Triangles: Exercise - 6.4

Q.1 Let ΔABC~ΔDEF and their areas be, respectively, 64 cm² and 121 cm². If EF = 15.4 cm, Find BC.

Sol. Given: Area (ΔABC)/Area (ΔDEF) $=BC^2/EF^2$

$\Rightarrow 64/121 = BC^2 / (15.4)^2$

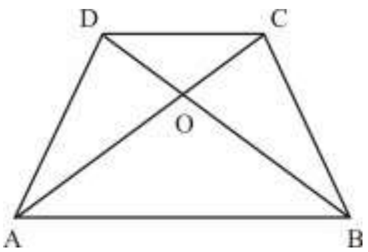
$\Rightarrow 8/11 = BC/15.4$ (By taking square root both sides.)

$\Rightarrow BC = (\frac{8}{11} \times 15.4)$ cm = 11.2 cm

Thus BC =11.2 cm.

Q.2 Diagonals of a trapezium ABCD with AB || DC intersect each other at the point O. If AB = 2 CD, find the ratio of the areas of triangles AOB and COD.

Sol.



Given: In a trapezium ABCD, AB ||DC and AB = 2CD

Now, In ΔAOB and ΔCOD,

∠AOB=∠COD (Vertically opposite angles)

and, ∠OAB=∠OCD (Alternate angles)

So, from AA-criterion of similarity,

ΔAOB~ΔCOD

Therefore

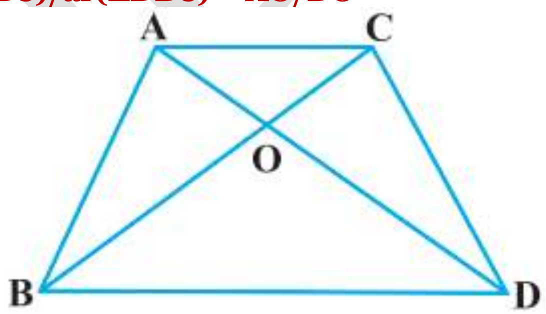
$\Rightarrow$ Area (ΔAOB)/Area (ΔCOD) = AB^2/DC^2

$\Rightarrow$ Area(ΔAOB)/Area(ΔCOD) = $(2DC)^2/DC^2$ (Since AB = 2CD)

= 4/1

Hence, area (ΔAOB) : area (ΔCOD) = 4 : 1

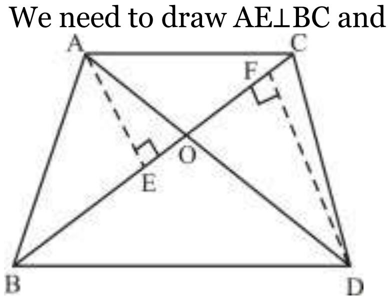
Q.3 In figure, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that ar(ΔABC)/ar(ΔDBC) = AO/DO



Sol. Given: Two triangles, ΔABC and ΔDBC which have same base BC

To prove: Area (ΔABC)/Area (ΔDBC) = AO/DO

Construction: We need to draw AE⊥BC and DF⊥BC in given figure.



Proof: In ΔAOE and ΔDOF, we have

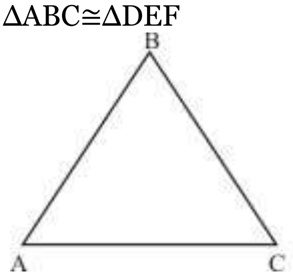
∠AEO = ∠DFO = 90° (Since, in construction of AE⊥BC and DF⊥BC)

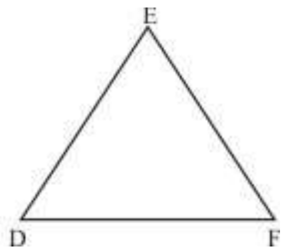
$\angle AOE = \angle DOF$ (Vertically opposite angles)
So, from AA-criterion of similarity,
$\Delta AOE \sim \Delta DOF$
Then,
$\Rightarrow$ $AE/DF = AO/OD$ .... (i)

Now, Area(ΔABC)/Area(ΔDBC) = $\dfrac{\frac{1}{2} x\, BC x AE}{\frac{1}{2} x\, BC x DF}$

$\Rightarrow$Since, $AE/DF = AO/OD$ from (i)
So, Area(ΔABC)/Area(ΔDBC) = AO/OD.................. Hence Proved.

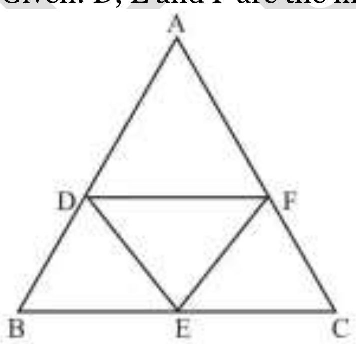
Q.4 If the areas of two similar triangles are equal, prove that they are congruent.
Sol. Given: Two triangles, ΔABC and ΔDEF and $\Delta ABC \sim \Delta DEF$
and their areas, Area (ΔABC) = Area (ΔDEF)
To prove: $\Delta ABC \cong \Delta DEF$





Proof : Since, $\Delta ABC \sim \Delta DEF$
Then,
$\Rightarrow \angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$
and, $AB/DE = BC/EF = AC/DF$
To prove $\Delta ABC \cong \Delta DEF$, we need to prove that
$AB = DE$, $BC = EF$ and $AC = DF$
Since, Area (ΔABC) = Area (ΔDEF)
$\Rightarrow$ Area(ΔABC)/Area(ΔDEF) = 1
$\Rightarrow AB^2/DE^2 = BC^2/EF^2 = AC^2/DF^2 = 1$
$\Rightarrow AB/DE = BC/EF = AC/DF = 1$
$\Rightarrow AB = DE$, $BC = EF$, $AC = DF$
So, from the SSS criterion of congruent,
$\Delta ABC \cong \Delta DEF$..................Hence proved.

