Surface Area and Volume: Exercise - 13.1

Q.1 2 cubes each of volume 64cm³ are joined end to end. Find the surface area of the resulting cuboid.

Sol. Let b cm be the length of each edge of the cube.

So, Volume = $64cm^3$

$\Rightarrow a^3 = 64$

$\Rightarrow a = 4$

According to question, when two cubes of equal volumes (i.e., equal edges) are joined end to end, we get a cuboid such that its.

l = Length = 4cm + 4cm = 8 cm

b = Breadth = 4 cm

and h = Height = 4 cm

So, Surface area of the cuboid = 2(lb + bh + hl)

$= 2(8 \times 4 + 4 \times 4 + 4 \times 8)\ cm^2$

$= 2(32 + 16 + 32)\ cm^2$

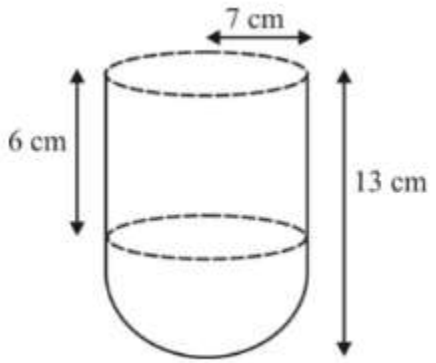
$= (2 \times 80)\ cm^2 = 160\ cm^2$

Q.2 A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.

Sol. Given: Radius of hemisphere, r = 7 cm,

Height of cylinder, h = (13 – 7) cm = 6 cm

So, radius of the base of cylindrical part is also r cm.



Surface area (vessel) = Curved surface area (cylindrical part) + Curved surface area (hemispherical part)

$= (2\pi rh + 2\pi r^2)\ cm^2$

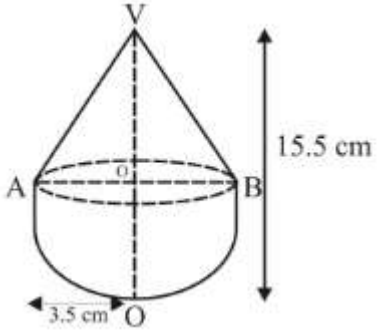
$= 2\pi r\ (h + r)\ cm^2$

$= 2 \times \frac{22}{7} \times 7 \times 13\ cm^2$

$= 572\ cm^2$

Q.3 A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

Sol. Figure according to question:



Given: VO = 15.5 cm, OA = OO' = 3.5 cm
Radius of the base of cone, r = OA = 3.5 cm
Height of conical part of the toy, h = VO = VO' – OO
= (15.5 – 3.5) cm = 12 cm
And radius of the hemisphere = OA = r = 3.5 cm

$$\text{Slant height, } \ell = \sqrt{OA^2 + OV^2}$$
$$= \sqrt{(3.5)^2 + 12^2}$$
$$= 12.5\text{cm}$$

Total surface area of the toy = Curved surface area of cone + Curved surface area of hemisphere
$$= \pi r\ell + 2\pi r^2$$
$$= \frac{22}{7} \times 3.5 \times (12.5 + 2\times 3.5)\ \text{cm}^2$$
$$= \frac{22}{7} \times 3.5 \times 19.5\text{cm}$$
$$= 214.5\text{cm}^2$$

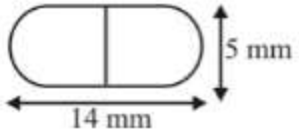
Q.4 A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter of the hemisphere can have? Find the surface area of the solid?

Sol. Given: The greatest diameter of a hemisphere can have = 7 cm.
Surface area of the solid after surmounted hemisphere
$$= 6\ell^2 - \pi R^2 + 2\pi R^2$$
$$= 6\ell^2 + \pi R^2$$
$$= 6(7)^2 + \frac{22}{7} \times \left(\frac{7}{2}\right)^2$$
$$= 6 \times 49 + 11 \times \frac{7}{2}$$
$$= 294 + 38.5 = 332.5\ \text{cm}^2$$

Q.5 A hemispherical depression is cut of one face of a cubical wooden block such the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

Sol. Let, Edge of the cube = ℓ
Then, diameter of the hemisphere = ℓ
So, Radius of the hemisphere = $\ell/2$
Area of the remaining solid after cutting out the hemispherical depression.
$$= 6\ell^2 - \pi\left(\frac{\ell}{2}\right)^2 + 2\pi\left(\frac{\ell}{2}\right)^2$$
$$= 6\ell^2 + \pi\left(\frac{\ell}{2}\right)^2$$
$$= 6\ell^2 + \pi \times \frac{\ell^2}{4} = \frac{\ell^2}{4}(24 + \pi)$$

Q.6 A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends (see figure). The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area.

