Triangles: Exercise - 6.5

Q.1 Sides of triangles are given below. Determine which of them are right triangles. In case of a right triangle, write the length of its hypotenuse.

(i) 7 cm, 24 cm, 25 cm
(ii) 3 cm, 8 cm, 6 cm
(iii) 50 cm, 80 cm, 100 cm
(iv) 13 cm, 12 cm, 5 cm

Sol. **(i)** Suppose the sides of the triangle, a = 7 cm, b = 24 cm and c = 25 cm
the larger side is c = 25 cm
As we know that for the right angle triangles:
$(\text{Perpendicular})^2 + (\text{base})^2 = (\text{Largest side or hypotenuse})^2$
So, $a^2 + b^2 = 7^2 + 24^2$
$= 49 + 576$
$= 625 = c^2$
Therefore, the triangle with the given sides is a right triangle.
The length of the hypotenuse = 25 cm.

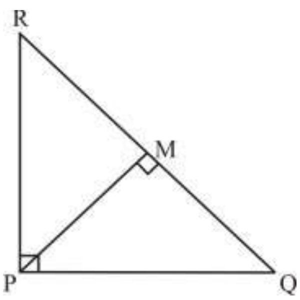
(ii) Suppose the sides of the triangle, a = 3 cm, b = 8 cm and c = 6 cm
the larger side is b = 8 cm
As we know that for the right angle triangles:
$(\text{Perpendicular})^2 + (\text{base})^2 = (\text{Largest side or hypotenuse})^2$
So, $a^2 + c^2 = 32 + 62$
$= 9+36$
$= 45 \neq b^2$
Therefore, the triangle with the given sides is not a right triangle.

(iii) Suppose the sides of the triangle, a = 50 cm, b = 80 cm and c = 100 cm
the larger side is c = 100 cm
As we know that for the right angle triangles:
$(\text{Perpendicular})^2 + (\text{base})^2 = (\text{Largest side or hypotenuse})^2$
$a^2 + b^2 = 502 + 802$
$= 2500 + 6400$
$= 8900 \neq c^2$
Therefore, the triangle with the given sides is not a right triangle.

(iv) Suppose the sides of the triangle, a = 13 cm, b = 12 cm and c = 5 cm
The larger side is a = 13 cm
As we know that for the right angle triangles:
$(\text{Perpendicular})^2 + (\text{base})^2 = (\text{Largest side or hypotenuse})^2$
$b^2 + c^2 = 12^2 + 5^2$
$= 144 + 25$
$=169 = a^2$
Therefore, the triangle with the given sides is a right triangle.
The length of hypotenuse = 13 cm.

Q.2 PQR is a triangle right angled at P and M is a point on QR such that PM⊥QR. Show that $PM^2 = QM.MR$.

Sol.



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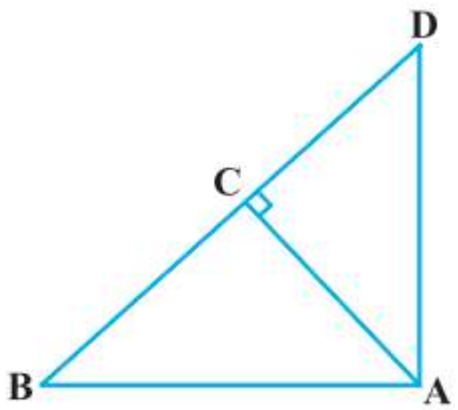
Given: PQR is a triangle right angled at P and $PM \perp QR$.
To prove: $PM^2 = QM.MR$
Proof: Since $PM \perp QR$
So, ΔPQM and ΔPRM are the right angle triangles.
In ΔPQM,
$PQ^2 = PM^2 + QM^2$
Or, $PM^2 = PQ^2 - QM^2$ ....**(i)**
Now, in ΔPMR,
$PR^2 = PM^2 + MR^2$
Or, $PM^2 = PR^2 - MR^2$ ....**(ii)**
Now, Adding equation, **(i)** and **(ii)**,
$2PM^2 = (PQ^2 + PM^2) - (QM^2 + MR^2)$
$= QR^2 - QM^2 - MR^2$ (since, $QR^2 = PQ^2 + PR^2$)
$= (QM + MR)^2 - QM^2 - MR^2$
$= 2QM \times MR$
$\therefore PM^2 = QM \times MR$............................. Hence Proved

Q.3 In figure, ABD is a triangle right angled at A and $AC \perp BD$. Show that
(i) $AB^2 = BC.BD$
(ii) $AC^2 = BC.DC$
(iii) $AD^2 = BD.CD$



