Surface Areas and Volumes: Exercise 13.6

Assume $\pi = \frac{22}{7}$, unless stated otherwise.

Q.1 The circumference of the base of a cylindrical vessel is 132 cm and its height is 25 cm. How many liters of water can it hold? ($1000\ cm^3 = 1\ell$)

***Sol.* Given:** Circumference of the base = 132 cm
If r cm is the radius of the base and h cm is the height of the cylinder.
Then circumference,

$\Rightarrow \quad 2\pi r = 132$

$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 132$

$\Rightarrow \quad r = (132 \times 72 \times 22)$ cm

$\Rightarrow \quad r = 21$cm

Now, volume of the cylinder = $\pi r^2 h\ cm^3$

$= (\frac{22}{7} \times 21 \times 21 \times 25)\ cm^3$

$= 34650\ cm^3$

So, vessel can hold = $\frac{34650}{1000}$ litres

= 34.65 litres

Thus, the cylindrical vessel can hold 34.65 litres of water.

Q.2 The inner diameter of a cylindrical wooden pipe is 24 cm and its outer diameter is 28 cm. The length of the pipe is 35 cm. Find the mass of the pipe, if 1 cm^3 of wood has a mass of 0.6 g.

***Sol.* Given:** Height of the cylindrical pipe, h = 35 cm
External radius, R = (28/2) cm = 14cm
Internal radius = (24/2) cm = 12cm
So, volume of the wood used in making the pipe = Volume (external cylinder) – Volume (internal cylinder)

$= \pi R^2 h - \pi r^2 h$

$= \pi (R^2 - r^2) h$

$= \frac{22}{7} \times (14^2 - 12^2) \times 35\ cm^3$

$= \frac{22}{7} \times 26 \times 2 \times 35\ cm^3$

$= 5720\ cm^3$

Since, mass of 1 cm^3 volume of wood = 0.6 g
So, mass of 5720 cm^3 volume of wood = (5720×0.6) g

$= \frac{5720 \times 0.6}{1000}$ kg

= 3.432 kg

Thus, mass of the pipe is 3.432 kg.

Q.3 A soft drink is available in two packs - (i) a tin can with a rectangular base of length 5 cm and width 4 cm, having a height of 15 cm and (ii) a plastic cylinder with circular base of diameter 7 cm and height 10 cm. Which container has greater capacity and by how much?

Sol. **(i) Given:** Dimension of cuboidal can, let length l = 5 cm, width b = 4 cm and height h = 15 cm.
Therefore, capacity of tin can = $\ell \times b \times h$ cm^3

$= (5 \times 4 \times 15)\ cm^3$

$= 300 cm^3$

(ii) Given: Dimension of cylindrical can, let radius r = 7/2 = 3.5 cm and height h = 10 cm
Therefore, capacity of plastic cylinder = $\pi r^2 h\ cm^3$

$$= \left(\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 10\right) cm$$

$= 385\ cm^3$

Therefore, the plastic cylinder has greater capacity than tin rectangular base can by (385 - 300) = 85 cm^3.

Q.4 If the lateral surface of a cylinder is 94.2 cm^2 and its height is 5 cm , then find (i) radius of its base (ii) its volume (Use π = 3.14)

Sol. **Given:** lateral surface of a cylinder = 94.2 cm^2 and let height h = 5 cm.
(i) Suppose r is the radius of the base of cylinder. Then,
Lateral surface = 94.2 cm^2

$\Rightarrow 2\pi r h = 94.2$

$\Rightarrow 2 \times 3.14 \times r \times 5 = 94.2$

$$\Rightarrow r = \frac{94.2}{2 \times 3.14 \times 5}$$

$\Rightarrow r = 3$

Hence, the radius of its base is 3 cm.

(ii) Now, volume of the cylinder = $\pi r^2 h$

$= (3.14 \times 3^2 \times 5)$ cm3

$= 141.3\ cm^3$

Thus, the volume of the cylinder is 141.3 cm^3.

Q.5 It costs Rs 2200 to paint the inner curved surface of a cylindrical vessel 10 m deep. If the cost of painting is at the rate of Rs. 20 per m^2, Find.
(i) Inner curved surface area of the vessel,
(ii) Radius of the base,
(iii) Capacity of the vessel.

Sol. **Given:** Cost of paint the inner curved surface of cylindrical vessel = Rs. 2200 and let deep h = 10 m.

