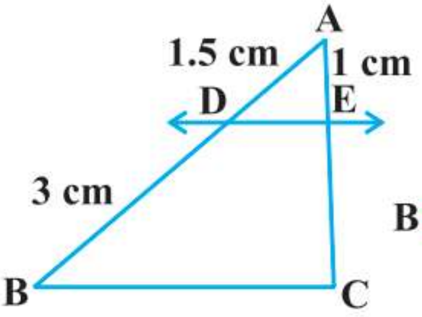
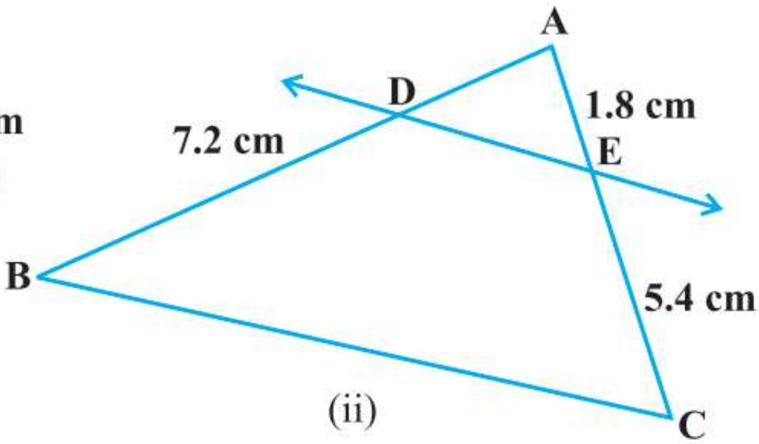
Triangles: Exercise - 6.2

Q.1 In figure, (i) and (ii), DE || BC. Find EC in (i) and AD in (ii).



(i)



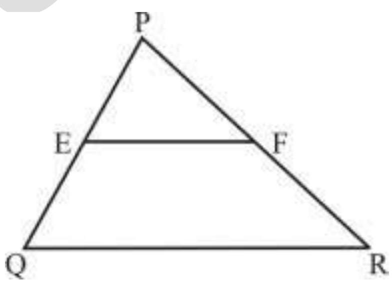
(ii)

Sol.
(a) From fig. (i),
since, DE || BC,
Then, AD/DB = AE/EC
⇒ 1.5/3 = 1/EC
⇒ EC = 3/1.5
= 30/15= 2 cm.

(b)From fig. (ii),
since, DE || BC,
AD/DB = AE/EC
⇒AD/7.2 = 1.8/5.4
$\Rightarrow AD = \dfrac{18x72}{54x10} = 2.4$ cm.

Q.2 E and F are points on the sides PQ and PB respectively of a Δ PQR. For each of the following cases, state whether EF || QR:
(i) PE = 3.9 cm, EQ = 4 cm, PF = 3.6 cm and FR = 2.4 cm.
(ii) PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm
(iii) PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.36 cm

Sol.



(i) Given: PE = 3.9 cm,
EQ = 4 cm,
PF = 3.6 cm
and FR = 2.4 cm

Now, PE/EQ = 3.9/4 = 0.97 cm
and, PF/FR = 3.6/2.4 = 3/2=1.5 cm
⇒ Since, PE/EQ ≠ PF/FR
Therefore, EF is not parallel to QR.

(ii) Given: PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm

Now, PE/EQ = 4/4.5 = 40/45 = 8/9

and, PF/FR = 8/9

So, from above,

⇒ PE/EQ = PF/EQ

Thus, EF divides sides PQ and PR of ΔPQR in the same ratio.

Therefore, from the converse of Basic Proportionality Theorem, EF || QR

(iii) Given: PQ = 1.28 cm, PR = 2.56 cm

PE = 0.18 cm and, PF = 0.36 cm

EQ = PQ – PE

= (1.28 – 0.18) cm.

= 1.10 cm

and, ER = PR – PF

= (2.56 – 0.36)

= 2.20 cm

Now, PE/EQ = 0.18/1.10 = 18/110 = 9/55

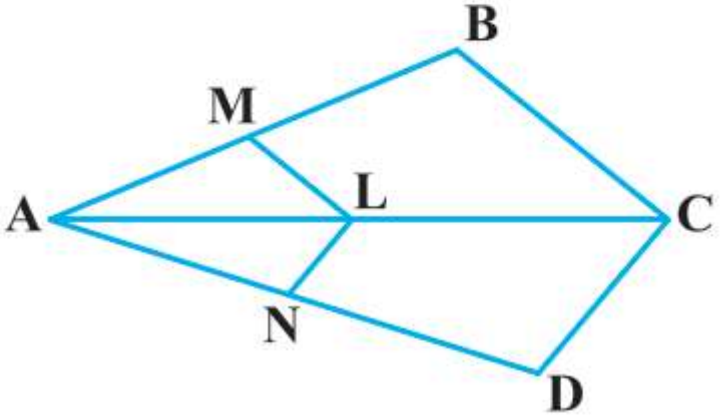
and, PF/FR = 0.36/2.20 = 36/220 = 9/55

⇒ PE/EQ = PF/FR

Therefore, EF divides sides PQ and PR of ΔPQR in the same ratio.