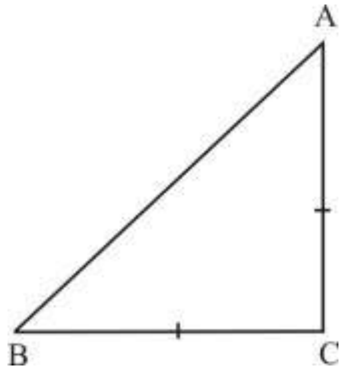
***Sol.* Given:** ABD is a triangle right angled at A, in which $AC \perp BD$.
To prove:
(i) $AB^2 = BC.BD$
(ii) $AC^2 = BC.DC$
(iii) $AD^2 = BD.CD$
Proof:
(i) Since $AC \perp BD$
So, both triangles ΔABC and ΔADC are right angle triangles
Therefore $\Delta ABC \sim \Delta ADC$ and also each triangle is similar to ΔABD.
Since, $\Delta ABC \sim \Delta ABD$
So, $AB/BD = BC/AB$
$\Rightarrow AB^2 = BC.BD$...............Hence Proved.

(ii) Since, $\Delta ABC \sim \Delta ADC$
So, $AC/BC = DC/AC$
$\Rightarrow AC^2 = BC.DC$ ....Hence Proved.

(iii) Since, $\Delta ACD \sim \Delta ABD$
$AD/CD = BD/AD$
$\Rightarrow AD^2 = BD.CD$...............Hence Proved.

Q.4 ABC is an isosceles triangle right angled at C. Prove that $AB^2 = 2AC^2$.

Sol.



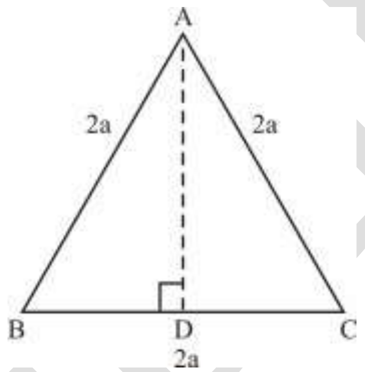
Since, ABC is an isosceles right triangle and right angled at C.
So, AC = BC
So, By the Pythagoras theorem:
$AB^2 = AC^2 + BC^2$
$\Rightarrow AB^2 = AC^2 + AC^2$ (Since, BC = AC)
$\Rightarrow AB^2 = 2AC^2$ ....Hence Proved.

Q.5 ABC is an isosceles triangle with AC = BC. If $AB^2 = 2AC^2$, prove that ABC is a right triangle.

Sol. Given: ABC is an isosceles triangle with AC = BC and $AB^2 = 2AC^2$
So, By the Pythagoras theorem:
$\Rightarrow \quad AB^2 = AC^2 + AC^2$
$\Rightarrow \quad AB^2 = AC^2 + BC^2$ (Since, AC = BC)
Thus, ΔABC is right angled at C.

Q.6 ABC is an equilateral triangle of side 2a. Find each of its altitudes.

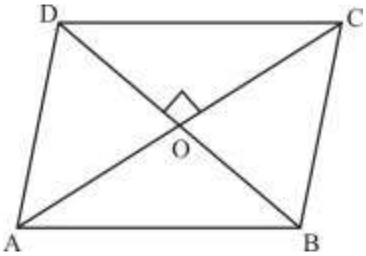
Sol.



Suppose ΔABC be an equilateral triangle of side 2a units.
Construction: We need to draw AD⊥BC and D is the mid-point of BC.
$\Rightarrow BD = \frac{1}{2} BC = \frac{1}{2} \times 2a = a$
Since AD⊥BC, so ΔABD is a right triangle, right angled at D.
So, By the Pythagoras theorem:
$AB^2 = AD^2 + BD^2$
$\Rightarrow (2a)^2 = AD^2 + (a)^2$
$\Rightarrow AD^2 = 4a^2 - a^2 = 3a^2$
So, its altitude, $AD = \sqrt{3}\, a$.

Q.7 Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

Sol. Let ABCD is a rhombus and its diagonals AC and BD intersect each other at O.



