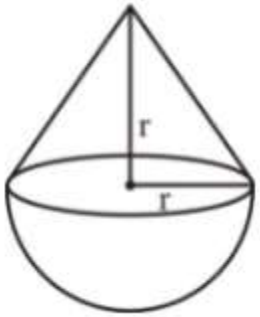
Surface Area and Volume: Exercise - 13.2

Q.1 A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π.

Sol. Given: radius = 1 cm, height of the cone is equal to its radius i.e. 1 cm



Volume of the solid = Volume of the cone + Volume of the hemisphere

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi R^3$$

Since, h = r and R = r

$$= \frac{1}{3}\pi r^2 \times r + \frac{2}{3}\pi r^3$$

$$= \frac{\pi}{3} r^3 (1 + 2)$$

$$= \frac{\pi}{3} r^3 \times 3$$

$$= \pi r^3$$

Since r = 1 cm

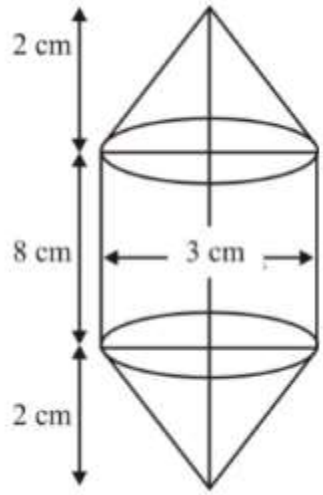
$$= \pi (1)^3$$

$$= \pi \text{ cm}^3$$

Q.2 Rachel an engineering student was asked to make a model shaped like a cylinder with two cones attached at its two ends by using thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model the Rachel made. (Assume the outer and inner dimensions of the model be nearly the same).

Sol. Given: Diameter of the model = 3 cm; length of diameter = 12 cm

Height of the cone = 2 cm



Therefore,

Volume (model) = Volume (cylindrical portion) + Volume (two conical ends)

$$= \pi r^2 h_1 + 2 \times \frac{1}{3} \pi r^2 h_2$$

$$= \pi r^2 (h_1 + \frac{2}{3} h_2)$$

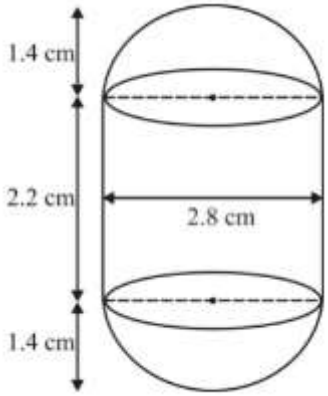
Since, r = $\frac{3}{2}$ cm; h_1 = 8cm and h_2 = 2 cm

$$= \frac{22}{7} \times (\frac{3}{2})^2 \times (8 + \frac{2}{3} \times 2) \text{ cm}^3$$

$$= 66 \text{cm}^3 \text{ (approx)}$$

Q.3 A gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemsipherical ends with length 5 cm and diameter is 2.8 cm (see figure).

Sol. Given: Number of gulabjamun = 45; length = 5 cm and diameter = 2.8 cm.



So,
Volume (one gulab jamun) = Volume (cylindrical portion) + Volume (hemispherical ends)

$$= \pi r^2 h + 2(\frac{2}{3} \pi r^3)$$

$$= \pi r^2 (h + \frac{4}{3} r),$$

Since, r = 1.4 cm, h = 2.2 cm

$$= \frac{22}{7} \times (1.4)^2 \times (2.2 + \frac{4}{3} \times 1.4) \text{ cm}^3$$

