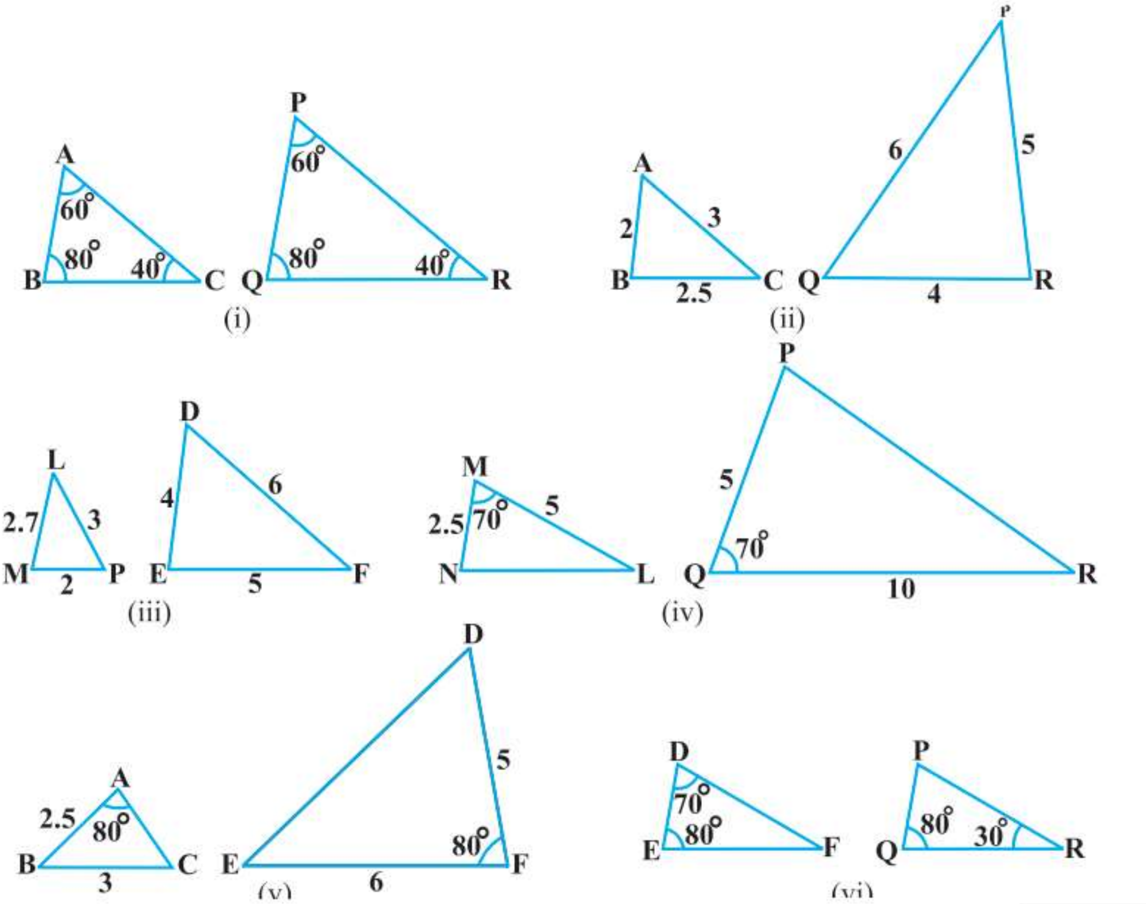
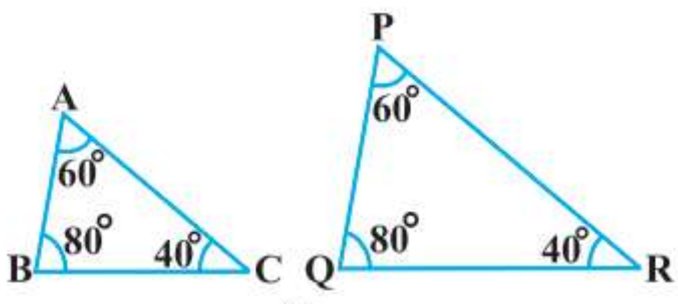
Triangles: Exercise - 6.3

Q.1 State which pairs of triangle in figure, are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form:



Sol.
(i)



(i)

