Surface Area and Volume: Exercise - 13.4

Q.1 A drinking glass is in the shape of a frustum of a cone of height 14 cm. The diameters of its two circular ends are 4 cm and 2 cm. Find the capacity of the glass.

Sol. Given: Height of glass =14 cm; Diameters of two circular ends = 4 cm and 2 cm.

So, Capacity of the glass, $V = \frac{\pi \times h}{3} \times (R^2 + r^2 + Rr)$

Since, R = 2 cm , r = 1 cm , h = 14 cm

So, $V = \frac{\frac{22}{7}}{3} \times (2^2 + 1^2 + 2\times1)$

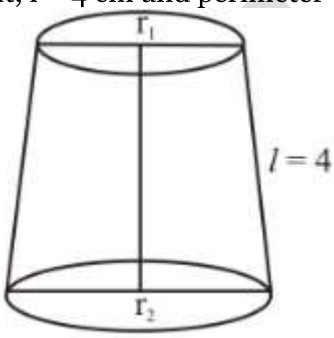
$= \frac{44}{3} \times (4+1+2)$

$= \frac{44}{3} \times 7$

$= \frac{308}{3}$ cm^3

Q.2 The slant height of a frustum of a cone is 4 cm and the perimeters (circumference) of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.

Sol. Given: Slant height, l = 4 cm and perimeter of the circular ends = 18 cm and 6 cm.



First circular end, $2\pi r_1 = 6$

$\Rightarrow \pi r_1 = 3$

and second circular end, $2\pi r_2 = 18$

$\Rightarrow \pi r_2 = 9$

Now, Curved surface of the frustum $= (\pi r_1 + \pi r_2)\,\ell$

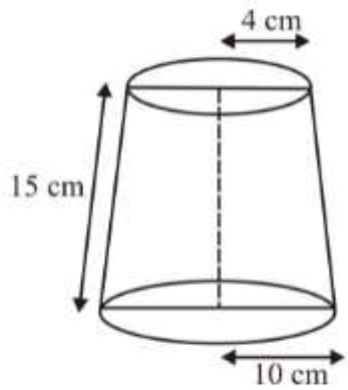
$= (3+9) \times 4$ cm^2

$= 12\times4$ cm^2

$= 48$ cm^2

Q.3 A fez, the (cap) used by the Turks, is shaped like the frustum of a cone (see figure). If its radius on the open side is 10 cm, radius at the upper base is 4 cm and its slant height is 15 cm, find the area of material used for making it.

Sol. Given: Radius on open side, R = 10 cm, radius at upper base, r = 4 cm and slant height, l = 15 cm



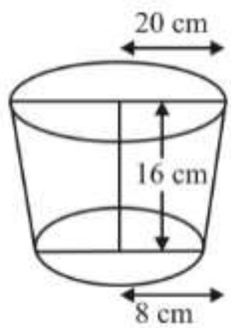
Area of the material used for fez = Surface area of frustum + surface of top circular section

$$= \pi(R + r)\,\ell + \pi r^2$$
$$= \frac{22}{7}(10+4)15 + \frac{22}{7} \times 4 \times 4$$
$$= \left(\frac{22}{7} \times 14 \times 15 + \frac{352}{7}\right) \text{cm}^2$$
$$= \frac{4972}{7} \text{cm}^2$$

Thus, the area of material used for making it = $\frac{4972}{7}$ cm²

Q.4 A container opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm, respectively. Find the cost of milk which can completely fill the container, at the rate of Rs 20 per litre. Also find the cost of metal sheet used to make the container, if it costs Rs 8 per 100 cm². (Take π=3.14).