Thus, from the converse of Basic Proportionality Theorem, EF || QR

Q.3 In figure, if LM || CB and LN || CD, prove that AM/AB = AN/AD.



Sol. Given: LM || CB and LN || CD

To prove: AM/AB = AN/AD

Proof:

Firstly, in ΔABC:

LM || CB (given)

So, from Proportionality Theorem,

AM/AB = AL/AC.............(i)

Now, in ΔACD:

LN || CD (given)

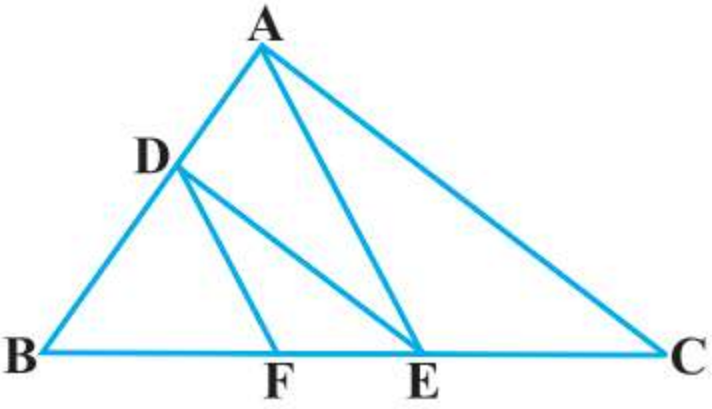
So, from Basic Proportionality Theorem,

AL/AC = AN/ AD .... (ii)

From (i) and (ii),

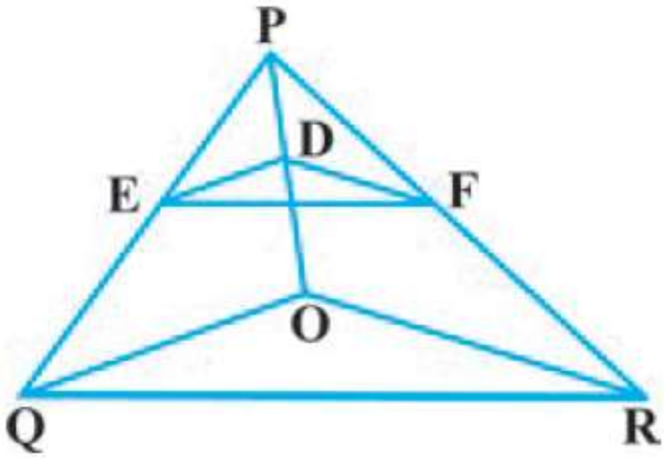
AM/AB = AN/AD   Hence proved.

Q.4 In figure, DE || AC and DF || AE. Prove that BF/FE = BE/EC



Sol. Given: DE || AC and DF || AE
To prove: BF/FE = BE/EC
Proof:
Firstly, in ΔBCA,
DE || AC (given)
So, from Basic Proportionality Theorem
BE/EC = BD/DA .... (i)
Now, in Δ BEA,
DF || AE (given)
So, from Basic Proportionality Theorem
BF/FE = BD/DA .... (ii)
From (i) and (ii),
BF/FE = BE/EC Hence Proved.

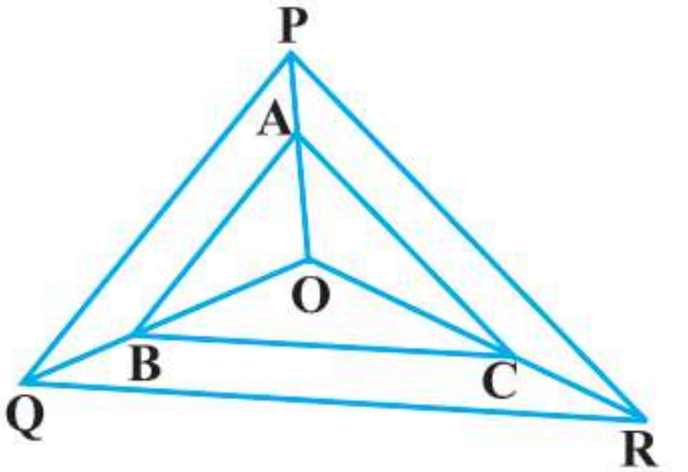
Q.5 In figure, DE || OQ and DF || OR. Show that EF || QR.



Sol. Given: DE || OQ and DF || OR
To prove: EF || QR
Proof:
Firstly, in ΔPQO,
DE || OQ (given)
So, from Basic Proportionality Theorem,
PE/EQ = PD/DO .... (i)
Now, in ΔPOR, we have
DF || OR (given)
So, from Basic Proportionality Theorem,
PD/DO = PF/FR .... (ii)
From (i) and (ii)

PE/EQ = PF/FR
⇒ Thus, EF||QR (From the converse of Basic Proportionality Theorem)

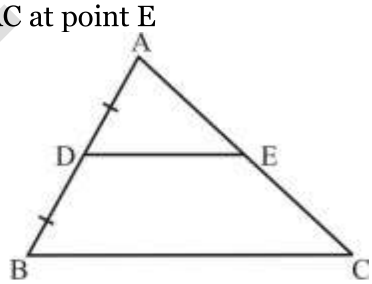
Q.6 In figure, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR.



