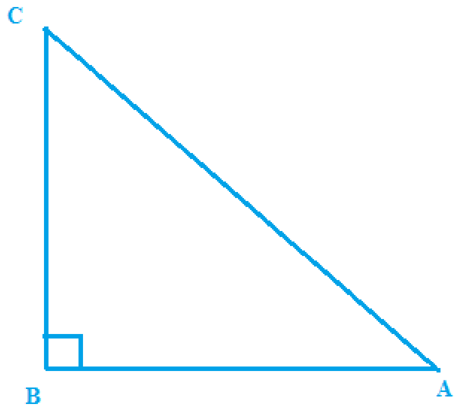
Triangles: Exercise 7.4

Q.1 Show that in a right angled triangle, the hypotenuse is the longest side.

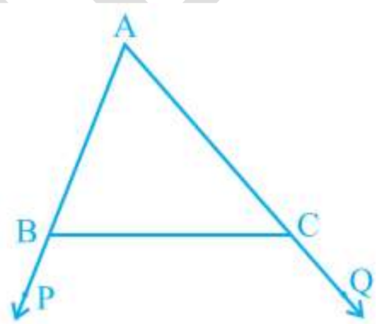
Sol. Suppose, ABC is a right angled triangle in which ∠ABC = 90°

Since, ∠ABC + ∠BCA + ∠CAB = 180° (From angle-sum property of triangle)



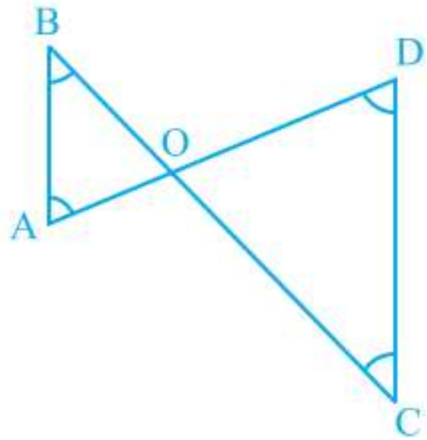
⇒ 90° + ∠BCA + ∠CAB = 180°
⇒ ∠BCA + ∠CAB = 90°
⇒ So, ∠BCA and ∠CAB are acute angles.
⇒ i.e. ∠BCA < 90° and ∠CAB < 90°
⇒ Also ∠BCA < ∠ABC and ∠CAB < ∠ABC
⇒ AC > AB and AC > BC (Because side opposite to greater angle is larger.)
Thus, in a right angled triangle, the hypotenuse is the longest side of the triangle.
Hence proved.

Q.2 In figure, sides AB and AC of Δ ABC are extended to points P and Q respectively. Also ∠PBC < ∠QCB. Show that AC > AB.



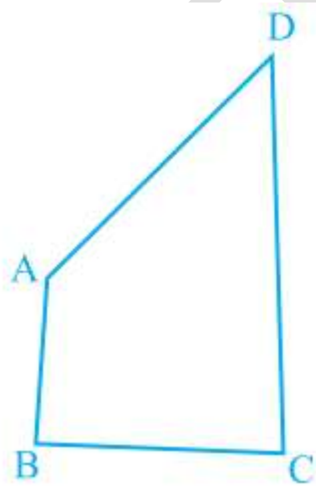
Sol. Given: ∠PBC < ∠QCB
By multiplying by '–'both sides,
⇒ –∠PBC > –∠QCB
By adding 180° on both sides,
⇒ 180° – ∠PBC > 180° – ∠QCB
Here, ∠PBC and ∠ABC as well as, ∠QCB and ∠ACB are linear pair angles.
⇒ ∠ABC > ∠ACB
⇒ Thus, AC > AB (Since, side opposite to greater angle is larger.)
Hence Proved.

Q.3 In figure, ∠B < ∠A and ∠C < ∠D. Show that AD < BC.

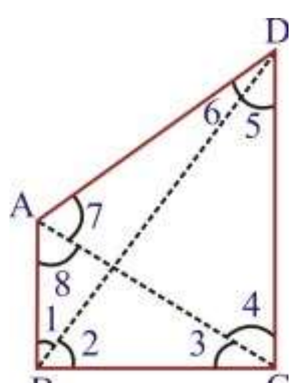


Sol. Given: ∠B < ∠A and ∠C < ∠D
So, AO < BO.................. (i)
and OD < OC.......................(ii) (Since, side opposite to greater angle is larger.)
By adding (i) and (ii),
AO + OD < BO + OC
⇒ AD < BC.......Hence Proved.

