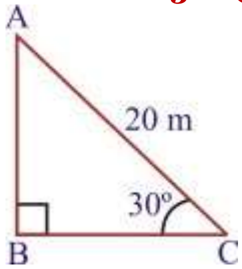
Applications of Trigonometry: Exercise 9.1

Q.1 A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30° (see figure).



Sol. From the figure, we can see that triangle ABC is a right angled triangle. In which ∠C = 30°, and
sin30° = p/h = AB/AC

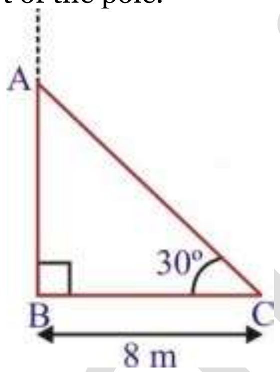
AB / 20 = 12

⇒ AB = 12 × 20 = 10

Thus, Height of the pole is 10 m.

Q.2 A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8m. Find the height of the tree.

Sol. According to statement, Figure shows the given situation, where AC shows broken part and AB is the remaining part of the pole.



In figure, triangle ABC is a right angled triangle. So, in ΔABC

tan30° = AB/BC

$AB / 8 = \frac{1}{\sqrt{3}}$

$\Rightarrow AB = 8 \text{ x } \frac{1}{\sqrt{3}}$ .... (i)

Now, again in ΔABC,

sec30° = AC / BC

$AC / 8 = \frac{2}{\sqrt{3}}$

$\Rightarrow AC = \frac{2}{\sqrt{3}} \times 8 = \frac{16}{\sqrt{3}}$ .... (ii)

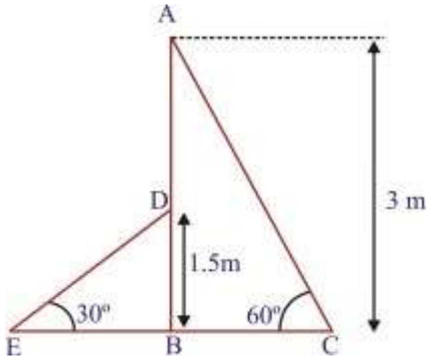
So, height of the tree = AB + AC

$= \frac{8}{\sqrt{3}} + \frac{16}{\sqrt{3}} = \frac{24}{\sqrt{3}}$

$= 8\sqrt{3}$ m

Q.3 A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m, and is inclined at an angle of 30° to the ground, whereas for elder children, she wants to have a steep slide at a height of 3m, and inclined at an angle of 60° to the ground. What should be the length of the slide in each case?

Sol. According to statement, Figure shows given situation, where ED shows slide for the children below the age of 5 year and AC shows the slides for elder children.



In right angled ΔBDE,

$\operatorname{cosec} 30^\circ = h/p = DE/BD$

$DE/1.5 = 2$

$\Rightarrow DE = 2 \times 1.5 = 3.0$

Now, in right angled ΔABC,

$\operatorname{cosec} 60^\circ = AC/AB$

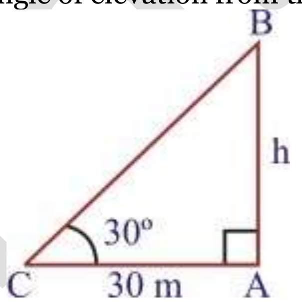
$AC/3 = \dfrac{2}{\sqrt{3}}$

$\Rightarrow AC = \dfrac{2}{\sqrt{3}} \times 3 = 2\sqrt{3}$

Thus, the length of slides are 3m and $2\sqrt{3}$ m.

Q.4 The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower, is 30°. Find the height of the tower.

Sol. Suppose, AB is the tower of height h m and C is a point on ground at a distance of 30 m from the foot of the tower. The angle of elevation from the point C is given as 30°.



In right angled ΔCAB,

$\tan 30^\circ = AB/CA$

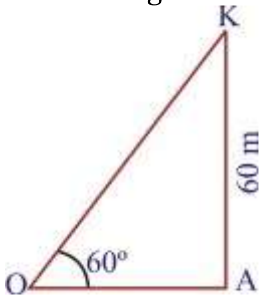
$\Rightarrow h/30 = \dfrac{1}{\sqrt{3}}$

$\Rightarrow h = \dfrac{30}{\sqrt{3}} = 10\sqrt{3}$

Thus, the height of the tower is $10\sqrt{3}$ m.