In ΔABC and ΔPQR,

$\angle A=\angle P=60^\circ$,

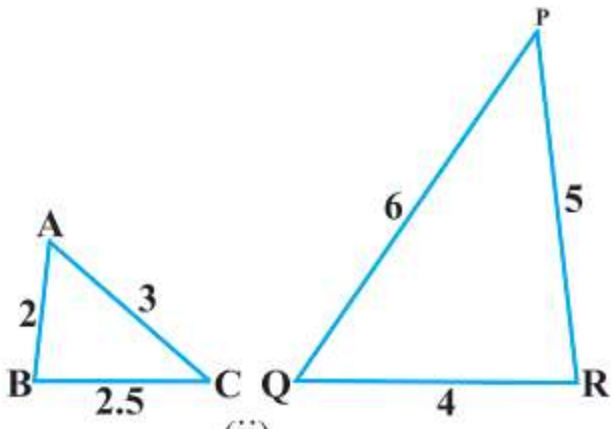
$\angle B=\angle Q=80^\circ$

and $\angle C=\angle R=40^\circ$

So, from AAA criterion of similarity,

$\Delta ABC \sim \Delta PQR$ ....Hence Proved.

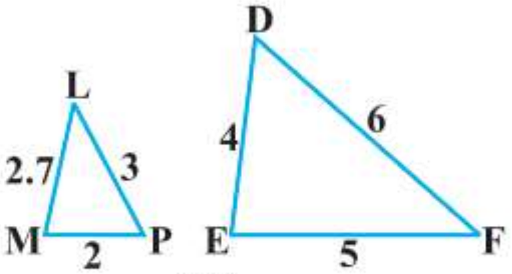
(ii)



(ii)

In ΔABC and ΔPQR,
AB/QR = BC/RP = CA/PQ = 1/2
SO, from SSS criterion of similarity,
ΔABC~ΔQRP...............Hence Proved.

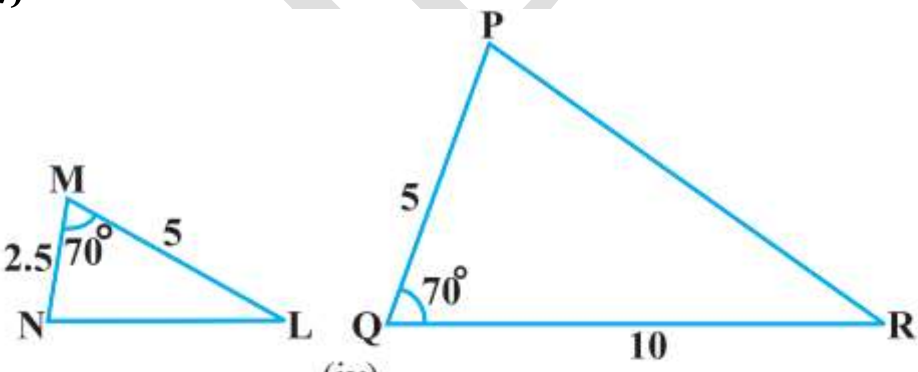
(iii)



(iii)

In ΔLMP and ΔDEF,
LM/DE =2.7/4 = 0.675
MP/EF = 2/5 = 0.4
LP/DF = 3/6 = 0.5
Since, LM/DE ≠ MP/EF ≠ LP/DF
So, the two triangles are not similar.

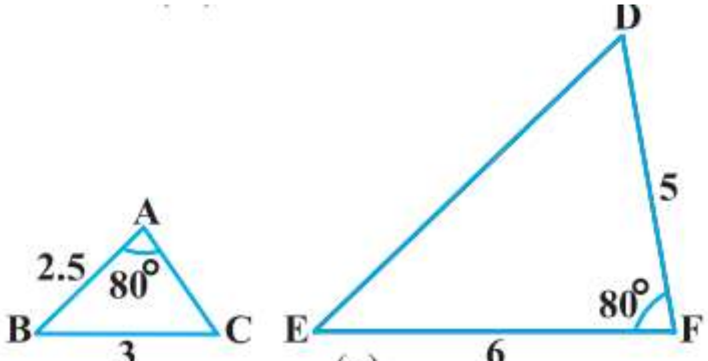
(iv)



(iv)

In ΔMNL and ΔQPR,
∠M=∠Q=70∘ (From Figure)
But, MN/PQ ≠ ML/QR
Since, both the triangles do not satisfy SAS criterion of similarity. So, these two triangles are not similar.