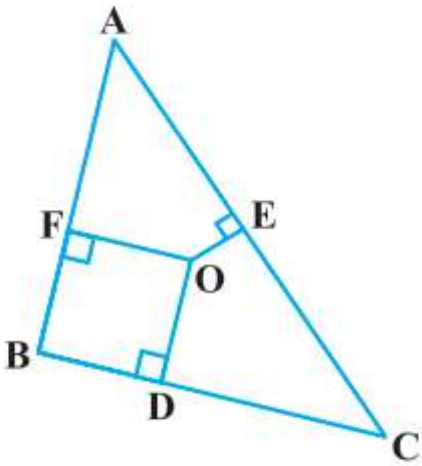
To Prove: $AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$
Proof:
Since, as we know that the diagonals of a rhombus bisect each other at right angles.
So, $\angle AOB = \angle BOC = \angle COD = \angle DOA = 90^\circ$.................(i)
and, AO = CO, BO = OD..................(ii)
Since, AOB is a right triangle, right angled at O.
So, By the Pythagoras theorem:

$AB^2 = OA^2 + OB^2$

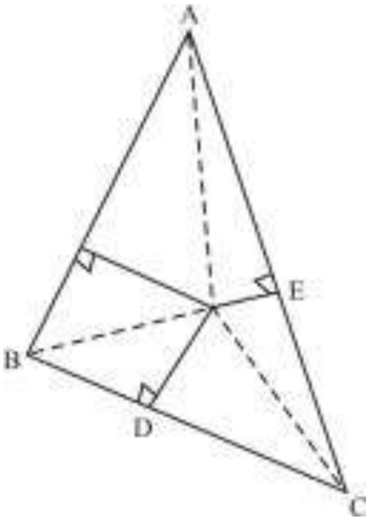
$\Rightarrow AB^2 = (\frac{1}{2}AC)^2 + (\frac{1}{2}BD)^2$ (from (ii))
$\Rightarrow 4AB^2 = AC^2 + BD^2$ .... (iii)
Similarly,
$4BC^2 = AC^2 + BD^2$ .... (iv)
$4CD^2 = AC^2 + BD^2$ .... (v)
and $4AD^2 = AC^2 + BD^2$ ....(vi)
By adding (iii), (iv), (v) and (vi),
$4(AB^2 + BC^2 + CD^2 + AD)^2 = 4(AC^2 + BD^2)$
$\Rightarrow AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$ ....Hence proved.

Q.8 In figure, O is a point in the interior of a triangle ABC, OD⊥BC, OE⊥AC and OF⊥AB. Show that

(i) $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$
(ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$



***Sol.* Given:** O is a point in the interior of a triangle ABC, OD⊥BC, OE⊥AC and OF⊥AB.
Construction: Join the points AO, BO and CO.



To Prove:

(i) $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$

Proof:

In right angled ΔOFA, ΔODB and ΔOEC,

So, By the Pythagoras theorem:

$OA^2 = AF^2 + OF^2$ ....(i)

$OB^2 = BD^2 + OD^2$..................(ii)

and, $OC^2 = CE^2 + OE^2$..................(iii)

Now adding all these equations,

$OA^2 + OB^2 + OC^2 = AF^2 + BD^2 + CE^2 + OF^2 + OD^2 + OE^2$

$\Rightarrow OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$........Hence Proved.

(ii) To Prove: $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$

Proof:

In right angled ΔODB and ΔODC,

So, By the Pythagoras theorem:

$OB^2 = OD^2 + BD^2$............(iv)

and, $OC^2 = OD^2 + CD^2$ ....(v)

Subtract (v) from (iv),

$\Rightarrow OB^2 - OC^2 = BD^2 - CD^2$ .... (vi)

Similarly,

$OC^2 - OA^2 = CE^2 - AE^2$ .... (vii)

and, $OA^2 - OB^2 = AF^2 - BF^2$ .... (viii)

Now, adding equations (vi), (vii) and (viii),

$(OB^2 - OC^2) + (OC^2 - OA^2) + (OA^2 - OB^2)$

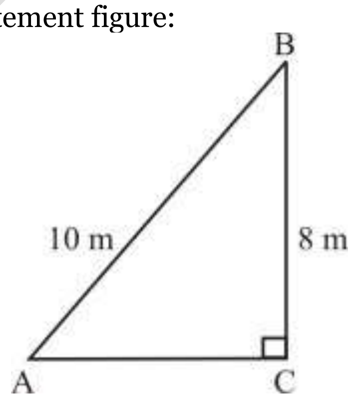
$= (BD^2 - CD^2) + (CE^2 - AE^2) + (AF^2 - BF^2)$

$\Rightarrow (BD^2 + CE^2 + AF^2) - (AE^2 + CD^2 + BF^2) = 0$

$\Rightarrow AF^2 + BD^2 + CE^2 = AE^2 + BF^2 + CD^2$...................Hence Proved.