Q.5 D, E and F are respectively the mid-points of sides AB, BC and CA of ΔABC. Find the ratio of the areas of ΔDEF and ΔABC.
Sol. Given: D, E and F are the mid-points of the sides AB, BC and CA of ΔABC respectively.



Since, D and E are the mid-points of the sides AB and BC.
So, $DE \parallel AC$
$\Rightarrow$ $DE \parallel FC$ .... (i)
Since, D and F are the mid-points of the sides AB and AC.
So, $DF \parallel BC$
$\Rightarrow$ $DF \parallel EC$ .... (ii)
From (i) and (ii),

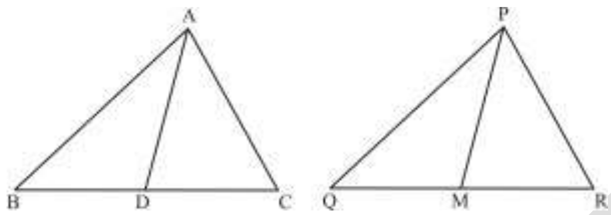
DECF is a parallelogram.

Similarly, ADEF is a parallelogram.
Now, in ΔDEF and ΔABC,
$\angle DEF = \angle A$ (Opposite angles of ||gm ADEF)
and, $\angle EDF = \angle C$ (Opposite angles of ||gm DECF)
So, from AA-criterion of similarity,
$\Delta DEF \sim \Delta ABC$
then
$\Rightarrow$ Area(ΔDEF)/Area(ΔABC) = DE^2/AC^2

$$= \frac{\frac{1}{2}AC^2}{AC^2} = 1/4$$

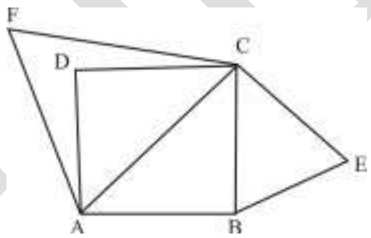
Thus, Area (ΔDEF) : Area (ΔABC) = 1 : 4.

Q.6 Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.
Sol.



Given: Two triangles, such that $\Delta ABC \sim \Delta PQR$, AD and PM are the medians of ΔABC and ΔPQR respectively.
To prove : Area(ΔABC)/Area(ΔPQR) = AD^2/PM^2
Proof : Since $\Delta ABC \sim \Delta PQR$ (given)
So, Area(ΔABC) / Area(ΔPQR) = $(AB)^2/(PQ)^2$ .... (i)
But, AB / PQ = $AD_{\text{med}} / PM_{\text{med}}$ .... (ii)
Therefore, from (i) and (ii),
Area(ΔABC) / Area(ΔPQR) = AD^2 / PM^2 ....Hence proved.

Q.7 Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of the diagonals.
Sol.



Given: A square ABCD. Equilateral ΔBEC which has been drawn on side and Equilateral ΔAFC, which has been drawn on one side of the diagonals AC

To prove : Area (ΔBCE) = $\frac{1}{2}$ (Area ΔACF)

Proof : Since, Both the triangles are equilateral triangles. So, being equilateral so similar by AAA criterion of similarity-
$\Delta BCE \sim \Delta ACF$
Then,
$\Rightarrow$ Area (ΔBCE) / Area(ΔACF) = BC^2 / AC^2
$\Rightarrow$ Area(ΔBCE) / Area(ΔACF) = $BC^2 / (\sqrt{2}BC)^2$ (Since, Diagonal = $\sqrt{2}$ side) $\Rightarrow AC = \sqrt{2}\,BC$
Thus,

$\Rightarrow$ Area (ΔBCE) / Area (ΔACF) = $\frac{1}{2}$

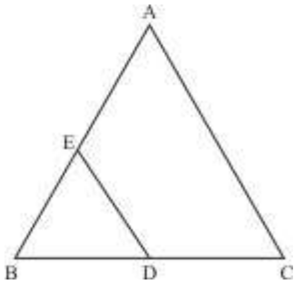
Tick the correct answer and justify:

Q.8 ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Ratio of the areas of triangles ABC and BDE is

(a) 2 : 1 **(b) 1 : 2**

(c) 4 : 1 **(d) 1 : 4**

Sol.



Since ΔABC and ΔBDE are equilateral. So, being equilateral so similar by AAA criterion of similarity-

ΔABC~ΔBDE ....(i)

⇒ Area(ΔABC) / Area(ΔBDE) = BC^2 / BD^2
⇒ Area(ΔABC) / Area(ΔBDE) = $(2BD)^2$ / BD^2 (Since, D is the mid-point of BC. So, BC = 2BD)
Thus
⇒ Area(ΔABC) / Area(ΔBDE) = 4/1
Thus correct Option: (C)

Q.9 Sides of two similar triangles are in the ratio 4 : 9. Areas of these triangles are in the ratio.

(A) 2 : 3 **(B) 4 : 9**

(C) 81 : 16 **(D) 16 : 81**

Sol. Since, as we know that the ratio of the areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides. So,
ratio of areas = $(4)^2 : (9)^2 = 16:81$
Thus correct Option: (D)