Sol. Given: R = 20 cm, r = 8 cm and h = 16 cm

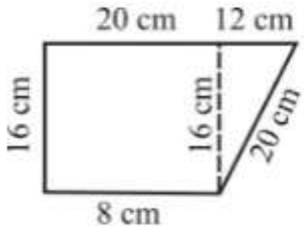


Then, Capacity of the container = Volume of the frustum

$$= \frac{1}{3}\pi h\,(R^2 + r^2 + Rr)$$
$$= \frac{1}{3} \times 3.14 \times 16(20^2 + 8^2 + 20 \times 8) \text{ cm}^3$$
$$= \frac{50.24}{3} \times (400+64+160) \text{ cm}^3$$
$$= \frac{50.24}{3} \times 624 \text{ cm}^3$$
$$= 50.24 \times 208 \text{ cm}^3$$
$$= \frac{10449.92}{1000} \text{ litres}$$

Since, Cost of milk @ Rs 20 per litre = Rs$(20\times\frac{10449.92}{1000})$

= Rs 208.99 ≈ Rs 209



Now, find the slant height, $\ell = \sqrt{16^2 + 12^2}$

= 20cm

Curved surface area = $\pi(R + r)\ell$

$= \frac{22}{7} \times (20+8) \times 20\ cm^2$

$= \frac{22}{7} \times 28 \times 20\ cm^2$

$= 1758.4\ cm^2$

and area of the bottom = $\pi r^2 = \frac{22}{7} \times (8)^2$

$= \frac{22}{7} \times 64$

$= 200.96\ cm^2$

Thus, Total area of metal required = $1758.4\ cm^2 + 200.96 cm^2$

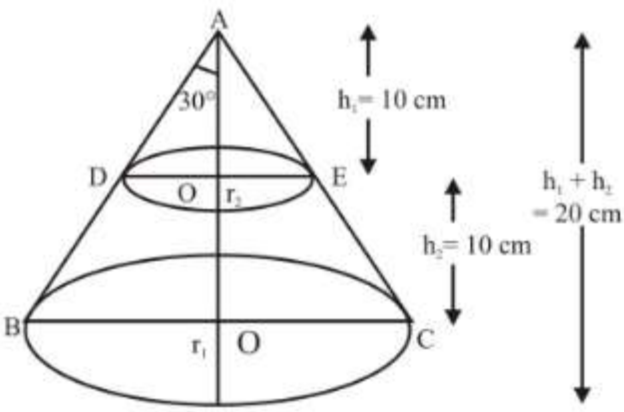
$= 1959.36\ cm^2$

Cost of metal sheet used to make the container @ Rs 8 per $100 cm^2$

=Rs $(\frac{8}{100} \times 1959.36)$ =Rs156.75

Q.5 A metallic right circular cone 20 cm high and whose vertical angle is 60⁰ is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained be drawn into wire of diameter $\frac{1}{16}$cm, find the length of the wire.

Sol. If ABC is the metallic cone, DECB is the required frustum.



From figure, the two radii of the frustum, $DO' = r_2$ and $BO = r_1$

Now, from the Triangles ADO' and ABO,

$r_2 = h_1 \tan 30^o = 10 \times \frac{1}{\sqrt{3}}$

$r_1 = (h_1 + h_2) \tan 30^o$

$= 20 \times \frac{1}{\sqrt{3}}$

Volume of the frustum DBCE = $\frac{\pi h_2}{3}$ $(r_1^2 + r_1 r_2 + r_2^2)$

$$= \frac{\pi h_2}{3} \times \left[\frac{400}{3} + \frac{200}{3} + \frac{100}{3}\right]$$

$$= \frac{\pi \times 10}{3} \times \frac{700}{3}$$

Volume of the wire of length l and diameter, D = $\pi(\frac{D}{2})^2 \times \ell$

$$= \frac{\pi D^2}{4} \times \ell \quad [V = \pi r^2 h]$$

Therefore, Volume (frustum) = Volume (wire drawn from it)

$\Rightarrow \quad \frac{\pi \times 10}{3} \times \frac{700}{3} = \frac{\pi D^2}{4} \times \ell \quad$ [since, D = $\frac{1}{16}$]

$\Rightarrow \quad \ell = \frac{10 \times 700 \times 4}{3x3} \times 16 \times 16$

$= 7964.44$ m