Q.9 A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from base of the wall.

Sol. According to statement figure:



Let AB be the ladder of length 10m, B be the window 8 m above the ground (i.e. CB =8m). So, ABC is a right triangle, right angled at C.

So, By the Pythagoras theorem:

$AB^2 = AC^2 + BC^2$

$\Rightarrow \quad 10^2 = AC^2 + 8^2$

$\Rightarrow \quad AC^2 = 100 - 64$

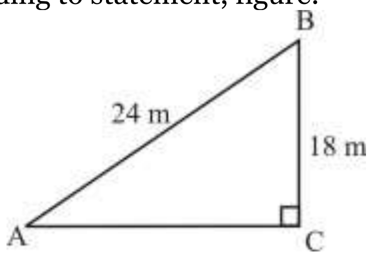
$\Rightarrow \quad AC^2 = 36$

$\Rightarrow \quad AC = 6$

Thus, the foot of the ladder is at 6 m distance from the base of the wall.

Q.10 A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

Sol. According to statement, figure:



Let AB be a guy wire of length 24cm attached to a vertical pole. BC =18 m be the height of pole. To keep the wire taut, let it be fixed to a stake at A. So, ABC is a triangle right angled at C.

So, By the Pythagoras theorem:

$AB^2 = AC^2 + BC^2$

$\Rightarrow \quad 24^2 = AC^2 + 18^2$

$\Rightarrow AC^2 = 576 - 324$

$\Rightarrow AC^2 = 252$

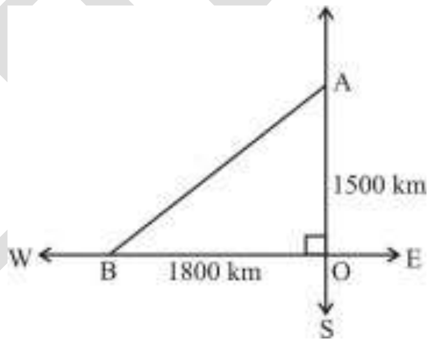
$\Rightarrow AC = \sqrt{252} = 6\sqrt{7}$

Thus, the stake should be placed at distance of $6\sqrt{7}$ m from the base of the pole.

Q.11 An aeroplane leaves an airport and flies due north at a speed of 1000 km per hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1200 km per hour. How far apart will be the two planes after 1½ hours?

Sol. Let the first aeroplane starts from origin O and flies due north at a speed of 1000 km/hr. So, distance covered in 3/2 hrs,

$$OA = (1000 \times \frac{3}{2})\text{ km} = 1500\text{ km.}$$



Let the second aeroplane starts from O at the same time and flies due west at a speed of 1200 km/hr. So, distance covered in 3/2 hrs,

$$OB = (1200 \times \frac{3}{2})\text{ km} = 1800\text{ km.}$$

According to question, required distance = BA.

In right angled ΔABC,

So, By the Pythagoras theorem:

$AB^2 = OA^2 + OB^2$

$= (1500)^2 + (1800)^2$

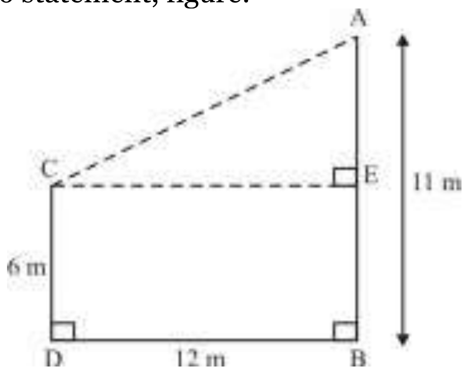
$= 2250000 + 3240000$

$= 5490000 = 9 \times 61 \times 100 \times 100$

$\Rightarrow \quad AB = 3 \times 100\sqrt{61} = 300\sqrt{61}$ = km.

Q.12 Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between the feet of the poles is 12 m, find the distance between their tops.

Sol. According to statement, figure:



Given: Pole of height, AB = 11 m and CD = 6 m. And distance between them, BD = 12 m.

Construction: Draw CE⊥AB and join AC,. This AC, we need to find out.

Therefore CE = DB = 12 m,

AE = AB – BE

= AB – CD

= (11–6) m = 5 m.

Now, in right angled ΔACE at right angled at E.