$$= \frac{22}{7} \times 1.96 \times (\frac{6.6 + 5.6}{3}) \text{ cm}^3$$

$$= \frac{22}{7} \times 1.96 \times \frac{12.2}{3} \text{ cm}^3$$

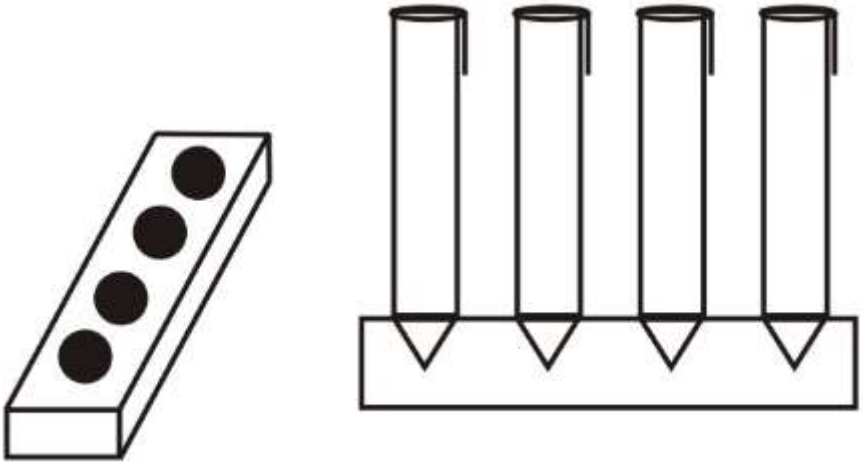
Now, volume of 45 gulab jamuns = $45\times\frac{22}{7}\times 1.96\times\frac{12.2}{3}$ cm3

According to question,
quantity of syrup in gulab jamuns = 30% of their volume

$$= \frac{30}{100}\times 45\times\frac{22}{7}\times 1.96\times\frac{12.2}{3}\ cm^3$$
$$= 338.184 cm3$$
$$= 338 cm^3 (approx)$$

Q.4 A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 m and the depth is 1.4 cm. Find the volume of wood in the entire stand (see figure).

Sol. Given: Dimension of cuboid = 15cm x 10 cm x 3.5 cm; radius of depression = 0.5 cm and depth = 1.4 cm

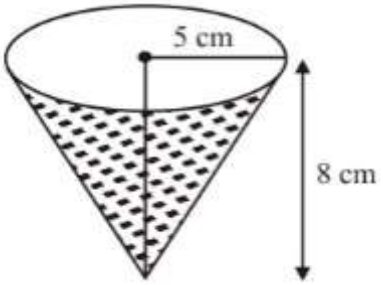


Volume (entire stand) = Volume (cuboid) – 4 × Volume (depression) (cone shaped)

$$= (15\times 10\times 3.5) - 4\times\left(\frac{1}{3}\times\frac{22}{7}\times 0.5\times 0.5\times 1.4\right) cm^3$$
$$= (525 - 1.47)\ cm^3$$
$$= 523.53\ cm^3$$

Q.5 A vessel in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one- fourth of the water flows out. Find the number of lead shots dropped in the vessel.

Sol. Given: Height of the conical vessel, h = 8 cm.
Its radius r = 5 cm



Volume (cone) = Volume (water in cone)

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 8 \text{ cm}^3$$

$$= \frac{4400}{21} \text{ cm}^3$$

Volume (water flows out) = Volume (lead shots)

$$= \frac{1}{4} \text{ of the volume of water in the cone}$$

$$= \frac{1}{4} \times \frac{4400}{21} \text{ cm}^3$$

$$= \frac{1100}{21} \text{ cm}^3$$

Since, radius of the lead shot = 0.5 cm

then, Volume of one spherical lead shot = $\frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times \frac{5}{10} \times \frac{5}{10} \times \frac{5}{10} \text{ cm}^3$$

$$= \frac{11}{21} \text{ cm}^3$$

Thus,

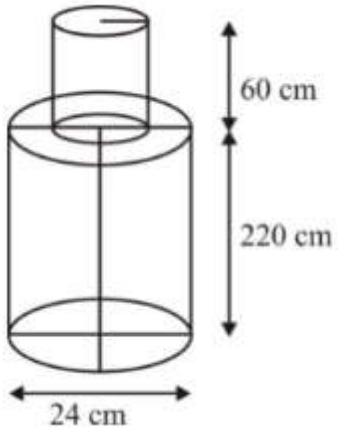
Number of lead shots dropped into the vessel = $\frac{\textit{Volume of water flows out}}{\textit{Volume of one lead shot}}$

$$= \frac{1100}{21} \div \frac{11}{21}$$

$$= \frac{1100}{21} \times \frac{21}{11} = 100$$

Q.6 A solid iron pole consists of a cylindrical height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm³ of iron has approximately 8 g mass. (Use π = 3.14)