(v)



(v)

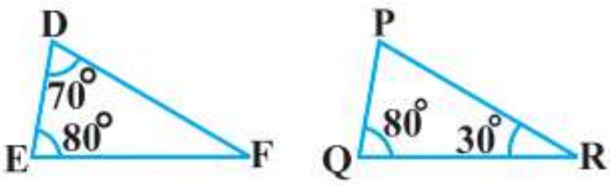
In ΔABC and ΔFDE,

$\angle A = \angle F = 80^\circ$

But, $AB/DF \neq AC/EF$ (Since AC is not given)

So, these two triangles are not similar.

(vi)



(vi)

In ΔDEF and ΔPQR,

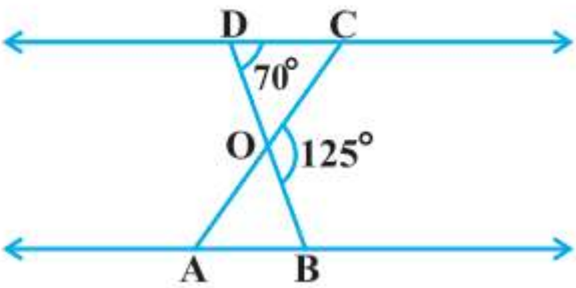
$\angle D = \angle P = 70^\circ$ (Since, $\angle P = 180^\circ - (80^\circ + 30^\circ) = 70^\circ$]

$\angle E = \angle Q = 80^\circ$

So, from AAA criterion of similarity, both triangles,

$\Delta DEF \sim \Delta PQR$

Q.2 In figures, $\Delta ODC \sim \Delta OBA$, $\angle BOC = 125^\circ$ and $\angle CDO = 70^\circ$. Find $\angle DOC$, $\angle DCO$ and $\angle OAB$.



Sol.

Since BD and OC are the lines which intersect each other at point O.

So, $\angle DOC + \angle BOC = 180^\circ$

$\Rightarrow \angle DOC + 125^\circ = 180^\circ$

$\Rightarrow \angle DOC = 180^\circ - 125^\circ = 55^\circ$

Now, in ΔCDO,

$\angle CDO + \angle DOC + \angle DCO = 180^\circ$ (Since, Sum of all the angles of a triangle are 180°)

$\Rightarrow 70^\circ + 55^\circ + \angle DCO = 180^\circ$

$\Rightarrow \angle DCO = 180^\circ - 125^\circ = 55^\circ$

Since, $\Delta ODC \sim \Delta OBA$

$\angle OBA = \angle ODC$,

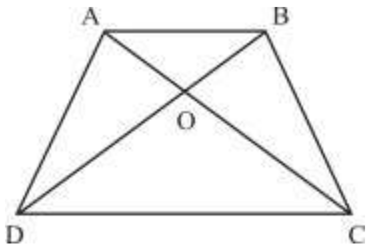
$\angle OAB = \angle OCD$

⇒ ∠OBA=70° and ∠OAB=55°
Therefore, ∠DOC=55°, ∠DCO=55° and ∠OAB=55°

Q.3 Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at the point O. Using a similarity criterion for two triangles, show that OA/OC = OB/OD.

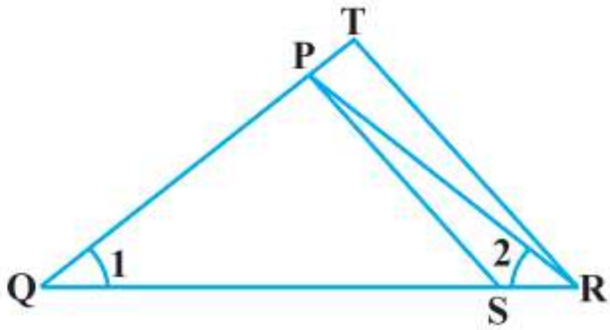
Sol.

Given: A trapezium ABCD, in which AB || DC.
To prove: OA/OC = OB/OD
Proof: In ΔOAB and ΔOCD,
∠AOB = ∠COD [Vert. Opposite angles]
∠OAB = ∠OCD [Alternate angles]
and ∠OBA = ∠ODC [Alternate angles]



So, from AAA criterion of similarity,
ΔOAB~ΔODC
Thus, OA/OC = OB/OD....................Hence proved.

