizontal distance of the fly from her after 12 seconds?



Sol. According to statement the rough figure is:



So, we need to find AC.
From Pythagoras theorem,

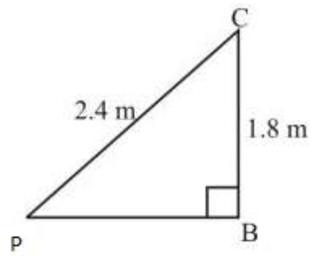
$AC^2 = (2.4)^2 + (1.8)^2$

$\Rightarrow \quad AC^2 = 5.76 + 3.24 = 9.00$

$\Rightarrow \quad AC = 3$ m

So, length of string she have out = 3 m.
According to question, length of the string pulled at the rate of 5 cm/sec in 12 seconds.
So length of string in 12 sec. = (5 × 12)
= 60 cm = 0.60 m
Then, remaining string left out = 3 – 0.6 = 2.4 m

In 2nd case: for horizontal distance PB,



From Pythagoras theorem,

$PB^2 = PC^2 - BC^2$
$= (2.4)^2 - (1.8)^2$
$= 5.76 - 3.24 = 2.52$

$\Rightarrow$So, $PB = \sqrt{2.52} = 1.59$ approx

Thus, the horizontal distance of the fly from Nazima after 12s
= (1.59 + 1.2) m
= 2.79 m approx.