So, By the Pythagoras theorem:

$AC^2 = CE^2 + AE^2$

$= (12)^2 + (5)^2$

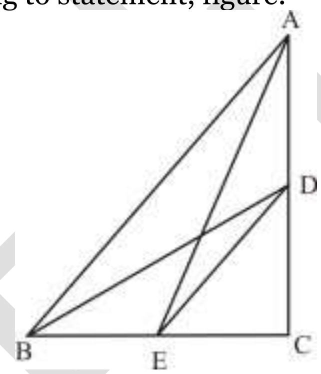
$= 144 + 25 = 169$

$\Rightarrow \quad AC = \sqrt{169} = 13$

Thus, the distance between the tops of the two poles = 13 m.

Q.13 D and E are points on the sides CA and CB respectively of a triangle ABC right angled at C. Prove that $AE^2 + BD^2 = AB^2 + DE^2$.

Sol. According to statement, figure:



Now, in right angled ΔACE,

$AE^2 = AC^2 + CE^2$ ....(i)

and in right angled ΔDCB,

$BD^2 = DC^2 + BC^2$ ....(ii)

By adding (i) and (ii)

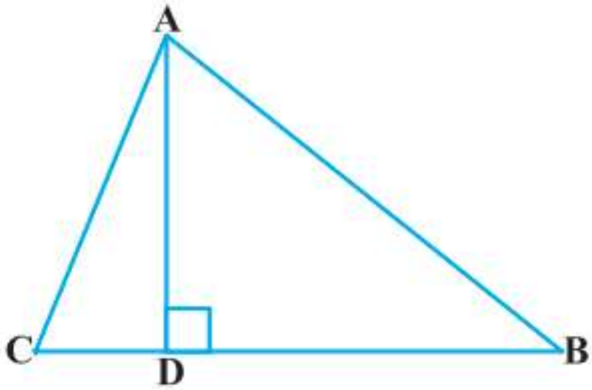
$\Rightarrow AE^2 + BD^2 = (AC^2 + CE^2) + (DC^2 + BC^2)$

$\Rightarrow AE^2 + BD^2 = (AC^2 + BC^2) + (DC^2 + CE^2)$

(From Pythagoras theorem, $AC^2 + BC^2 = AB^2$ and $DC^2 + CE^2 = DE^2$)

$\Rightarrow AE^2 + BD^2 = AB^2 + DE^2$ ....Hence Proved.

Q.14 The perpendicular from A on side BC of a ΔABC intersects BC at D such that DB = 3 CD (see figure). Prove that $2AB^2 = 2AC^2 + BC^2$.



Sol. Given: DB = 3CD
Since, BC = DB + CD
So,
⇒ BC = 3CD + CD (Since, BD = 3CD)
⇒ BC = 4CD

$CD = \frac{1}{4} BC$ ....(i)

DB = 3CD From (i)

$= \frac{3}{4} BC$ .... (ii)

Since, in ΔABD is a right triangle, right angled at D.
So, By the Pythagoras theorem:

$AB^2 = AD^2 + DB^2$ .... (iii)

Similarly, In right angled triangle ΔACD,

$AC^2 = AD^2 + CD^2$ .... (iv)

(iii) – (iv) ,
$AB^2 - AC^2 = DB^2 - CD^2$

⇒ $AB^2 - AC^2 = (\frac{3}{4} BC)^2 - (\frac{1}{4} BC)^2$ [Using (i) & (ii)]

⇒ $AB^2 - AC^2 = (1/2)BC^2$
⇒ $AB^2 - AC^2 = (½)BC^2$
⇒ $2AB^2 - 2AC^2 = BC^2$
⇒ $2AB^2 = 2AC^2 + BC^2$................Hence Proved.

Q.15 In an equilateral triangle ABC, D is a point on side BC such that BD= $\frac{1}{3}$ BC. Prove that $9AD^2 = 7AB^2$

Sol. Given: ΔABC is an equilateral triangle and D be a point on BC such that

$BD = \frac{1}{3} BC$

To prove: $9AD^2 = 7AB^2$
Construction: Draw AE⊥BC. Join the points A and D.
Proof:

