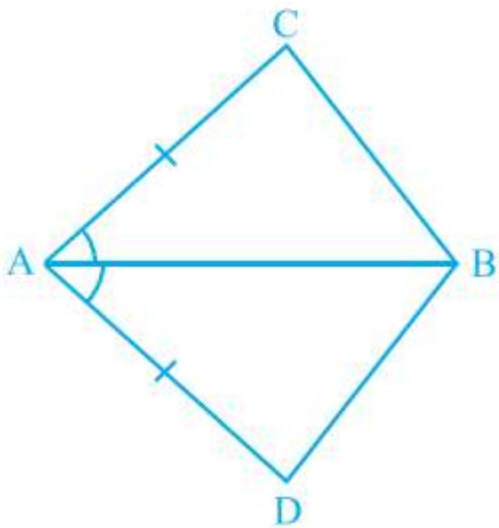
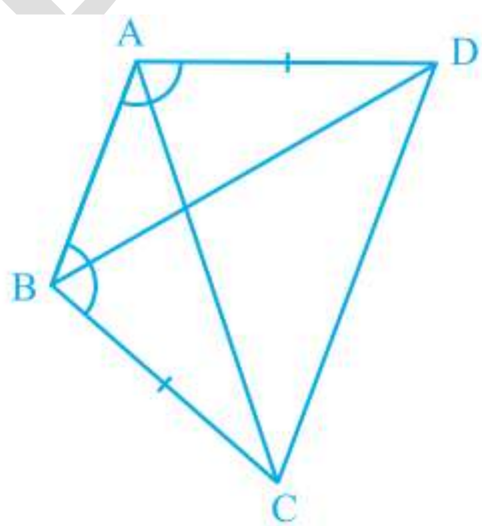
Triangles: Exercise 7.1

Q.1 In quadrilateral ACBD, AC = AD and AB bisects ∠A (see figure). Show that ΔABC ≅ ΔABD. What can you say about BC and BD?



Sol. In triangles ΔABC and ΔABD,
AC = AD (Given)
∠CAB =∠BAD (Since, AB bisects ∠A)
and AB = AB (Common side)
So, from SAS congruence criterion,
ΔABC ≅ ΔABD
⇒ BC = BD (Corresponding parts of congruent triangles)

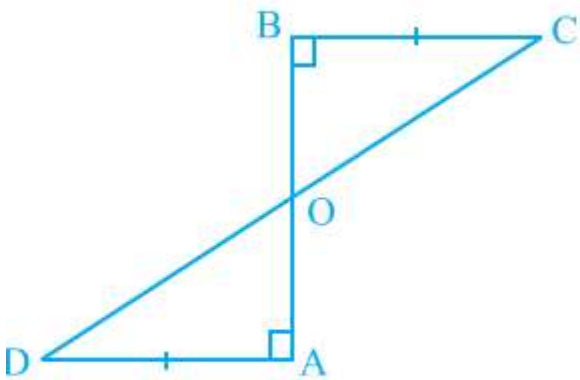
Q.2 ABCD is a quadrilateral in which AD = BC and ∠DAB = ∠CBA (see figure). Prove that
(i) ΔABD ≅ ΔBAC
(ii) BD = AC
(iii) ∠ABD = ∠BAC



Sol. In triangles ΔABD and ΔBAC,
AD = BC (Given)
∠DAB = ∠CBA (Given)
AB = AB (Common side)

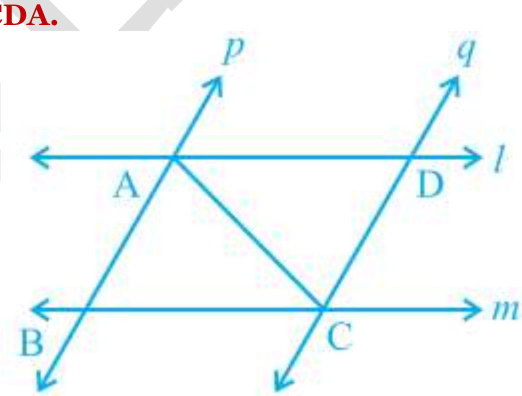
So, from SAS criterion of congruence,
(i) ΔABD≅ΔBAC………… Hence Proved.
(ii)⇒ BD = AC (Since, corresponding parts of congruent triangles)
(iii) and, ∠ABD=∠BAC, (Since, corresponding parts of congruent triangles)

Q.3 AD and BC are equal perpendiculars to a line segment AB (see figure). Show that CD bisects AB.