Q.4 In figure, QR/QS = QT/PR and ∠1=∠2. Show that ΔPQS~ΔTQR

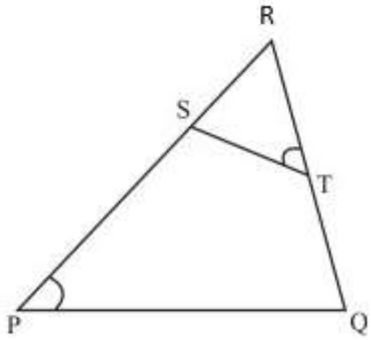


Sol.

Given: QR/QS = QT/PR
⇒ QT/QR = PR/QS .... (i)
Also, ∠1=∠2
⇒ PR = PQ .... (ii) (since sides opp. to equal angles are equal)
From (i) & (ii),
QT/QR = PQ/QS
⇒PQ/QT = QS/QR .... (iii)
Hence in ΔPQS and ΔTQR,
PQ/QT = QS/QR
and ∠PQS=∠TQR=∠Q
So, from SAS criterion of similarity,
ΔPQS~ΔTQR.................Hence proved.

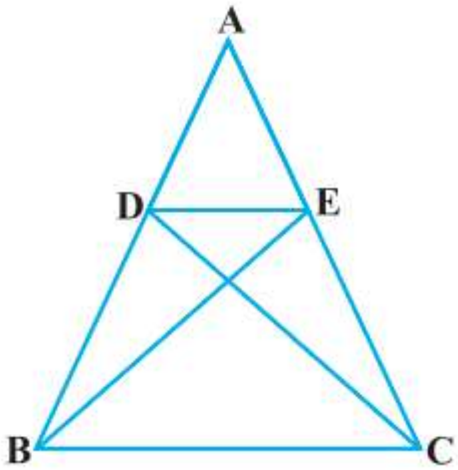
Q.5 S and T are points on sides PR and QR of ΔPQR such that ∠P = ∠RTS. Show that ΔRPQ~ΔRTS.

Sol.



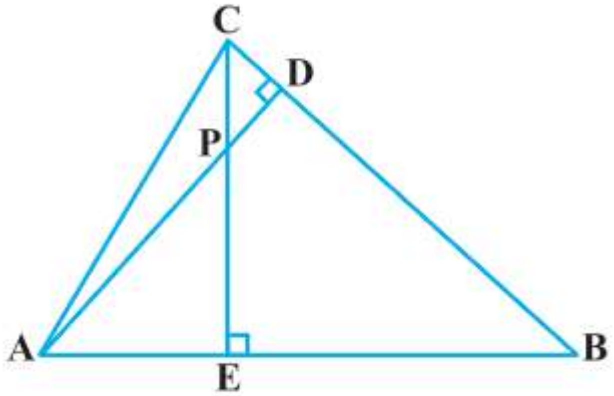
Given: ∠P = ∠RTS
Firstly, in Δs RPQ and RTS,
∠RPQ=∠RTS (given)
∠PRQ=∠TRS (Common angle)
So, from AA criterion of similarity,
ΔRPQ~ΔRTS................Hence Proved.

Q.6 In figure, if ΔABE≅ΔACD, show that ΔADE~ΔABC.



Sol.
Given: ΔABE≅ΔACD
So, AB = AC (since corresponding parts of congruent triangles are equal.)
and, AE = AD
⇒ AB/AD = AC/AE
⇒ AB/AC = AD/AE .... (i)
Now, In ΔADE and ΔABC,
AB/AC = AD/ AE from (i)
and, ∠BAC = ∠DAE (Common angle)
Hence, from SAS criterion of similarity,
ΔADE~ΔABC......................Hence Proved.

Q.7 In figure, altitudes AD and CE of ΔABC intersect each other at the point P, Show that:



(i) ΔAEP~ΔCDP
(ii) ΔABD~ΔCBE
(iii) ΔAEP~ΔADB
(iv) ΔPDC~ΔBEC

Sol. Given: Altitudes AD and CE of ΔABC

(i) In ΔAEP and ΔCDP,
∠AEP=∠CDP=90∘ (Since CE⊥AB & AD⊥BC)
∠APE = ∠CPD [Vertically opposite angles)
So, from AA - criterion of similarity,
ΔAEP~ΔCDP................Hence Proved.

