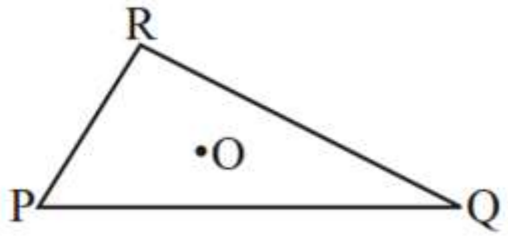
Triangle and Its Properties: Exercise 6.4

Q.1 Is it possible to have a triangle with the following sides?
(i) 2 cm, 3 cm, 5 cm (ii) 3 cm, 6 cm, 7 cm (iii) 6 cm, 3 cm, 2 cm
Sol: If sum of any two of these numbers is greater than the third then triangle will be possible.
(i) Given: 2 cm, 3 cm, 5 cm
$2 + 3 = 5 = 5$(third side)
From above, it is not possible to have a triangle with the sides 2cm, 3cm and 5cm.

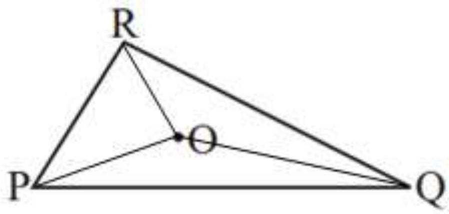
(ii) Given: 3 cm, 6 cm, 7 cm
$3 + 6 = 9 > 7$ (third side)
$6 + 7 = 13 > 3$ (third side)
$3 + 7 = 10 > 6$ (third side)
From above, it is possible to have a triangle with the sides 3 cm, 6 cm, and 7 cm.

(iii) Given: 6 cm, 3 cm, 2 cm
$6 + 3 = 9 > 2$
$3 + 2 = 5 < 6$
From above, it is not possible to have a triangle with the sides 3 cm, 6 cm, and 7 cm.

Q.2 Take any point O in the interior of a triangle PQR. Is
(i) OP + OQ > PQ?
(ii) OQ + OR > QR?
(iii) OR + OP > RP?



***Sol:* Given:** Take any point O in the interior of a triangle PQR.
Firstly we join the interior point O to P, Q and R.

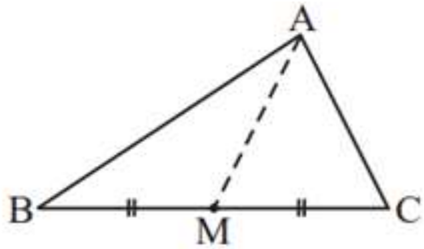


As we know that in a triangle, sum of any two sides is always greater than the third side.
So,
(i) In ΔOPQ has sides OP, OQ and PQ.
So, yes $OP + OQ > PQ$

(ii) In ΔOQR has sides OR, OQ and QR.
So, yes $OQ + OR > QR$

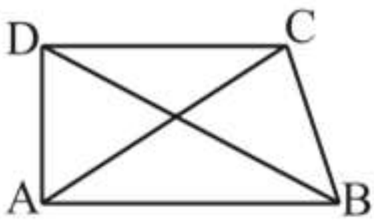
(iii) In ΔORP has sides OR, OP and PR.
So, yes $OR + OP > RP$

Q.3 AM is a median of a triangle ABC. Is AB + BC + CA > 2 AM?
(Consider the sides of triangles ΔABM and ΔAMC.)



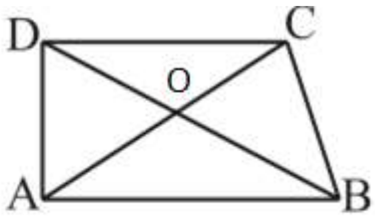
Sol: As we know that in a triangle, sum of any two sides is always greater than the third side.
Now, in ΔABM,
AB + BM > AM ... (i)
And in ΔACM,
AC + CM > AM ... (ii)
On adding equation (i) and (ii),
AB + BM + AC + CM > AM + AM
Since from the figure, BC = BM + CM
AB + BC + AC > 2 AM
Thus, the given expression is possible.

Q.4 ABCD is a quadrilateral. Is AB + BC + CD + DA > AC + BD?



Sol: As we know that in a triangle, sum of any two sides is always greater than the third side.
Now in ΔABC,
AB + BC > CA ... (i)
Then in ΔBCD,
BC + CD > DB ... (ii)
In ΔCDA,
CD + DA > AC ... (iii)
And in ΔDAB
DA + AB > DB ... (iv)
On adding (i), (ii), (iii) and (iv),
AB + BC + BC + CD + CD + DA + DA + AB > CA + DB + AC + DB
2AB + 2BC + 2CD + 2DA > 2CA + 2DB
Now, take common 2 on both the side,
2(AB + BC + CA + DA) > 2(CA + DB)
AB + BC + CA + DA > CA + DB
Thus, the given expression is possible.

Q.5 ABCD is quadrilateral. Is AB + BC + CD + DA < 2 (AC + BD)?
Sol: Let ABCD be the quadrilateral and its diagonals intersect at point O.



As we know that in a triangle, sum of any two sides is always greater than the third side.
Firstly, in ΔPAB,
PA + PB < AB … (i)
In ΔPBC,
PB + PC < BC … (ii)
In ΔPCD,
PC + PD < CD … (iii)
And in ΔPDA
PD + PA < DA … (iv)
On adding (i), (ii), (iii) and (iv),
PA + PB + PB + PC + PC + PD + PD + PA < AB + BC + CD + DA
2PA + 2PC + 2PB + 2PD < AB + BC + CD + DA
2(PA + PC) + 2(PB + PD) < AB + BC + CD + DA
Since from the figure, AC = PA + PC and BD = PB + PD
Then,
2AC + 2BD < AB + BC + CD + DA
2(AC + BD) < AB + BC + CD + DA
Thus, the given expression is possible.

Q.6 The lengths of two sides of a triangle are 12 cm and 15 cm. Between what two measures should the length of the third side fall?

Sol: As we know that in a triangle, sum of any two sides is always greater than the third side.
Since, two sides of triangle are 12 cm and 15 cm.
So, the length of third side should be less than the sum of other two sides,
12 + 15 = 27 cm.
And the third side is cannot not be less than the difference of the two sides,
So, 15 – 12 = 3 cm
Therefore, the length of the third side should be fall between the length 3 cm and 27 cm.