Q.5 A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.

Sol. Let K be the position of the kite at height 60 m above the ground and let OK = x m be length of the string. The inclination of the string with the ground is 60° i.e. ∠KOA = 60°



Now, in right angled ΔAOK,

$\sin 60^\circ = P/h = AK/OK$

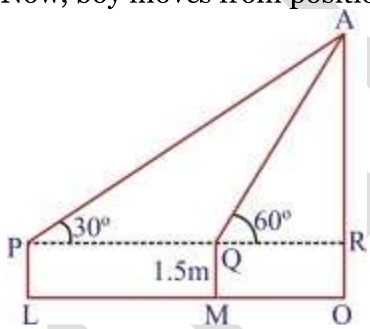
$\Rightarrow 60/x = \dfrac{\sqrt{3}}{2}$

$\Rightarrow x = 60 \times \dfrac{2}{\sqrt{3}} = \dfrac{120}{\sqrt{3}} = 40\sqrt{3}$

Thus, the length of the string is $40\sqrt{3}$ m.

Q.6 A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

Sol. Let PL be the initial position of the man of height 1.5m and OA be the building of height 30m such that ∠APR=30°. Now, boy moves from position PL to MQ such that ∠AQR=60°.



In right angled triangle ΔARQ,

$\cot 60^\circ = b / P = QR / AR$

$\Rightarrow QR / AR = \dfrac{1}{\sqrt{3}}$

$\Rightarrow QR = AR\ \dfrac{1}{\sqrt{3}} = \sqrt{3}$ .... (i)

Now in right angled triangle ΔARP

$\cot 30^\circ = b / P = PR/AR$

$\Rightarrow PR/AR = \sqrt{3}$

$\Rightarrow PR = \sqrt{3}\,AR$ .... (ii)

From (i) and (ii),

$PQ = PR - QR$

$= \sqrt{3}\,AR - \dfrac{AR}{\sqrt{3}}$

$= \dfrac{(3-1)AR}{\sqrt{3}}$

$$= \frac{2\sqrt{3}AR}{3}$$

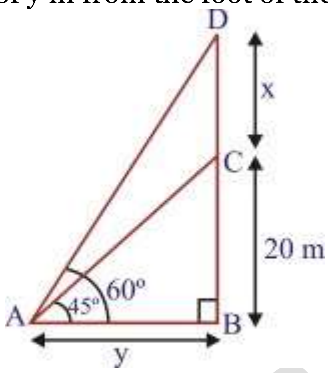
Since, AR = 30 – 1.5 = 28.5 m

$$= \frac{2\sqrt{3} \text{ x } 28.5}{3} = 19\sqrt{3}$$

Thus, the distance walked by the man towards the building is $19\sqrt{3}$.

Q.7 From a point on the ground the angles of elevation of the bottom and top of a tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.

Sol. Suppose, BC is the building of height 20 m and CD is the tower of height x m and A is a point on the ground at a distance of y m from the foot of the building.



In right angled triangle ΔABC,

$\tan 45^\circ = P/b = BC/AB$

$\Rightarrow 20/y = 1$

$\Rightarrow y = 20$

AB = 20 m

Now, in right angled triangle ΔABD,

$\tan 60^\circ = P/b = BD/AB$

$$\Rightarrow \frac{20+x}{20} = \sqrt{3}$$

$\Rightarrow 20+x = 20\sqrt{3}$

$\Rightarrow x = 20(\sqrt{3} - 1)$

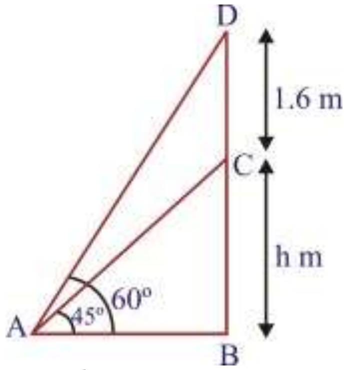
$\Rightarrow x = 20(1.732 - 1)$

$\Rightarrow = 20 \times .732 = 14.64$

Thus, the height of the tower is 14.64 m.