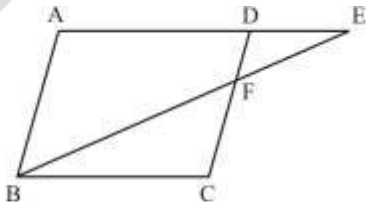
(ii) In Δs ABD and CBE,
∠ABD = ∠CBE (Common angle)
∠ADB = ∠CEB=90° (Since AD⊥BC and CE⊥AB)
So, from AA-criterion of similarity,
ΔABD~ΔCBE......................... Hence Proved.

(iii) In ΔAEP and ΔADB,
∠AEP =∠ADB=90∘ (Since AD⊥BC & CE⊥AB)
∠PAE = ∠DAB (Common angle)
So, from AA-criterion of similarity,
ΔAEP~ΔADB......................... Hence Proved.

(iv) In ΔPDC and ΔBEC,
∠PDC = ∠BEC =90∘ (Since AD⊥BC and CE⊥AB)
∠PCD=∠ECB (Common angle)
So, from AA-criterion of similarity,
ΔPDC~ΔBEC............... Hence Proved.

Q.8 E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that ΔABE~ΔCFB.

Sol.

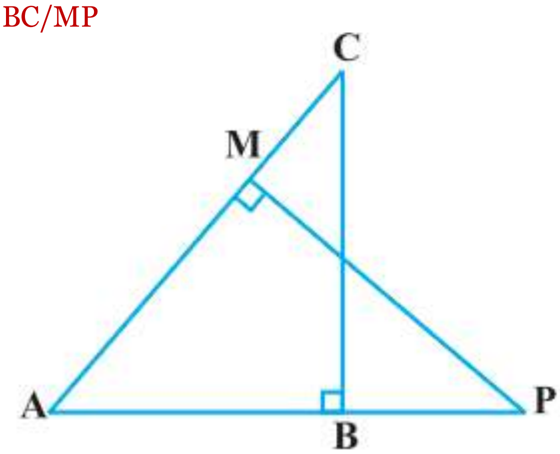


Given: a parallelogram ABCD and BE intersects CD at F.
To prove: ΔABE~ΔCFB
Proof: In ΔABE and ΔCFB,
∠AEB = ∠CBF (Alternate angles)
∠A=∠C (Opposite angles of a parallelogram)
So, from AA-criterion of similarity,
ΔADE~ΔCFB........... Hence Proved.

Q.9 In figure, ABC and AMP are two right triangles, right angled at B and M respectively. Prove that:

(i) ΔABC~ΔAMP

(ii) CA/PA = BC/MP



Sol. Given: ABC and AMP are two right triangles, right angled at B and M respectively.

(i) In ΔABC and ΔAMP,

∠ABC = ∠AMP = 90^o (Given)

and, ∠BAC = ∠MAP (Common angles)

So, from AA, criterion of similarity,

ΔABC~ΔAMP........................Hence Proved.

(ii) Since ΔABC~ΔAMP , (We have proved above.)

⇒ CA/PA = BC/ MP................ Hence Proved.

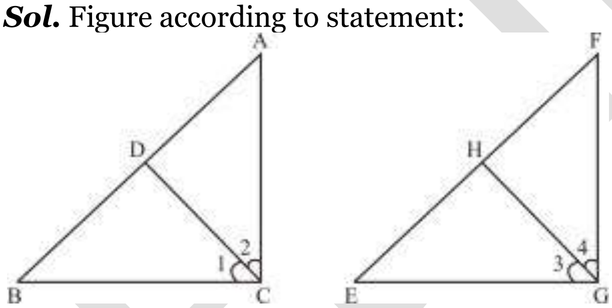
Q.10 CD and GH are respectively the bisectors of ∠ACB and ∠EGF such that D and H lie on sides AB and FE of ΔABC and ΔEFG respectively. If ΔABC~ΔFEG, show that:

(i) CD/GH = AC/FG

(ii) ΔDCB~ΔHGE

(iii) ΔDCA~ΔHGF

Sol. Figure according to statement:



Firstly we need to proof part (iii).

Given: ΔABC~ΔFEG

⇒ then, ∠A = ∠F .... (i)

and, $\frac{1}{2}$∠C = ∠G

⇒ $\frac{1}{2}$∠C = $\frac{1}{2}$∠G

⇒ So, ∠1=∠3 and ∠2=∠4 .... (ii)

Since, CD and GH are bisectors of ∠C and ∠G respectively.

