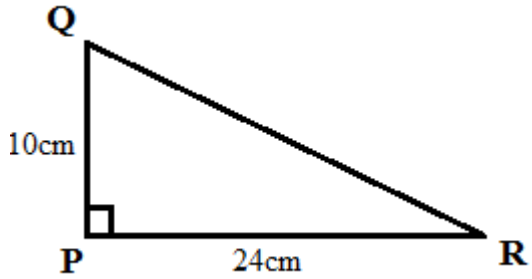
Triangle and Its Properties: Exercise 6.5

Q.1 PQR is a triangle, right-angled at P. If PQ = 10 cm and PR = 24 cm, find QR.

Sol: Let PQR be a triangle, right angled triangle at P.



Given: PQ = 10 cm and PR = 24 cm.
From the Pythagoras Theorem,
(Hypotenuse)^2 = (Perpendicular)^2 + (Base)^2
$QR^2 = PQ^2 + PR^2$

$QR^2 = 10^2 + 24^2$

$QR^2 = 100 + 576$

$QR^2 = 676$

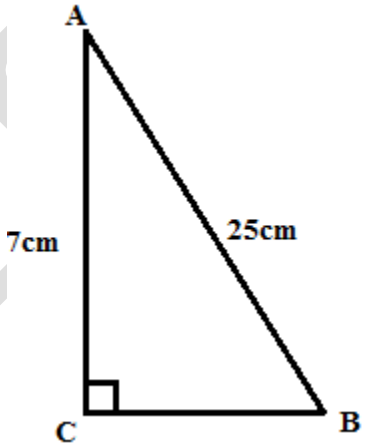
$QR = \sqrt{676}$

QR = 26 cm

Thus, the length of QR = 26 cm.

Q.2 ABC is a triangle, right-angled at C. If AB = 25cm and AC = 7 cm, find BC.

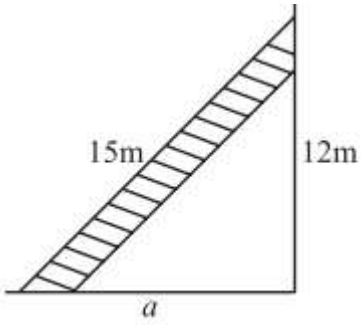
Sol: Let ABC be a triangle, right angled triangle at A.



Given: AB = 25 cm and AC = 7 cm.
From the Pythagoras Theorem,
(Hypotenuse)^2 = (Perpendicular)^2 + (Base)^2
$AB^2 = AC^2 + BC^2$
$25^2 = 7^2 + BC^2$
$625 = 49 + BC^2$

On transposing 49 from RHS to LHS,
$BC^2 = 625 - 49$
$BC^2 = 576$
$BC = \sqrt{576}$
$BC = 24$ cm
Thus, the length of the BC = 24 cm.

Q.3 A 15 m long ladder reached a window 12 m high from the ground on placing it against a wall at a distance a. Find the distance of the foot of the ladder from the wall.



Sol: Given: ladder length = 12 m and height of window from the ground = 12 cm
Let a be the distance of the foot of the ladder from the wall.
From the Pythagoras Theorem,
$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$
$15^2 = 12^2 + a^2$
$225 = 144 + a^2$
By transposing 144 from RHS to LHS it becomes – 144
$a^2 = 225 - 144$
$a^2 = 81$
$a = \sqrt{81}$
$a = 9$ m
Thus, the distance of the foot of the ladder from the wall = 9 m

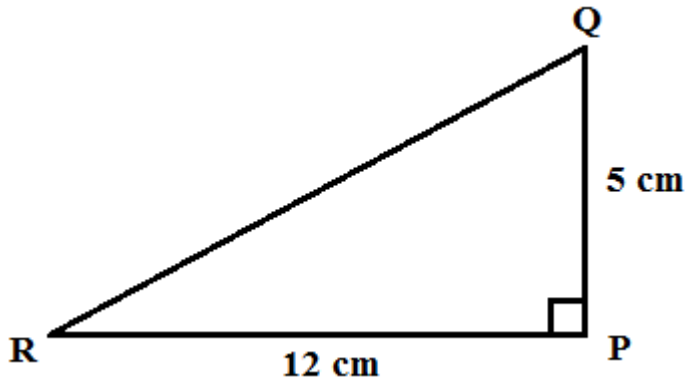
Q.4 Which of the following can be the sides of a right triangle?
(i) 2.5 cm, 6.5 cm, 6 cm.
(ii) 2 cm, 2 cm, 5 cm.
(iii) 1.5 cm, 2cm, 2.5 cm.
In the case of right-angled triangles, identify the right angles.
Sol:
(i) Given: Sides of triangle 2.5 cm, 6.5 cm and 6 cm.
Since, for right angled triangle, sum of square of two side of triangle is equal to the square of third side.
Let the largest value of length is the hypotenuse, b = 6.5 cm
From the Pythagoras Theorem,
$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$
$b^2 = a^2 + c^2$
$6.5^2 = 2.5^2 + 6^2$
$42.25 = 6.25 + 36$
$42.25 = 42.25$
So from above, given triangle is right-angled triangle and Right angle must be the opposite of the greater side of length 6.5 cm.

(ii) Given: Sides of triangle 2 cm, 2 cm and 5 cm.

Since, for right angled triangle, sum of square of two side of triangle is equal to the square of third side.
Let the largest value of length is the hypotenuse, c = 5 cm
From the Pythagoras Theorem,
$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$
$c^2 = a^2 + b^2$
$5^2 = 2^2 + 2^2$
$25 = 4 + 4$
$25 \neq 8$
So from above, given triangle is not right-angled triangle.