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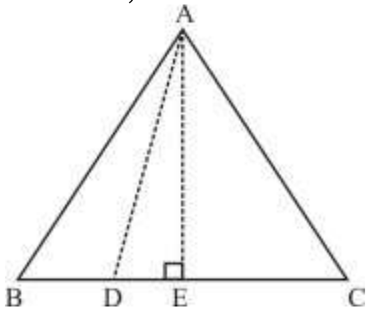
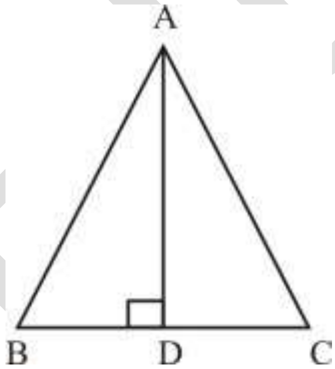
In ΔAEB and ΔAEC,



AB = AC (since, ΔABC is equilateral)
$\angle AEB = \angle AEC$ ($AE \perp BC$)
and, AE = AE (common)
So, from SAS-criterion of similarity,
$\Delta AEB \sim \Delta AEC$
$\Rightarrow$ BE = EC
Therefore,
$BD = \frac{1}{3}BC$, DC =BC and, $BE = EC = \frac{1}{2}BC$ .... (i)
Since, $\angle C = 60^o$
So, ΔADC is an acute triangle.
$AD^2 = AC^2 + DC^2 - 2\,DC \times EC$
$\Rightarrow AD^2 = AC^2 + (\frac{2}{3}BC)^2 - 2\times\frac{2}{3}BC \times \frac{1}{2}BC$ (From (i))
$\Rightarrow AD^2 = AC^2 + \frac{4}{9}BC^2 - \frac{2}{3}BC^2$
$\Rightarrow AD^2 = AB^2 + \frac{4}{9}AB^2 - \frac{2}{3}AB^2$ (Since AB = BC = AC]
$\Rightarrow AD^2 = (7/9)AB2$
$\Rightarrow$ 9AD2=7AB2, ...Hence Proved.

Q.16 In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.
Sol.



To prove: $3AB^2 = 4AD^2$
Proof:
Let ABC be an equilateral triangle and $AD \perp BC$.
In ΔADB and ΔADC,
AB = AC (Since equilateral triangle)
$\angle B = \angle C$ (Each = 60°)
and, $\angle ADB = \angle ADC$ (Each = 90°, since $AD \perp BC$)
So, from RHS criterion of congruence,
$\Delta ADB \cong \Delta ADC$
$\Rightarrow$ BD = DC
$\Rightarrow$ BD = DC = (1/2)BC
Since ΔADB is a right triangle, from Pythagoras theorem,

$AB^2 = AD^2 + BD^2$

$\Rightarrow AB^2 = AD^2 + (\frac{1}{2}BC)^2$

$\Rightarrow AB^2 = AD^2 + \frac{1}{4}BC^2$

$\Rightarrow AB^2 = AD^2 + \frac{1}{4}AB^2$ (Since, BC = AB)

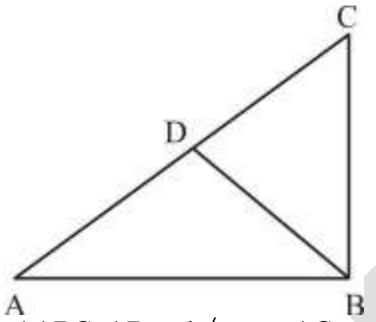
$\Rightarrow \frac{3}{4}AB^2 = AD^2$

$\Rightarrow 3AB^2 = 4AD^2$ ....Hence Proved.

Q.17 Tick the correct answer and justify: In ΔABC, AB=6√3 cm, AC = 12 cm and BC = 6 cm. The angles A and B are respectively :

(a) 90° and 30°
(b) 90° and 60°
(c) 30° and 90°
(d) 60° and 90°

Sol.



Given: In ΔABC, AB = 6√3 cm, AC = 12 cm and BC = 6 cm .... (i)
Now applying Pythagoras theorem:

$AB^2 + BC^2 = (6\sqrt{3})^2 + (6)^2$
$= 36 \times 3 + 36$
$= 108 + 36$
$= 144 = (AC)^2$

Therefore, ΔABC is a right triangle, right angled at B.
So, ∠B = 90°
Suppose point D is the mid-point of AC. And mid-point of the hypotenuse of a right triangle is equidistant from the vertices.
i.e. AD = BD = CD
⇒ CD = BD = 6 cm (Since CD =1/2AC)
Also BC = 6 cm
So, in ΔBDC,
BD = CD = BC
⇒ Therefore, ΔBDC is equilateral where ∠ACB = 60°
Now for ∠A = 180° – (∠B+∠C)
= 180° – (90° + 60°) = 30°
Therefore, ∠A=30° and ∠B=90°
So, correct option: (C)

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