So, in ΔDCA and ΔHGF,

∠A=∠F ....From (i)

∠2=∠4 ....From (ii)

Thus, from AA-criterion of similarity,

ΔDCA~ΔHGF...............Hence Proved (iii)

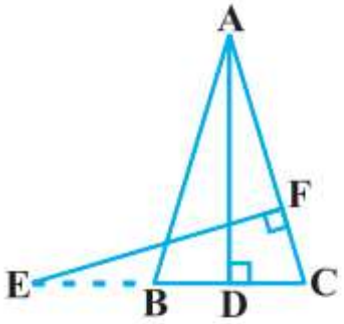
Now, as we have proved that ΔDCA ~ ΔHGF

⇒ AC/FG = CD/GH

⇒ CD/GH = AC/FG.................Hence Proved (i)

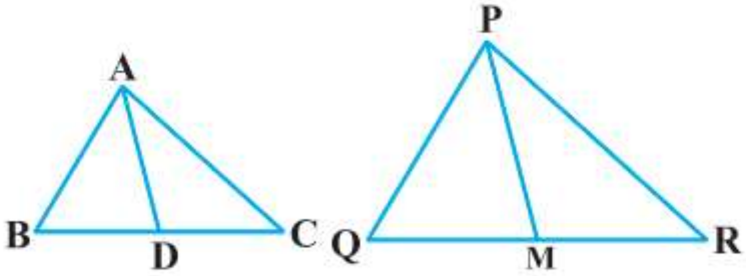
Now, in ΔDCB and ΔHGE,
∠1 =∠3 ....From (ii)
∠B =∠E ....(Since ΔABC ~ ΔFEG)
So, from AA-criterion of similarity,
ΔDCB ~ ΔHGE.............Hence Proved (ii).

Q.11 In figure, E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If AD⊥BC and EF⊥AC, prove that ΔABD~ΔECF.



Sol. Given: ΔABC is isosceles with AB = AC
So, ∠B = ∠C ....(i)
Now, in ΔABD and ΔECF,
∠ABD=∠ECF (from (i))
∠ADB = ∠EFC = 90° (Since AD⊥BC and EF⊥AC)
So, from AA-criterion of similarity,
ΔABD~ΔECF.................Hence Proved.

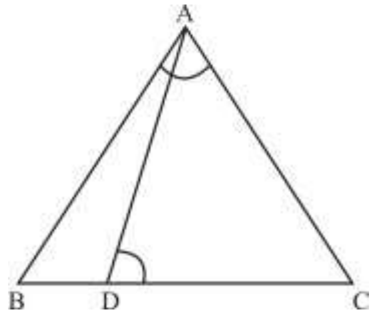
Q.12 Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of ΔPQR (see figure). Show that ΔABC~ΔPQR.



Sol.
Given: Here, AD is the median of Δ ABC and PM is the median of Δ PQR
AB/PQ = BC/QR = AD/PM............(i)
To prove : ΔABC~ΔPQR
Proof : BD = $\frac{1}{2}$ BC (given)
and, QM = $\frac{1}{2}$ QR (given)
Put value of BC and QR in (i)
⇒ AB/PQ = 2BD/2QM = AD/PM
⇒ AB/PQ = BD/QM = AD/PM
So, from SSS-criterion of similarity,
ΔABD~ΔPQM
⇒ ∠B =∠Q (Since, similar triangles have corresponding angles are equal.)
and, AB/PQ = BC/QR (given)
So, from SAS-criterion of similarity,
ΔABC~ΔPQR................Hence Proved

Q.13 D is a point on the side BC of a triangle ABC such that ∠ADC=∠BAC. Show that CA^2 = CB.CD.

Sol.