Sol. Since, AB and CD intersect each other at O. So,
∠AOD = ∠BOC …. (i) (Vertically opposite angles)
Now, In Triangles ΔAOD and ΔBOC,
∠AOD = ∠BOC (From eq. (i))
Since, AD and BC are equal perpendiculars to a line segment AB
∠DAO = ∠OBC (Each = 90^o)
and , AD = BC (Given)
So, from AAS congruence criterion,
ΔAOD ≅ ΔBOC
⇒ OA = OB
(Since, corresponding parts of congruent triangles)
Therefore. O is the mid- point of AB.
Thus, CD bisects AB……… Hence Proved.

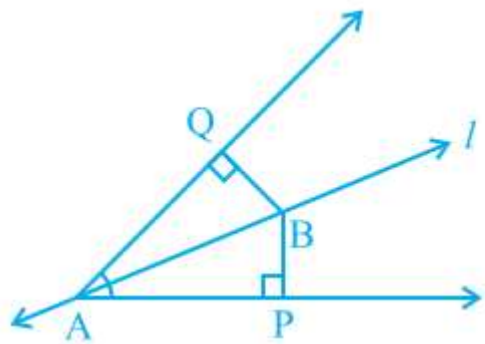
Q.4 *l* and m are two parallel lines intersected by another pair of parallel lines p and q (see figure). Show that ΔABC ≅ ΔCDA.



Sol. Given that *l* and m are two parallel lines intersected by another pair of parallel lines p and q. So, AD || BC and AB || CD
It means ABCD is a parallelogram.
So, AB = CD and BC = AD
Now, in Triangles ΔABC and ΔCDA,
AB = CD (Already Proved)
BC = AD (Proved above)
and AC = AC (Common Side)

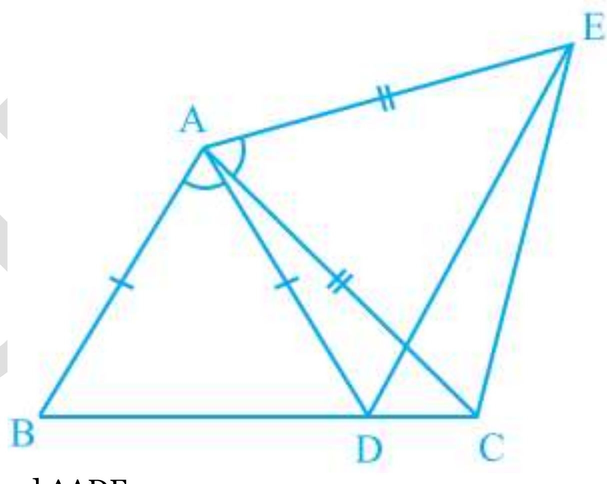
So, from SSS criterion of congruence.
ΔABC ≅ ΔCDA..............Hence Proved.

Q.5 Line *l* is the bisector of an angle A and B is any point on l. BP and BQ are perpendiculars from B to the arms of ∠A (see figure). Show that:
(i) ΔAPB ≅ ΔAQB
(ii) BP = BQ or B is equidistant from the arms of ∠A.



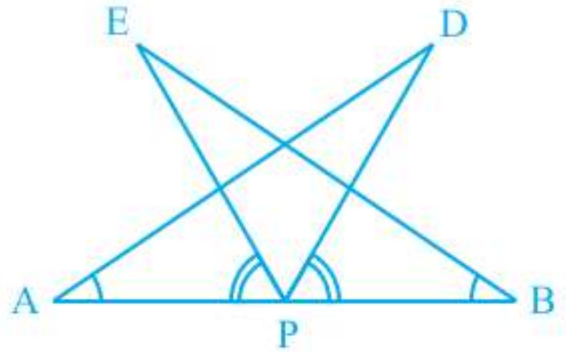
Sol. Firstly, In triangles ΔAPB and ΔAQB,
∠APB = ∠AQB (Given, each = 90°)
∠PAB = ∠QAB [Since, *l* bisects ∠PAQ]
AB = AB (Common side)
So, from AAS congruence criterion,
(i) ΔAPB ≅ ΔAQB..............Hence Proved
(ii) And BP = PQ (Corresponding parts of congruent triangles)
It means that B is equidistant from the arms of ∠A.