(i) Therefore, inner curved surface area of the vessel = $\dfrac{Total\ cost\ of\ painting}{Rate\ of\ painting}$

$$= \frac{2200}{20}$$

$= 110\ m^2$

(ii) Suppose, r is the radius of the base of the cylindrical vessel.
So, inner curved surface area = $2\pi r h$

$110 = 2 \pi r h$

$\Rightarrow \quad 110 = 2 \times \frac{22}{7} \times r \times 10$

$\Rightarrow \quad r = \frac{110 \times 7}{2 \times 22 \times 10}$

$\Rightarrow \quad r = \frac{7}{4}$

$\Rightarrow \quad r = 1.75$

Therefore, the radius of the base is 1.75 m.

(iii) Now, capacity of the vessel = $\pi r^2 h$

$= (\frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \times 10)\ m^3$

$= 96.25\ m^3$

Thus, the capacity of vessel is 96.25 m^3.

Q.6 The capacity of a closed cylindrical vessel height 1 m is 15.4 litres. How many square metres metal sheet would be needed to make it?

Sol. **Given:** Capacity of a closed cylindrical vessel = 15. 4 litres

Since, 1 m^3= 1000 liters

So, Capacity of a closed cylindrical vessel = $(15.4 \times \frac{1}{1000})\ m^3$

$= 0.0154\ m^3$

Suppose, r is the radius of the base and h =1 m be the height of the vessel.

Then, Volume of cylindrical vessel = $\pi r^2 h$

$= \pi r^2 \times 1$

$= \pi r^2$

Therefore, $0.0154 = \pi r^2$

$\Rightarrow \quad \frac{22}{7} \times r^2 = 0.0154$

$\Rightarrow \quad r^2 = 0.0154 \times \frac{7}{22}$

$r^2 = 0.0049$

$\Rightarrow \quad r = \sqrt{0.0049} = 0.07$

Therefore, the radius of the base of vessel is 0.07 m.

Now, metal sheet needed to make the vessel will be equal to total surface area of the vessel.

So, total surface area = $2\pi rh + 2\pi r^2$

$= 2\pi r\,(h + r)$

$= 2 \times \frac{22}{7} \times 0.07 \times (1 + 0.07)\ m^2$

$= 44 \times 0.01 \times 1.07\ m^2$

$= 0.4708\ m^2$

Thus, metal sheet would be needed to make the cylindrical vessel is 0.4708 m^2.

Q.7 A lead pencil consists of a cylinder of wood with a solid cylinder of graphite filled in the interior. The diameter of the pencil is 7 mm and the diameter of the graphite is 1 mm. If the length of the pencil is 14 cm, find the volume of the wood and that of the graphite.

Sol. **Given:** Dimension of cylindrical shaped pencil, diameter of the graphite cylinder = 1 mm = $\frac{1}{10}$ cm, radius = $\frac{1}{20}$ cm and length = 14 cm

So, volume of the graphite cylinder = $\pi r^2 h$

$$= \left(\frac{22}{7} \times \frac{1}{20} \times \frac{1}{20} \times 14\right) \text{cm}^3$$

$$= 0.11 \text{ cm}^3$$

Now, dimension of pencil, diameter of the pencil = 7mm = $\frac{7}{10}$ cm

Therefore, radius of the pencil = $\frac{7}{20}$ cm and, length of the pencil = 14 cm

So, Volume of the pencil = $\pi r^2 h$

$$= \left(\frac{22}{7} \times \frac{7}{20} \times \frac{7}{20} \times 14\right) \text{cm}^3$$

$$=5.39\text{cm}3$$

Volume of wood = Volume (pencil) – Volume (graphite)

$$= (5.39 - 0.11) \text{ cm}^3$$

$$= 5.28 \text{ cm}^3$$

Thus, volume of wood is 5.28 cm^3

Q.8 A patient in a hospital is given soup daily in a cylindrical bowl of diameter 7 cm. If the bowl is filled with soup to a height of 4 cm, how much soup the hospital has to prepare daily to serve 250 patients?

Sol. Given: Dimension of the cylindrical bowl, diameter = 7 cm, Radius = 7/2cm
and Height = 4 cm

So, soup saved in on serving = Volume of the bowl

$$= \pi r^2 h$$

$$= \left(\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 4\right) \text{cm}^3$$

$$= 1.54 \text{ cm}^3$$

Therefore, soup for 250 patients = (250 × 1.54) cm^3

$$= 38500 \text{ cm}^3$$

Since, 1000 cm^3 = 1 ℓ

So, 38500 cm^3 = $\frac{1}{1000}$ x 38500 = 38.5ℓ

Thus, the hospital required 38.5 ℓ soup daily to serve 250 patients.