Surface Areas and Volumes: Exercise 13.7

Assume $\pi = \frac{22}{7}$, unless stated otherwise.

Q.1 Find the volume of the right circular cone with
(i) Radius 6 cm , height 7 cm
(ii) Radius 3.5 cm , height 12 cm

Sol.

(i) Given: Dimension of right circular cone, let radius r = 6 cm and height h = 7 cm

Therefore, volume of the cone = $\frac{1}{3}\pi r^2 h$

$= (\frac{1}{3} \times \frac{22}{7} \times 6 \times 6 \times 7)\ cm^3$

$= 264\ cm^3$

(ii) Given: Dimension of right circular cone, let radius r = 3.5 cm and height h = 12 cm

Therefore, volume of the cone = $\frac{1}{3}\pi r^2 h$

$= (\frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 12)\ cm^3$

$= 154\ cm^3$.

Q.2 Find the capacity in litres of a conical vessel with
(i) Radius 7 cm , slant height 25 cm
(ii) Height 12 cm, slant height 13 cm

Sol.

(i) Given: Dimension of conical vessel, let radius r = 7 cm and slant height l = 25 cm.
Let h be the height of the cone.
Then, $h^2 = \ell^2 - r^2$

$h^2 = 25^2 - 7^2$

$h^2 = 625 - 49$

$h^2 = 576$

$\Rightarrow \quad h = \sqrt{576} = 24cm$

Therefore, volume of the conical vessel = $\frac{1}{3}\pi r^2 h$

$= (\frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 24)\ cm^3$

$= 1232\ cm^3$

since, $1000\ cm^3 = 1\ell$

Therefore, capacity of the vessel in litres = $(\frac{1232}{1000})\ \ell = 1.232\ \ell$

(ii) Given: Dimension of conical vessel, let height h = 12 cm and slant height l = 13 cm.
Let r be the radius of the base of the cone.
Then, $r^2 = \ell^2 - h^2$

$= 13^2 - 12^2$
$= 169 - 144$
$= 25$

$\Rightarrow \quad r = \sqrt{25} = 5\text{cm}$

Therefore, volume of the conical vessel $= \frac{1}{3}\pi r^2 h$

$= (\frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 12)\ \text{cm}^3$

$= \frac{2200}{7}\ \text{cm}^3$

Since, $1000\ \text{cm}^3 = 1\ell$

So, capacity of the vessel in litres $= (\frac{2200}{7} \times \frac{1}{1000})\ \ell$

$= \frac{11}{35}\ \ell$

Q.3 The height of a cone is 15 cm. If its volume is 1570 cm³, find the radius of the base. (Use π = 3.14)

Sol. **Given: Dimension of cone, let height** h = 15 cm and volume V = 1570 cm^3
Let r be the radius of the base of cone.
So, Volume of the cone = 1570 cm^3

$\Rightarrow \quad \frac{1}{3}\pi r^2 h = 1570$

$\Rightarrow \quad \frac{1}{3} \times 3.14 \times r^2 \times 15 = 1570$

$\Rightarrow \quad r^2 = \frac{1570}{3.14x5} = 100$

$\Rightarrow \quad r = \sqrt{100} = 10$

Hence, the radius of the base of cone = 10 cm.

Q.4 If the volume of a right circular cone of height 9 cm is 48 π cm³, find the diameter of its base.

Sol. **Given:** Dimension of right circular cone, height h = 9 cm and volume V = $48\pi\ \text{cm}^3$.
Let r be the radius of the base of the cone.
So, volume of cone = $48\pi\ \text{cm}^3$,

$\Rightarrow \frac{1}{3}\pi r^2 h = 48\pi$

$\Rightarrow \frac{1}{3} \times r^2 \times 9 = 48$

$\Rightarrow 3r^2 = 48$
$\Rightarrow r^2 = 16$
$\Rightarrow r = 4$

Therefore, the diameter of the base of the cone = 2 x 4 = 8 cm

Q.5 A conical pit of top diameter 3.5 m is 12 m deep. What is its capacity in kilolitres?

***Sol.* Given:** Dimension of conical pit, Diameter of the top = 3.5 m
So, radius r = (3.5/2) m = 1.75 m and depth of the pit h = 12 m

Therefore, volume of conical pit = $\frac{1}{3}\pi r^2 h$

$$= \left(\frac{1}{3} \times \frac{22}{7} \times 1.75 \times 1.75 \times 12\right) m^3$$

$$= 38.5\ m^3$$

Since, $1m^3$ = 1 kilolitres
So Capacity of pit = 38.5 kilolitres.