Q.8 A statue 1.6 m tall stands on the top a pedestal. From a point on the ground the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.

Sol. Suppose, BC be the pedestal of height h m and CD is the statue of height 1.6 m and A is a point on the ground such that ∠CAB=45° and ∠DAB = 45°



In right angled triangle ΔABC,

$\cot 45^o = b / P = AB / BC$

$\Rightarrow AB / h = 1$

$\Rightarrow AB = h$ ... (i)

Now in right angled triangle,

$\tan 60^o = BD / AB$

$$\Rightarrow \frac{BC + CD}{AB} = \sqrt{3}$$

$$\Rightarrow \frac{h + 1.6}{h} = \sqrt{3}$$

$\Rightarrow h + 1.6 = h\sqrt{3}$

$\Rightarrow h(\sqrt{3} - 1) = 1.6$

$$\Rightarrow h = \frac{1.6}{\sqrt{3} - 1}$$

$$= \frac{1.6}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$

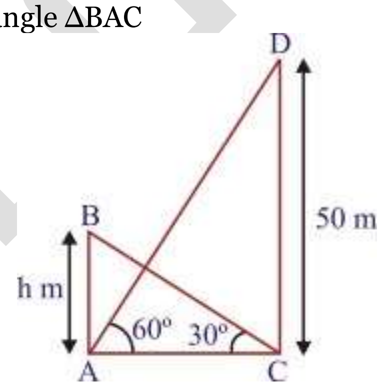
$= 0.8(\sqrt{3} + 1)$

Thus, the height of the pedestal is $0.8(\sqrt{3} + 1)$ m.

Q.9 The angle of elevation of the top of the building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.

Sol. Suppose AB is the building of height h m and CD is the tower of height 50 m. Since, the building subtends an angle of 30° at C i.e. ∠ACB=30° and Tower subtends angle of 60° i.e. ∠CAB=60°.

In right angled triangle ΔBAC



$\cot 30^o = B/P = AC/AB$

$\Rightarrow AC / h = \sqrt{3}$

$\Rightarrow AC = \sqrt{3}h$ .... (i)

Now in right angled triangle ΔDCA

$DC/AC = \tan 60^o$

$\Rightarrow 50/AC = \sqrt{3}$

$\Rightarrow AC = 50\sqrt{3}$ .... (ii)

By equating the values of AC,

$\sqrt{3}\ h = 50 / \sqrt{3}$

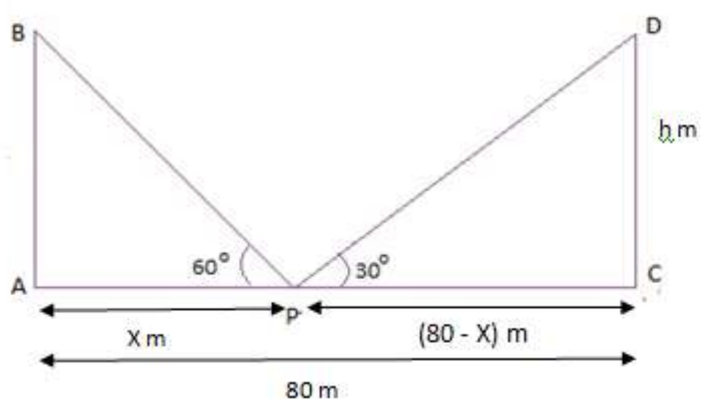
$\Rightarrow h = \dfrac{50}{\sqrt{3}} \times \dfrac{1}{\sqrt{3}}$

$= 50/3$

Thus, the height of the building is 50/3 m.

Q.10 Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angle of elevation of the top of the poles are 60° and 30° respectively. Find the height of the poles and the distances of the point from the poles.

