Surface Area and Volume: Exercise - 13.3

Q.1 A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.

Sol. Given: Radius of sphere = 4.2 cm, radius of cylinder = 6 cm

Since, Volume of the sphere = $\frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \pi \times (4.2)^3 \text{ cm}^3$$

Let h be the height of a cylinder of radius 6 cm.
Then, its volume = $\pi (6)^2 h$ cm^3
$= 36\pi h$ cm^3

Since, metallic sphere is reformed into the shape of a cylinder, so volume of sphere and cylinder remains the same.

$$36\pi h = \frac{4}{3} \times \pi \times 4.2 \times 4.2 \times 4.2$$

$$\Rightarrow \quad h = \frac{1}{36} \times \frac{4}{3} \times 4.2 \times 4.2 \times 4.2$$

$$\Rightarrow \quad h = 2.744 \text{ cm}$$

Thus, the height of the cylinder = 2.744 cm

Q.2 Metallic spheres of radii 6 cm, 8 cm and 10 cm, respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.

Sol. Given: Radii of the spheres are 6cm, 8 cm and 10 cm

So, Sum of the volumes of 3 gives spheres = $\frac{4}{3}\pi (r_1^3 + r_2^3 + r_3^3)$

$$= \frac{4}{3}\pi (6^3 + 8^3 + 10^3) \text{ cm}^3$$

$$= \frac{4}{3}\pi (216 + 512 + 1000) \text{ cm}^3$$

$$= \frac{4}{3}\pi (1728) \text{ cm}^3$$

If R is the radius of the new spheres whose volume is the sum of the volumes of 3 given spheres.
So,

$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi (1728)$$

$$\Rightarrow \quad R^3 = 1728$$

$$\Rightarrow \quad R^3 = (12)^3$$

$$\Rightarrow \quad R = 12$$

Thus, the radius of the resulting sphere is 12 cm.

Q.3 A 20 cm deep well with diameter 7 cm is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of the platform.

Sol. Given: depth of well = 20 cm; diameter = 7 cm and dimension of plate form = 22 m x 14m.
If h m is the required height of the platform. Then, the shape of the platform would be the shape of a cuboid of dimension 22 m × 14 m × h.
The volume of the platform = Volume of the earth dug out from the well.
Now, the volume of the earth = Volume of the cylindrical well

$$= \pi r^2 h$$

$$= \frac{22}{7} \times 12.25 \times 20 \text{ m}^3$$

$= 770 \text{ m}^3$

And, the volume of the platform = $22 \times 14 \times h \text{ m}^3$

Since, volume of the platform = Volume of the well

$$22 \times 14 \times h = 770$$

$$h = \frac{770}{22 \times 14}$$

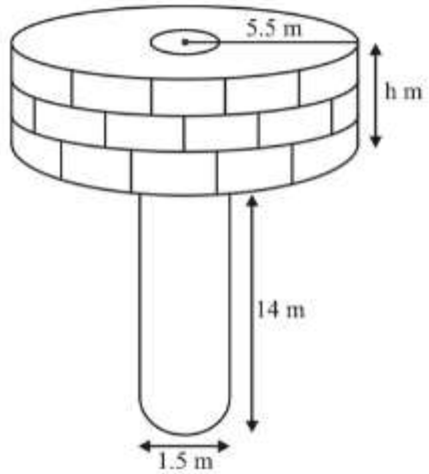
$$= 2.5 \text{ cm}$$

Hence, the height of the platform = 2.5 m

Q.4 A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 4 m to form an embankment. Find the height of the embankment.

Sol. Given: diameter of well = 3m; radius = 3/2 = 1.5m and depth = 14 cm

If h is the required height of the embankment.



Since, the shape of the embankment would be the shape of a cylinder with internal radius 1.5 m and external radius (4 + 1.5) m = 5.5 m

Therefore,

Volume of the embankment = volume of earth dug out from the well.

Volume of the earth = Volume of the cylindrical well

$$\pi \times (1.5)^2 \times 14 \text{ m}^3 = \pi (5.5^2 - 1.5^2) h \text{ m}^3$$

$$31.5\pi \text{ m}^3 = \pi (5.5 + 1.5)(5.5 - 1.5) h \text{ m}^3$$

Thus, $31.5\pi \text{ m}^3 = 28 \pi h \text{ m}^3$

$\Rightarrow$ $h = 31.5/28$

$= 1.125$

Thus, the required height of the embankment = 1.125 m

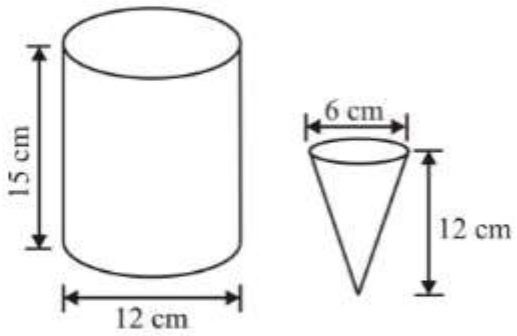
Q.5 A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice cream. The ice cream is to be filled into cones of height12 cm and diameter 6 cm, having hemispherical shape on the top. Find the number of such cones which can be filled with ice cream.

