Surface Areas and Volumes: Exercise 13.8

(Assume $\pi = \frac{22}{7}$, unless stated otherwise.)

Q.1 Find the volume of a sphere whose radius is:
(i) 7 cm (ii) 0.63 cm

Sol.

(i) Given: Dimension of sphere, let radius r = 7 cm

So, volume of the sphere = $\frac{4}{3}\pi r^3$

$= (\frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7)\ cm^3$

$= \frac{4312}{3}\ cm^3$

$= 1437\frac{1}{3}\ cm^3$

(ii) Given: Dimension of sphere, let radius r = 0.63 m

So, volume of the sphere = $\frac{4}{3}\pi r^3$

$= (\frac{4}{3} \times \frac{22}{7} \times 0.63 \times 0.63 \times 0.63)\ m^3$

$= 1.05\ m^3$ (approx)

Q.2 Find the amount of water displaced by a solid spherical ball of diameter.
(i) 28 cm (ii) 0.21 m

Sol.

(i) Given: Diameter of the spherical ball, d = 28 cm
So, Radius r = 28/2 = 14 cm
Amount of water displaced by the spherical ball will be equal to its volume,

So, volume of spherical ball = $\frac{4}{3}\pi r^3$

$= (\frac{4}{3} \times \frac{22}{7} \times 14 \times 14 \times 14)\ cm^3$

$= \frac{34496}{3}\ cm^3$

$= 11498\frac{2}{3}\ cm^3$

Thus, the amount of water displaced by a solid spherical ball = $11498\frac{2}{3}\ cm^3$

(ii) Given: Diameter of the spherical ball, d = 0.21 m
Therefore, Radius r = 0.21/2 = 0.105 m
Amount of water displaced by the spherical ball will be equal to its volume,

So volume of spherical ball = $\frac{4}{3}\pi r^3$

$$= \left(\frac{4}{3} \times \frac{22}{7} \times 0.105 \times 0.105 \times 0.105\right) m^3$$

$$= 0.004851\ m^3$$

Thus, the amount of water displaced by a solid spherical ball = $0.004851\ m^3$

Q.3 The diameter of a metallic ball is 4.2 cm. What is mass of the ball, if the density of the metal is 8.9 g per cm^3?

***Sol.* Given:** Diameter of the ball, d = 4.2 cm
So, Radius = 4.2/2 = 2.1 cm

Therefore, volume of the ball = $\frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times 2.1 \times 2.1 \times 2.1\ cm^3$$

$$= 38.808\ cm^3$$

Since, density of the metal = 8.9g per cm^3
As we know that Density = Mass/Volume
Thus, mass of the ball = (38.808 × 8.9) g
= 345.3912 g

Q.4 The diameter of the moon is approximately one - fourth the diameter of the earth. What fraction of the volume of the earth is the volume of the moon?

Sol. Let r be the diameter of the moon. So, the radius of the moon = r/2
Now, diameter of the earth is 4r, so its radius = 4r/2 = 2r.

Let V_1 be the volume of the moon = $\frac{4}{3}\pi\left(\frac{r}{2}\right)^3$

$$= \frac{4}{3}\pi r^3 \times \frac{1}{8}$$

$$\Rightarrow \quad 8V_1 = \frac{4}{3}\pi r^3 \ldots \text{(i)}$$

and, V_2 be the volume of the earth = $\frac{4}{3}\pi (2r)^3$

$$= \frac{4}{3}\pi r^3 \times 8$$

$$\Rightarrow \quad \frac{V_2}{8} = \frac{4}{3}\pi r^3 \ldots \text{(ii)}$$

Therefore, from (i) & (ii),

$$8V_1 = \frac{V_2}{8}$$

$$\Rightarrow V_1 = \frac{1}{64} V_2$$

Thus, the volume of the moon is $\frac{1}{64}$ time of the volume of the earth.

Q.5 How many litres of milk can a hemispherical bowl of diameter 10.5 cm hold?

***Sol.* Given:** Diameter of a hemispherical bowl d = 10.5 cm

So, radius r = 10.5/2 cm = 5.25cm

Therefore, volume of the bowl = $\frac{2}{3}\pi r^3$

$$= (\frac{2}{3} \times \frac{22}{7} \times 5.25 \times 5.25 \times 5.25)\ cm^3$$

$$= 303.1875\ cm^3$$

Since, 1000 cm^3 = 1 litre

So, 303.18 cm^3 = $\frac{303.18}{1000}$ = 0.303 litre

Thus, the hemispherical bowl can hold the milk = 0.303 litre.

Q.6 A hemispherical tank is made up of an iron sheet 1 cm thick. If the inner radius is 1 m, then find the volume of the iron used to make the tank.

Sol. Let R cm be the external radius and r cm be internal radius of the hemispherical vessel.