Sol. Given: AB || PQ and AC || PR
To prove: BC || QR
Proof:
Firstly, in ΔOPQ,
AB || PQ (given)
So, from Basic Proportionality Theorem,
OA/AP = OB/BQ .... (i)
Now, In Δ OPR,
AC || PR (given)
So, from Basic Proportionality Theorem,
OA/AP = OC/CR .... (ii)
From (i) and (ii),
OB/BQ = OC/CR.............(iii)
Thus, From the (iii), B and C are points dividing the sides OQ and OR in the same ratio. So, from the converse of Basic Proportionality Theorem,
BC || QR...........Hence Proved

Q.7 Using Theorem 6.1, prove that a line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX).

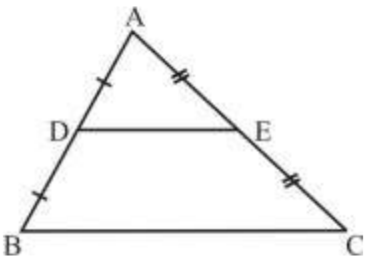
Sol. Given: In a ΔABC, D is the mid-point of side AB and the line DE is drawn through point D which is parallel to BC, meeting AC at point E



To Prove: AE = EC
Proof: In ΔABC
DE || BC (given)
So, from Basic Proportionality Theorem,
AD/DB = AE/EC .... (i)
But, AD = DB (Given: D is the mid-point of AB)

⇒ So, AD/DB = 1
Now, from (i), AE/EC = 1
⇒ AE = EC................(ii)
Thus, From (ii), E is the mid-point of the third side AC.

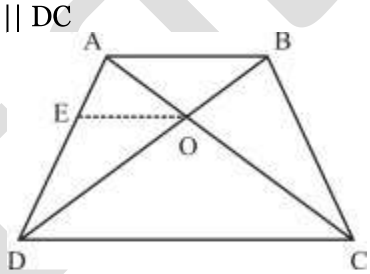
Q.8 Using Theorem 6.2, prove that the line joining the mid-points of any two sides of a triangle is, parallel to the third side. (Recall that you have done it in Class IX).
Sol.
Given: In a ΔABC, D and E are the mid-points of sides AB and AC respectively.



To prove: DE || BC
Proof: In ΔABC
Since, AD = DB and AE = EC (given)

So, AD/DB = 1 ....(i)
AE/EC = 1..................(ii)
From (i) & (ii), AD/DB = AE/EC = 1
i.e. AD/DB = AE/EC
Therefore, in ΔABC, D and E are points dividing the sides AB and AC in the same ratio. So, from the converse of Basic Proportionality Theorem,
DE || BC

Q.9 ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that AO/BO = CO/DO
Sol.
Given: In a trapezium ABCD, AB || DC and its diagonals AC and BD intersect each other at point O.
To prove: AO/BO = CO/DO
Construction: Through point O, draw a line OE which is parallel to AB i.e.
OE || DC



Proof:
Firstly, In ΔADC, OE || DC (from construction)
So, from Basic Proportionality Theorem,
AE/ED = AO/CO .... (i)
Now, in ΔABD,
OE || AB (from construction)
So, from Basic Proportionality Theorem,
ED/AE = DO/BO
⇒AE/ED = BO/DO .... (ii)
From (i) and (ii),
AO/CO = BO/DO
⇒AO/BO = CO/DO Hence Proved.

Q.10 The diagonals of a quadrilateral ABCD intersect each other at the point O such that AO/BO = CO/DO. Show that ABCD is a trapezium.

Sol.

Given: A quadrilateral ABCD, diagonals AC and BD intersect each other at point O such that

AO/BO = CO/DO

i.e. AO/CO = BO/DO.................(i)

To prove : Quadrilateral ABCD is a trapezium.

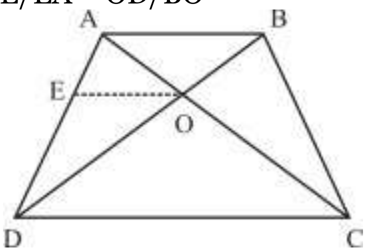
Construction: Through point O draw line OE which is parallel to AB meeting AD at E point. i.e. OE || AB ....(ii)

Proof: In ΔADB,

OE || AB (From construction)

Therefore by Basic Proportionality Theorem,

DE/EA = OD/BO



⇒EA/DE = BO/DO

⇒EA/DE = BO/DO = AO/CO (From (i))

⇒EA/DE = AO/CO

Therefore, in ΔADC, E and O are points that divide the sides AD and AC in the same ratio. So, from the converse of Basic Proportionality Theorem, we have

EO || DC

But, EO || AB (From (ii))

Hence, AB || DC.......................(iii)

Thus, from (iii), Quadrilateral ABCD is a trapezium.