Sol. Given: Diameter of cylinder = 12 cm or radius = 12/2 = 6 cm

Height of cylinder = 15 cm

Height of cone = 12 cm

Diameter of cone = 6 cm or radius = 6/2 = 3 cm



Now, Volume of the cylinder = $\pi r^2 h$

$= \pi (6)^2 \times 15$

So, volume of a cone having hemispherical shape on the top

$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

$= \frac{1}{3}\pi r^2 (h + 2r)$

$= \frac{1}{3}\pi (3)^2 (12 + 2\times3)$

$= \frac{1}{3}\pi \times 3^2 \times 18$

Let n be the number of cone that can be filled with ice cream.

Then, $\frac{1}{3}\pi\times3^2 \times 18 \times n = \pi \times 6^2 \times 15$

$n = \frac{\pi\times6\times6\times15}{\pi\times3\times3\times18\times3} = 10$

Q.6 How many silver coins, 1.75 cm in diameter and of thickness 2 mm, must be melted to form a cuboid of dimensions 5.5 cm × 10 cm × 3.5 cm?

Sol. Given: diameter of coin = 1.75 or radius = 1.75/2 cm = 0.875 cm

Since, the shape of the coin would be the shape of a cylinder of radius = 0.875 cm and height of 2mm = $\frac{2}{10}$ cm =.2cm

Now, its volume = $\pi r^2 h = \frac{22}{7} \times 0.875 \times 0.875 \times .2$ cm^3

= 0.48125 cm^3

And volume of the cuboid = 5.5 × 10 × 3.5 cm^3

=192.5 cm^3

Number of coins required to form the cuboid = $\frac{Volume\ of\ the\ cuboid}{Volume\ of\ the\ coin}$

$= \frac{192.5}{0.48125} = 400$

Thus, 400 coins must be melted to form a cuboid.

Q.7 A cylindrical bucket, 32 m high and with radius of base 18 cm, is filled with sand. The bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap.

Sol. Given: height of cylindrical bucket = 32 m and radius = 18 cm

Height of the conical heap = 24 cm

Since, volume (sand) = Volume (cylindrical bucket)

So, Volume of cylindrical bucket = $\pi r^2 h$

$= \pi \times 18 \times 18 \times 32\ cm^3$

Now, volume of the conical heap = $\frac{1}{3}\pi r^2 h$, where r = ?, h = 24 cm

$= \frac{1}{3}\pi r^2 \times 24\ cm^3$

$= 8\pi r^2$

The volume of the conical heap will be equal to that of sand.

Therefore, $8\pi r^2 = \pi \times 18 \times 18 \times 32$

$\Rightarrow r^2 = 18 \times 18 \times 4$

$= 18^2 \times 2^2$

$\Rightarrow r = 18 \times 2 = 36$

Now, slant height $\ell = \sqrt{r^2 + h^2}$

$\Rightarrow \ell = \sqrt{36^2 + 24^2}$

$\Rightarrow = 12\sqrt{13}$

Thus, the radius of the conical heap = 36 cm and slant height = $12\sqrt{13}$ cm

Q.8 Water in a canal 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/h. How much area will it irrigate in 30 minutes, if 8 cm of standing water is needed?

Sol. Given: Width of the canal = 6 m

Depth of the canal = 1.5 m

Length of water column per hour = 10 km

Length of water column in 30 minutes or $\frac{1}{2}$ hour = $\frac{1}{2} \times 10$km = 5000m

So, volume of water flown in 30 minutes = $1.5 \times 6 \times 5000\ m^3$

$= 45000\ m^3$

Since, 8 cm = $\frac{8}{100}$ m i.e., 0.08 m standing water is desired

Therefore, Area irrigated in 30 minutes = Volume/ Height

= 45000/0.08

= 562500 m^2.

Or 56.25 hectares

Q.9 A farmer connects a pipe of internal diameter 20 cm from a cannal into a cylindrical tank in his field, which is 10 m in diameter and 2m deep. If water flows through the pipe at the rate of 3 km/h, in how much time will the tank be filled?

Sol. Given: Diameter of the pipe = 20 cm

$\Rightarrow$ Radius of the pipe = 20/2 = 10 cm

Radius of tank = 5m = 500 cm and height = 2m = 200 cm

Length of water column per hour = 3 km = $3 \times 1000 \times 100$ cm

Volume of water flown in one hour = $\pi \times 100 \times 300000\ cm^3$

Tank to be filled = Volume of cylinder

$= \pi \times 500 \times 500 \times 200\ cm^3$

So, Time required to fill the tank = $\frac{Volume\ of\ tank}{Volume\ of\ water\ flown}$

$= \frac{\pi \times 500 \times 500 \times 200}{\pi \times 100 \times 300000}$ hrs

= 5/3hrs

= 1 hour 40 minutes

= 60 + 40 minutes

= 100 minutes.