Q.6 In figure AC = AE, AB = AD and ∠BAD =∠EAC. Show that BC = DE.



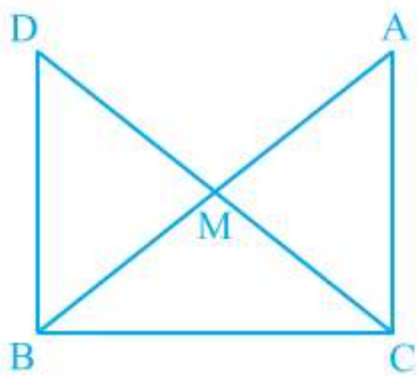
Sol. Firstly, In triangles ΔABC and ΔADE,
AB = AD (Given)
Since, ∠BAD = ∠EAC
Add ∠DAC both the sides,
⇒ ∠BAD + ∠DAC = ∠EAC+∠DAC
So, ∠BAC = ∠DAE
and, AC = AE (Given)
So, from SAS criterion of congruence,
ΔABC ≅ ΔADE
⇒ BC = DE (Corresponding parts of congruent triangles)

Q.7 AB is a line segment and P is its mid- point. D and E are points on the same side of AB such that ∠BAD =∠ABE and ∠EPA =∠DPB (see figure). Show that -
(i) ΔDAP ≅ ΔEBP
(ii) AD = BE.



Sol. Given: ∠EPA = ∠DPB
Add ∠DPE both sides,
⇒ ∠EPA + ∠DPE = ∠DPB + ∠DPE
⇒ ∠DPA = ∠EPB.............(i)
Now, in triangles ΔEBP and ΔDAP,
∠EPB = ∠DPA (From eq. (i))
BP = AP (Given)
and , ∠EBP = ∠DAP (Given)
So, from ASA criterion of congruence,
ΔEBP ≅ ΔDAP
⇒ BE = AD
AD = BE (Corresponding parts of congruent triangles)
Hence Proved.

Q.8 In right triangle ABC, right angled at C, M is the mid- point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B (see figure). Show that:
(i) ΔAMC ≅ ΔBMD
(ii) ∠DBC is a right angle
(iii) ΔDBC ≅ ΔACB
(iv) CM = $\frac{1}{2}$ AB



***Sol.* (i)** Firstly, In triangles ΔAMC and ΔBMD,
AM = BM (Since, M is the mid- point of hypotenuse AB)
∠AMC = ∠BMD (Vertically opposite angles)
and CM = MD (Given)
So, from SAS criterion of congruence,
ΔAMC≅ΔBMD............ Hence Proved.

(ii) Now, $\Delta AMC \cong \Delta BMD$

$\Rightarrow BD = CA$ and $\angle BDM = \angle ACM$..... (i) (Corresponding parts of congruent triangles)

Therefore, transversal line CD intersects CA and BD at C and D respectively.

$\angle BDM = \angle ACM$ (alternate interior angles).

So, BD || CA.

$\Rightarrow \angle CBD + \angle BCA = 180^\circ$ (Since sum of consecutive interior angles are supplementary)

$\Rightarrow \angle CBD + 90^\circ = 180^\circ$ (Since, $\angle BCA = 90^\circ$)

$\Rightarrow \angle CBD = 180^\circ - 90^\circ$

$\Rightarrow \angle DBC = 90^\circ$................Hence Proved.

(iii) In triangles, ΔDBC and ΔACB,

BD = CA (From eq. (i))

$\angle DBC = \angle ACB$ (Since, Each = 90° (Already proved))

BC = BC (Common Side)

So, from SAS criterion of congruence,

$\Delta DBC \cong \Delta ACB$................Hence Proved.

(iv) CD = AB (Corresponding parts of congruent triangles)

$\Rightarrow \frac{1}{2} CD = \frac{1}{2} AB$

$\Rightarrow CM = \frac{1}{2} AB$......................Hence Proved.