Since, thickness = 1 cm = 0.01 m

Therefore, external radius, R = 1m + 0.01m = 1.01

and internal radius, r = 1 m.

Therefore, volume of iron used = External volume – Internal volume

$$= \frac{2}{3}\pi R^3 - \frac{2}{3}\pi r^3$$

$$= \frac{2}{3}\pi (R^3 - r^3)$$

$$= \frac{2}{3} \times \frac{22}{7} \times [(1.01)^3 - (1)^3]\ m^3$$

$$= \frac{44}{21} \times (1.030301 - 1)m3$$

$$= (\frac{44}{21} \times 0.030301)\ m^3$$

$$= 0.06348\ m^3 \text{ (approx)}$$

Thus, volume of iron used = 0.06348 m^3

Q.7 Find the volume of a sphere whose surface area is 154 cm^2.

***Sol.* Given:** Surface area of sphere = 154 cm^2

Suppose r cm is the radius of the sphere.

Therefore, surface area = 154 cm^2

$\Rightarrow 4\pi r^2 = 154$

$\Rightarrow 4 \times \frac{22}{7} \times r^2 = 154$

$\Rightarrow r^2 = \frac{154x7}{4x22} = 12.25$

$\Rightarrow r = \sqrt{12.25} = 3.5cm$

Now, Volume of sphere = $\frac{4}{3}\pi r^3$

$= (\frac{4}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 3.5)\ cm^3$

$= 5393\ cm^3$

$= 17923\ cm^3$

The volume of a sphere = 17923 cm^3

Q.8 a dome of a building is in the form of a hemisphere. From inside it was white - washed at the cost of Rs. 498.96. If the cost of white- washing is Rs.2.00 per square metre, find the
(i) Inside surface area of the dome,
(ii) Volume of the air inside the dome.

***Sol.* Given:** White wash cost of inside the dome = Rs. 498.96
The cost of white - washing for per square meter = Rs.2.00

(i) Therefore, inside surface area of the dome = $\frac{Total\ cost\ of\ white\ washing}{Rate\ of\ white\ washing}$

$= \frac{498.96}{2}$

$= 249.48\ m^2$

(ii) Now, for volume
Suppose r is the radius of the dome.
So, Surface area = $2\pi r^2$

$\Rightarrow 249.48 = 2 \times \frac{22}{7} \times r^2$

$\Rightarrow r^2 = \frac{249.48 \times 7}{2 \times 22} = 39.69$

$\Rightarrow r = \sqrt{39.69} = 6.3m$

Now, volume of the air inside the dome = $\frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times 6.3 \times 6.3 \times 6.3\ m^3$

$= 523.9\ m^3$ (approx)

Thus, the volume of the air inside the dome is 523.9 m^3.

Q.9 Twenty seven solid iron spheres, each of radius r and surface area S are melted to form a sphere with surface area S'. Find the
(i) Radius r' of the new sphere,

(ii) Ratio of S and S'.

Sol. Given: radius of 27 solid iron sphere = r

(i) Therefore, volume of 27 solid sphere = $27\times\frac{4}{3}\pi r^3$ ... (i)

And volume of the new sphere of radius r′ = $\frac{4}{3}\pi r'^3$ ... (ii)

Since, 27 solid iron spheres are melted to form a sphere of radius r'.

Therefore, $\frac{4}{3}\pi r'^3 = 27\times\frac{4}{3}\pi r^3$

$\Rightarrow \quad r'^3 = 27r^3$

$\Rightarrow \quad r'^3 = (3r)^3$

$\Rightarrow \quad r' = 3r$

Thus, Radius r' of the new sphere = 3r

(ii) Now for Ratio of S and S':

Therefore, $\frac{S}{S'} = \frac{4\pi r^2}{4\pi r'^2}$

Since, r′ = 3r

$$\frac{S}{S'} = \frac{r^2}{(3r)^2}$$

$$\frac{S}{S'} = \frac{r^2}{9r^2} = \frac{1}{9}$$

Thus, the Ratio of S and S' is 1 : 9.

Q.10 A capsule of medicine is in the shape of a sphere of diameter 3.5 mm. How much medicine (in mm^3) is needed to fill this capsule?

***Sol.* Given:** a sphere of diameter = 3.5 mm

So, radius = 3.5/2 mm

= 1.75 mm

So, medicine required for its filling will be equal to volume of spherical capsule.

Thus, volume of spherical capsule = $\frac{4}{3}\pi r^3$

$$= \frac{4}{3}\times\frac{22}{7}\times 1.75\times 1.75\times 1.75 \text{ mm}^3$$

$= 22.46 \text{ mm}^3$ (approx)

Thus, medicine is needed to fill this capsule = 22.46 mm^3