Q.6 The volume of a right circular cone is 9856 cm^3. If the diameter of the base is 28 cm, find
(i) Height of the cone.
(ii) Slant height of the cone,
(iii) Curved surface area of the cone.

***Sol.* Given:** Dimension of right circular cone, diameter of the base = 28 cm, radius r =28/2 = 14 cm and volume V = 9856 cm^3.

(i) Let h be the height of the cone.
Therefore, volume of the cone = 9856 cm^3

$$\Rightarrow \frac{1}{3}\pi r^2 h = 9856$$

$$\Rightarrow \frac{1}{3} \times \frac{22}{7} \times 14 \times 14 \times h = 9856$$

$$\Rightarrow h = \frac{9856 \times 3 \times 7}{22 \times 14 \times 14}$$

$$h = 48\ cm$$

Hence, the height of the cone is 48 cm.

(ii) Let *l* be the slant height of the cone.
So, slanted height, $\ell^2 = h^2 + r^2$

$$= 48^2 + 14^2$$

$$= 2304 + 196$$

$$= 2500$$

$$\Rightarrow \ell = \sqrt{2500} = 50$$

Therefore, the slant height of the cone is 50 cm.

(iii) Now, Curved surface area = π x r x ℓ

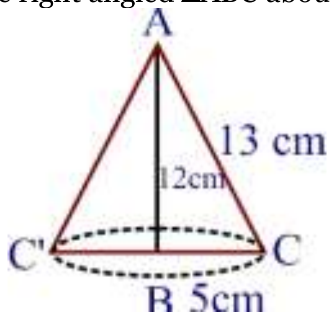
$$= \left(\frac{22}{7} \times 14 \times 50\right) cm^2$$

$$= 2200\ cm^2$$

Thus, curved surface area of the cone is 2200 cm^2.

Q.7 A right triangle ABC with sides 5 cm, 12 cm and 13 cm is revolved about the side 12 cm. Find the volume of the solid so obtained.

Sol. If we revolve the right angled ΔABC about the side let AB = 12 cm, we will obtain a cone:



So, dimension of cone, let radius r = 5 cm, height h = 12 cm and slant height l = 13 cm.

Therefore, volume of solid so obtained = $\frac{1}{3}\pi r^2 h$

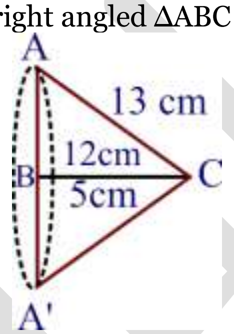
$= (\frac{1}{3} \times \pi \times 5^2 \times 12)\ cm^3$

$= 100\pi\ cm^3$

Thus, the volume of the solid = $100\pi\ cm^3$

Q.8 If the triangle ABC in the question 7 above is revolved about the side 5 cm, then find the volume of the solid so obtained. Find also the ratio of the volumes of the two solids obtained in question 7 and 8.

Sol. Now, If we revolve the right angled ΔABC about the side let BC = 5 cm, then obtain a cone:



So, dimension of the cone, let radius of base r = 12 cm, height h = 5 cm and slant height l = 13 cm

Therefore, volume of solid so obtained = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 12 \times 12 \times 5\ cm^3$

$= 240\pi\ cm^3$

Now, ratio of their volumes = $100\pi : 240\pi$

$= 5 : 12$

Q. 9 A heap of wheat is in the form of a cone whose diameter is 10.5 m and height is 3m. Find its volume. The heap is to be covered by canvas to protect it from rain. Find the area of the canvas required.

Sol. **Given:** Dimension of conical heap of wheat, let diameter d =10.5m, radius of the base r = (10.5/2) m = 5.25m and height of the cone h= 3m

So, volume of the conical heap = $\frac{1}{3}\pi r^2 h$

$= (\frac{1}{3} \times \frac{22}{7} \times 5.25 \times 5.25 \times 3)\ m^3$

$= 86.625\ m^3$

To find the required canvas for covering, we need to find out the slant height of the cone.

So, slant height l: $\ell^2 = h^2 + r^2$

$= 3^2 + (5.25)^2$

$= 9 + 27.5625$

$= 36.5625$

$\Rightarrow \quad \ell = \sqrt{36.5625}$

$= 6.0467$ (approx)

Thus, canvas required to cover the conical heap = Curved surface area

$= \pi r \ell$

$= (\frac{22}{7} \times 5.25 \times 6.0467)\ m^2$

$= 99.77\ m^2$ (approx.)