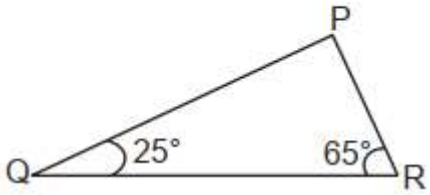
(iii) Given: Sides of triangle 1.5 cm, 2cm and 2.5 cm.
Since, for right angled triangle, sum of square of two side of triangle is equal to the square of third side.
Let the largest value of length is the hypotenuse, c = 2.5 cm
From the Pythagoras Theorem,
$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$
$2.5^2 = 1.5^2 + 2^2$
$6.25 = 2.25 + 4$
$6.25 = 6.25$
So from above, given triangle is right-angled triangle and Right angle must be the opposite of the greater side of length 2.5 cm.

Q.5 A tree is broken at a height of 5 m from the ground and its top touches the ground at a distance of12 m from the base of the tree. Find the original height of the tree.
Sol: Since, a tree is broken at a height of 5 m from the ground and its top touches the ground at a distance of12 m from the base of the tree.
Let PQR be a right angled triangle which shows free body diagram of given situation.
PQ = 5 cm (height from the ground)
PR = 12 cm (length of top of tree from the base of the tree)



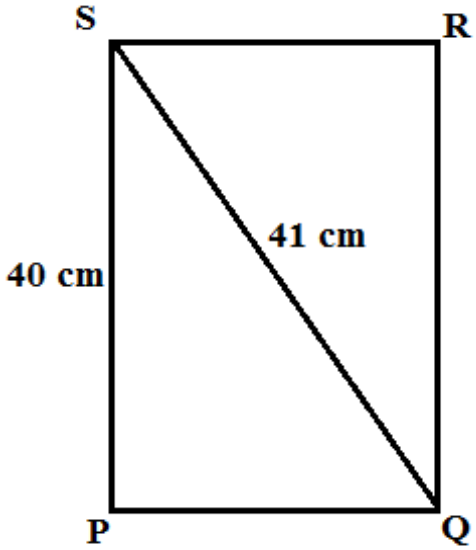
From the Pythagoras Theorem,
$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$
$RQ^2 = PQ^2 + PR^2$
$RQ^2 = 5^2 + 12^2$
$RQ^2 = 25 + 144$
$RQ^2 = 169$
$RQ = \sqrt{169}$
$RQ = 13$ m
Thus, the original height of the tree = PQ + RQ
= 5 + 13
= 18 m

Q.6 Angles Q and R of a ΔPQR are 25° and 65°.
Write which of the following is true:
(i) $PQ^2 + QR^2 = RP^2$
(ii) $PQ^2 + RP^2 = QR^2$
(iii) $RP^2 + QR^2 = PQ^2$



Sol: **Given:** In a triangle PQR, ∠Q = 25° and ∠R = 65°
Since, sum of the three interior angles of triangle is equal to 180°.
So, ∠P + ∠Q + ∠R = 180°
∠P + 25° + 65° = 180°
∠P + 90° = 180°
∠P = 180° - 90°
∠P = 90°
If ∠P = 90° then side QR will be hypotenuse.
So, From the Pythagoras Theorem,
$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$
$QR^2 = PQ^2 + PR^2$
Thus, option (ii) is true.

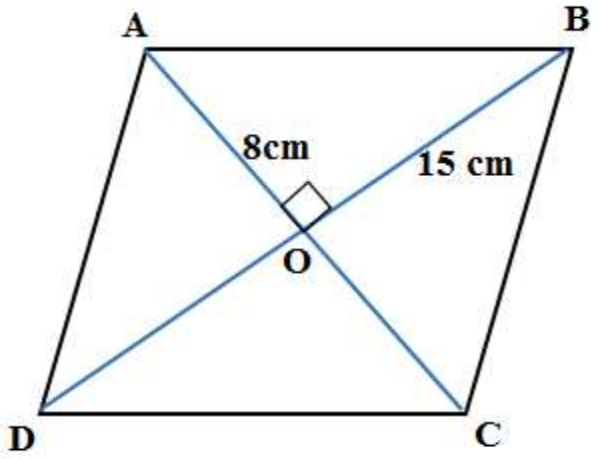
Q.7 Find the perimeter of the rectangle whose length is 40 cm and a diagonal is 41 cm.
Sol: Let PQRS be the rectangle with length = 40 and diagonal = 41 cm.



From the figure, triangle PQ is a right angled triangle.
So, From the Pythagoras Theorem,
$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$
$(QS)^2 = (PS)^2 + (PQ)^2$
$41^2 = 40^2 + BC^2$
$BC^2 = 41^2 - 40^2$
$BC^2 = 1681 - 1600$
$BC^2 = 81$

BC = √81
BC = 9 cm
Thus width of rectangle = 9 cm
And perimeter of rectangle = 2 (length + breadth)
= 2(40 + 9)
= 2 × 49
= 98 cm

Q.8 The diagonals of a rhombus measure 16 cm and 30 cm. Find its perimeter.
Sol: Let ABCD be the rhombus and its diagonals BD = 16 cm and AC = 30 cm.



Since in rhombus, all sides are equal and its diagonals bisect each other at 90°.
So, triangle AOB is right angled triangle.
Where AO = (AC/2) = 8 cm and OB = (BD/2) = 15 cm
$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$
$(AB)^2 = (AO)^2 + (OB)^2$
$(AB)^2 = (8)^2 + (15)^2$
$(AB)^2 = 64 + 225$
$(AB)^2 = 289$
AB = 17 cm
Thus, one side of the rhombus AB = 17 cm
And perimeter of rhombus = 4 × side of the rhombus
= 4 × 17
= 68 cm

Thus, perimeter of rhombus = 68 cm.