Sol. Suppose, AB and CD are the two poles of same height of h m and distance between them is 80m. Point P is a point on the road such that AP = x m and CP = (80 – x) m. Poles AB and CD make angle of elevation of the top of the poles are 60° and 30° i.e. ∠APB=60° and ∠CPD=30°



In right angled triangle Δ APB,

tan60° = P/B = AB/AP

$\Rightarrow h / x = \sqrt{3}$

$\Rightarrow h = \sqrt{3}\,x$ .... (i)

Now, in right angled triangle ΔCPD,

tan30° = P/B = CD/CP

$\Rightarrow \dfrac{h}{80-x} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow h = \dfrac{80-x}{\sqrt{3}}$ .... (ii)

By equating the values of h,

$\sqrt{3}\,x = \dfrac{80-x}{\sqrt{3}}$

$\Rightarrow 3x = 80 - x$

$\Rightarrow 4x = 80$

$\Rightarrow x = 20$

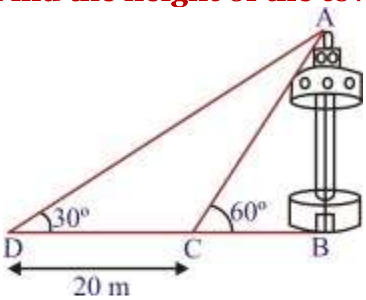
Putting x = 20 in equation (i),

$h = \sqrt{3} \times 20 = (1.732) \times 20 = 34.64$

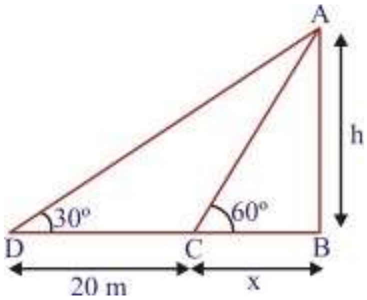
Hence, the required point P is at a distance of 20 m from the first pole and 60 m from the second pole. Thus, height of the pole is 34.64 m.

Q.11 A T.V. tower stands vertically on a bank of a river. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From a point

20 m away from this point on the same bank, the angle of elevation of the top of the tower is 30° (see figure). Find the height of the tower and the width of the river.



Sol. Suppose, AB is the T.V. tower of height h m standing on the bank of a river and C is the point on the opposite bank of the river such that BC = x. Point D is another point away from C such that CD = 20, and the angles of elevation of the top of the T.V. tower at C and D are 60° and 30° respectively i.e., $\angle ACB = 60^\circ$ and $\angle AOB = 30^\circ$



In right angled triangle Δ ABC,

tan60° = P/B = ABBC

$\Rightarrow h / x = \sqrt{3}$

$\Rightarrow h = \sqrt{3}\,x$........... (i)

Now, in right angled Δ ABD,

tan30° = P/B = AB/BD

$$\Rightarrow \frac{h}{x+20} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow h = \frac{20+x}{\sqrt{3}} \text{............ (ii)}$$

By equating values of h,

$$\sqrt{3}\,x = \frac{20+x}{\sqrt{3}}$$

$\Rightarrow 3x = x + 20$

$\Rightarrow 3x - x = 20$

$\Rightarrow 2x = 20$

$\Rightarrow x = 10$

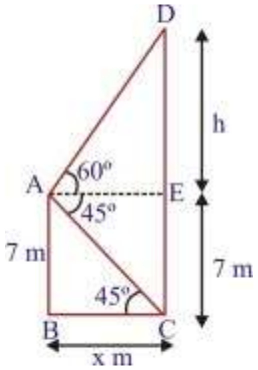
Now, putting x = 10 in (i),

$h = \sqrt{3} \times 10 = (1.732) \times 10 = 17.32$

Hence, the height of the T.V. tower is 17.32 m and the width of the river is 10 m.

Q.12 From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

Sol. Suppose, AB is the building of height 7 m and CD is the cable tower. It is given that the angle of elevation of the top D of the tower observed from A is 60° and the angle of depression of the base C of the tower observed from A is 45° i.e. $\angle EAD = 60^\circ$ and $\angle BCA = 45^\circ$.