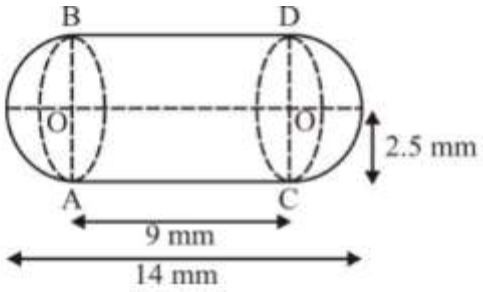


Sol. Given: Radius, r = $\frac{5}{2}$ mm = 2.5mm

and $h = (14 - 2\times\frac{5}{2})$ mm

$= (14 - 5)$mm $= 9$mm

Radius of hemisphere, $r = \frac{5}{2}$ cm



Now, surface area (capsule) = Curved surface (cylinder) + Surface area (two hemispheres)

$= (2\pi rh + 2\times 2\pi r^2)$ mm^2

$= 2\pi r\,(h + 2r)$ mm^2

$= 2 \times \frac{22}{7} \times \frac{5}{2} \times (9 + 2\times\frac{5}{2})$ mm^2

$= \frac{22}{7} \times 5 \times 14$ mm^2

$= 220$ mm^2

Q.7 A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of Rs 500 per m^2. (Note that the base of the tent will not covered with canvas).

Sol. Given: Height of cylindrical part, $h = 2.1$ m; Diameter of the cylinder part = 4 m; slant height, $\ell = 2.8$ m

We need to find out,

Total canvas used = Curved surface area of cylinder + Curved surface area of cone

$= (2\pi rh + \pi r\ell)$ m^2

$= \pi r\,(2h + \ell)$ m^2

$= \frac{22}{7} \times 2 \times (2 \times 2.1 + 2.8)$ m^2

$= \frac{22}{7} \times 2 \times 7$ m^2

$= 44$ m^2

Since, cost of 1m^2 the canvas for the tent = Rs 500

Therefore, cost of 44 m^2 the canvas for the tent = Rs 44×500 = Rs 22000

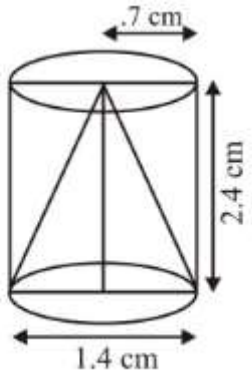
Q.8 From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2.

Sol. Given: Radius of the cylinder, $r = \frac{1.4}{2}$ cm $= 0.7$cm

Height of the cylinder, $h = 2.4$ cm

Radius of the cone, $r' = 0.7$ cm

Height of the cone, $h' = 2.4$ cm



Therefore, slant height of the cone = $\sqrt{(0.7)^2 + (2.4)^2}$ cm

$= \sqrt{0.49+5.76}$ cm

= 2.5 cm

Surface area (remaining solid) = Curved surface area (cylinder) + Curved surface (cone) + Area (upper circular base of cylinder)

$= 2\pi rh + \pi r\ell + \pi r^2$

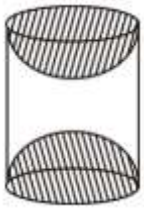
$= \pi r (2h + \ell + r)$

$= \frac{22}{7} \times 0.7 \times (2 \times 2.4 + 2.5 + .7)$ cm²

$= 2 \times 0.1 \times (4.8 + 2.5 + .7)$ cm²

$= 0.2 \times 8.0$ cm²

Q.9 A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in figure. If the height of the cylinder is 10 cm, and its base is of radius 3.5 m, find the total surface area of the article.



Sol. Given: height of the cylinder = 10 cm, radius = 3.5 m

So,

Surface area (article) = Curved surface area (cylinder) + 2 × Curved surface area (hemisphere).

$= 2\pi rh + 2 \times 2\pi r^2$

$= 2\pi r (h + 2r)$

$= 2 \times \frac{22}{7} \times 3.5 \times (10 + 2 \times 3.5)$ cm²

$= 2 \times \frac{22}{7} \times 3.5 \times 17$cm²

= 374 cm²