Q.4 AB and CD are respectively the smallest and longest sides of a quadrilateral ABCD (see figure). Show that ∠A > ∠C and ∠B > ∠D



***Sol.* Given:** AB and CD are respectively the smallest and longest sides of a quadrilateral ABCD.
Now, join AC and BD.



Since, AB is the smallest side of quadrilateral ABCD.
So, in Δ ABC,

Since, BC > AB

⇒So, $\angle 8 > \angle 3$ … (i) (Since, angle opposite to longer side is greater.)

Here, CD is the longest side of quadrilateral ABCD.

Now, in ΔACD,

CD > AD

⇒ So, $\angle 7 > \angle 4$ … (ii) (Since, angle opposite to longer side is greater.)

By adding (i) and (ii),

$\angle 8 + \angle 7 > \angle 3 + \angle 4$

⇒ $\angle A > \angle C$

Now again, in ΔABD,

AD > AB (Because, AB is the shortest side)

⇒ $\angle 1 > \angle 6$ … (iii)

And in ΔBCD,

CD > BC [Since CD is the longest side]

⇒ $\angle 2 > \angle 5$ … (iv)

By adding (iii) and (iv),

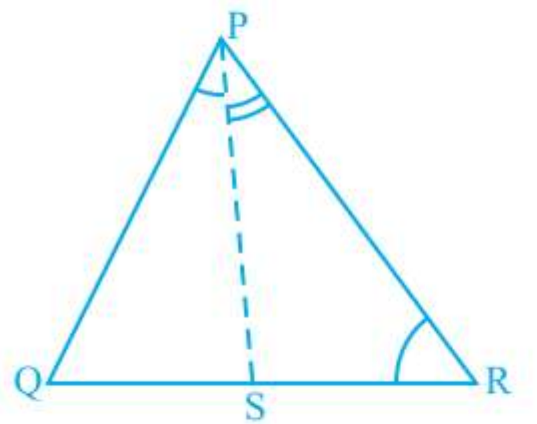
$\angle 1 + \angle 2 > \angle 5 + \angle 6$

⇒ $\angle B > \angle D$

Therefore, $\angle A > \angle C$ and $\angle B > \angle D$

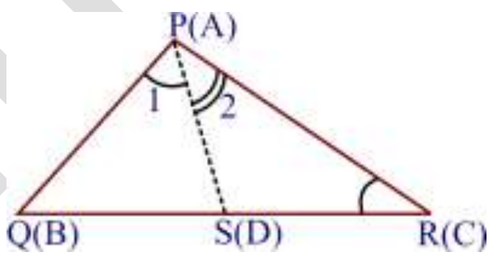
Hence Proved.

Q.5 In figure PR > PQ and PS bisects ∠QPR. Prove that ∠PSR > ∠PSQ.



Sol. Given: In Δ PQR, PR > PQ

⇒ So, $\angle PQR > \angle PRQ$ (Since, angle opposite to larger side is greater.)



By adding $\angle 1$ on both sides,

⇒ $\angle PQR + \angle 1 > \angle PRQ + \angle 1$

Since, PS is the bisector of $\angle P$. So, $\angle 1 = \angle 2$

⇒ $\angle PQR + \angle 1 > \angle PRQ + \angle 2$

Now, in ΔPQS and ΔPSR,

$\angle PQS + \angle 1 + \angle PSQ = 180^0$ and $\angle PRS + \angle 2 + \angle PSR = 180^0$

⇒ $\angle PQS + \angle 1 = 180^o - \angle PSQ$ and $\angle PRS + \angle 2 = 180^o - \angle PSR$

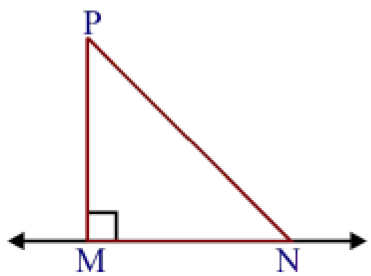
So, $180^0 - \angle PSQ > 180^o - \angle PSR$

⇒ $-\angle PSQ > -\angle PSR$

By adding '-'in both sides,
⇒ ∠PSQ < ∠PSR
i.e. ∠PSR > ∠PSQ..............Hence Proved.

Q.6 Show that of all line segments drawn from a given point not on it, the perpendicular line segment is the shortest.

Sol. Suppose, P is any point not on the straight line *l*. PM is perpendicular on line *l* and N is any point on line *l* other than M.



Now, In ΔPMN,
Since, ∠M = 90°
⇒ ∠MPN + ∠PNM = 90°
⇒ ∠P + ∠N = 90°
⇒ ∠N < 90°
⇒ ∠N < ∠M
⇒ PM < PN (Since, side opposite to greater angle is larger.)
Thus, PM is the shortest of all line segments from point P to line *l*.