Given: ∠ADC=∠BAC

Now, in ΔABC and ΔDAC,

∠ADC = ∠BAC ....(given)

and ∠C = ∠C ....(Common angle)

So, from AA-criterion of similarity,

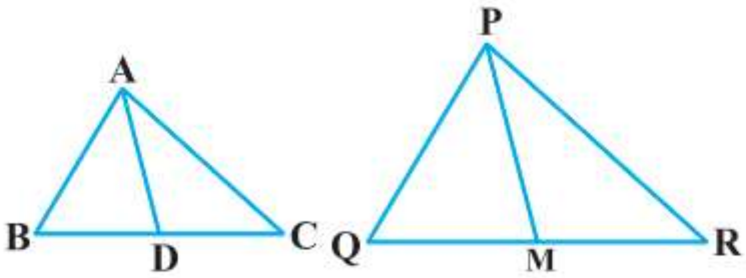
ΔABC~ΔDAC

⇒ AB/DA = BC/AC = AC/DC

⇒ CB/CA = CA/CD

⇒ CA^2 = CB × CD....................Hence Proved.

Q.14 Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that ΔABC~ΔPQR.



Sol.

Given: Here, AD is the median of Δ ABC and PM is the median of Δ PQR

AB/PQ = BC/QR = AD/PM............(i)

To prove : ΔABC~ΔPQR

Proof : BD = $\frac{1}{2}$ BC (given)

and, QM = $\frac{1}{2}$ QR (given)

Put value of BC and QR in (i)

⇒ AB/PQ = 2BD/2QM = AD/PM

⇒ AB/PQ = BD/QM = AD/PM

So, from SSS-criterion of similarity,

ΔABD~ΔPQM

⇒ ∠B =∠Q (Since, similar triangles have corresponding angles are equal.)

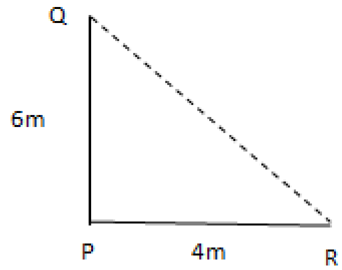
and, AB/PQ = BC/QR (given)

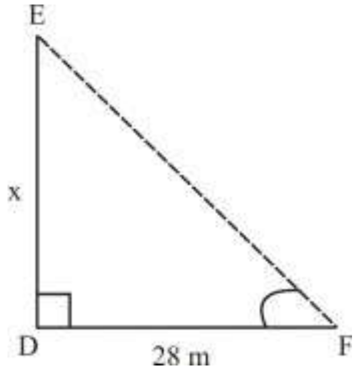
So, from SAS-criterion of similarity,

ΔABC~ΔPQR................Hence Proved

Q.15 A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Sol. Let PQ be the vertical pole and PR be the shadow on the ground. Also, let DE be the vertical tower and DF be its shadow on the ground. Join BC and EF in the figure. Let DE = x metres.





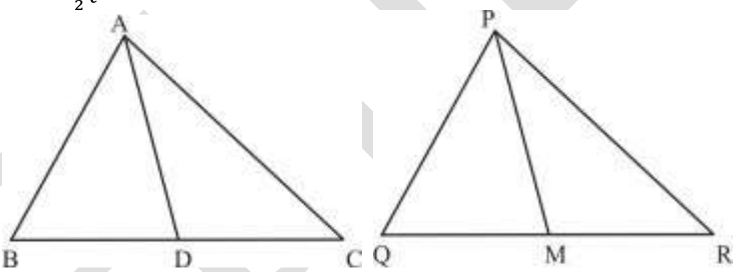
Given: PQ = 6 m, PR = 4 cm and DF = 28 m
Now, in ΔPQR and ΔDEF,

∠P = ∠D = 90°,
and ∠R =∠F (Since each is the angular elevation of the sun)
So, from AA-criterion of similarity,
ΔABC ~ ΔDEF
Then,
⇒ AB/DE = AC/DF
⇒ 6x =4/28
⇒ 6x = 1/7
⇒ x = 6×7 = 42
Thus, the height of the tower = 42 metres.

Q.16 If AD and PM are medians of triangles ABC and PQR respectively, where ΔABC~ΔPQR, prove that AB/PQ = AD/PM

Sol.
Given: AD and PM are the medians of ΔABC and ΔPQR respectively and ΔABC~ΔPQR.
To prove: AB/PQ = AD/PM
Proof: In ΔABD and ΔPQM,
∠B =∠Q (Since, ΔABC~ΔPQR)

AB/PQ = $\frac{\frac{1}{2}BC}{\frac{1}{2}QR}$



⇒ AB/PQ = BD/QM
So, from SAS - criterion of similarity

ΔABD~ΔPQM
⇒ AB/PQ = BD/QM = AD/PM
⇒ AB/PQ = AD/PM...................... Hence Proved.