Sol. Given: Height of first cylinder = 220 cm and diameter = 24 cm, Height of second surmounted cylinder = 60 cm and diameter = 8 cm;



Volume (solid iron pole) = Volume (cylindrical portion) + Volume (other cylindrical portion)

$= \pi r_1^2 h_1 + \pi r_2^2 h_2$

$= (3.14 \times (12)^2 \times 220 + 3.14 \times (8)^2 \times 60) \text{ cm}^3$

$= (3.14 \times 144 \times 220 + 3.14 \times 64 \times 60) \text{ cm}^3$

$= (99475.2 + 12057.6) \text{ cm}^3$

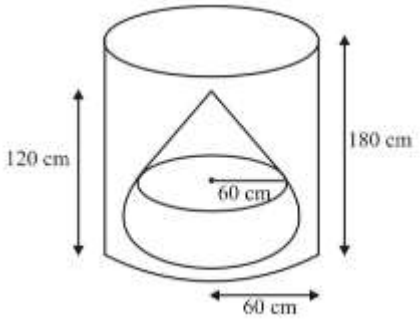
$= 111532.8 \text{ cm}^3$

Thus, the mass of the pole = (111532.8 × 8) grams

$$= \left(\frac{111532.8 \times 8}{1000}\right) \text{ kg} = 892.26 \text{ kg}$$

Q.7 A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water circular such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.

Sol. Given: height of cone = 120 cm; radius = 60 cm; radius of hemisphere = 60cm; radius of cylinder = 60 cm and height of cylinder = 180 cm



Since, volume of the cylinder = $\pi r^2 h$

$$= \frac{22}{7} \times (60)^2 \times 180 \text{ cm}^3$$

$$= \frac{22 \times 3600 \times 180}{7} \text{ cm}^3$$

$$= \frac{14256000}{7} \text{ cm}^3$$

Volume (solid) = Volume (cone) + Volume (hemisphere)

$$= \left(\frac{1}{3} \times \frac{22}{7} \times 60^2 \times 120 + \frac{2}{3} \times \frac{22}{7} \times 60^3\right) \text{cm}^3$$

$$= \left(\frac{3168000}{7} + \frac{3168000}{7}\right) \text{cm}^3$$

$$= \frac{6336000}{7} \text{ cm}^3$$

Volume (water left in the cylinder) = Volume (cylinder) – Volume (solid)

$$= \left(\frac{14256000}{7} - \frac{6336000}{7}\right) \text{cm}^3$$

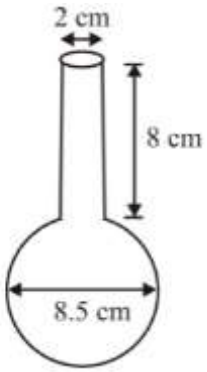
$$= \left(\frac{7920000}{7}\right) \text{cm}^3$$

$$= 1131428.57142 \text{ cm}^3$$

$$= 1.131 \text{m}^3 \text{ (approx)}$$

Q.8 A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter ; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm³. Check whether she is correct, taking the above as the inside measurements and π=3.14

Sol.



Volume (spherical vessel) = $\pi (1)^2 (8) + \frac{4}{3} \pi (4.25)^3$

$= 3.14 [8 + \frac{4}{3} \times 76.765625]$

$= 3.14 [8 + 102.353]$

$= 3.14 \times 110.353$

$= 346.51$

Thus, here answer is incorrect. Correct answer is $346.51 cm^3$