In right angled triangle Δ EAD,

$\tan 60^o = P/B = DE/EA$

$\Rightarrow h / x = \sqrt{3}$

$\Rightarrow h = \sqrt{3}\, x$ .... (i)

Now, in Δ ABC,

$\tan 45^o = P/B = AB/BC$

$\Rightarrow 7/x = 1$

$\Rightarrow x = 7$ .... (ii)

By putting x = 7 in (i),

$h = 7\sqrt{3}$

$\Rightarrow DE = 7\sqrt{3}$ m

Thus, CD = CE + ED

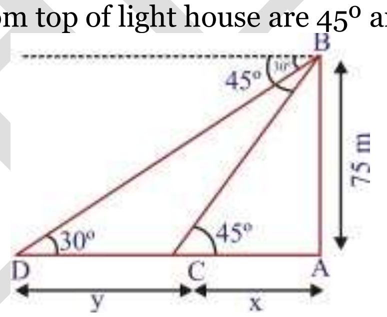
$= (7+7\sqrt{3})$ m

$= 7(\sqrt{3}+1)$ m

Therefore, the height of the cable tower is 19.124 m.

Q.13 As observed from the top of a 75 m tall lighthouse, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.

Sol. Suppose, AB is the lighthouse of height 75 m and two ships are at position on C and D such that the angles of depression from top of light house are 45° and 30° respectively as shown in figure.



Let AC = x and CD = y and $\angle BCA = 45^o$ and $\angle BDA = 30^o$

In right angled triangle Δ ABC,

$\tan 45^o = B/P = AB/AC$

$\Rightarrow 75/x = 1$

$\Rightarrow x = 75$ .... (i)

Now, in right angled ΔABD,

$\tan 30^o = AB/AD$

$$\Rightarrow \frac{75}{x+y} = \frac{1}{\sqrt{3}}$$

$\Rightarrow x+y = 75\sqrt{3}$ .... (ii)

From (i) and (ii),

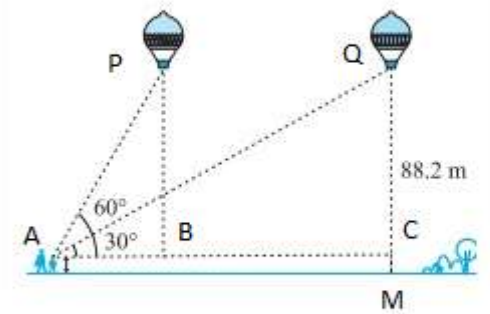
$75+y = 75\sqrt{3}$

$\Rightarrow y = 75(\sqrt{3} - 1)$

$\Rightarrow y = 75\,(1.732 - 1) = 75 \times 0.732 = 54.9$

Thus, the distance between the two ships is 54.9 m.

Q.14 A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°.

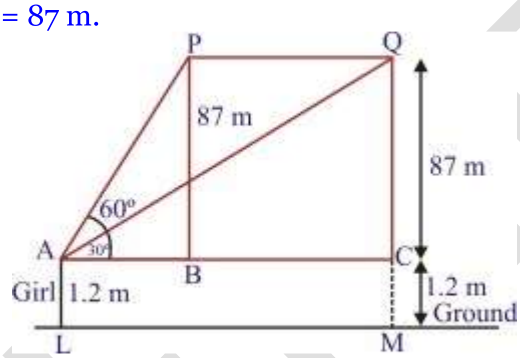


After some time, the angle of elevation reduces to 30°. Find the distance travelled by the balloon during the interval.

Sol. Suppose, P and Q are the two positions of the balloon and A is the point of observation. ABC is the horizontal through A. It is given that angles of elevation of the balloon in two position P and Q from ∠PAB = 60°, ∠QAB = 30° respectively. Since, MQ = 88.2 m

So, CQ = MQ – MC = (88.2 – 1.2) m

= 87 m.



⇒ In right angled triangle, Δ ABP,

tan60° = P/B = BP /AB

$\Rightarrow 87/AB = \sqrt{3}$

$\Rightarrow AB = \dfrac{87}{\sqrt{3}} = 29\sqrt{3}$ .... (i)

Now, in ΔACQ,

tan30° = P/B = CQ/AC

$\Rightarrow 87/AC = \dfrac{1}{\sqrt{3}}$

$\Rightarrow AC = 87\sqrt{3}$ .... (ii)

Therefore, PQ = BC = AC – AB

$= (873\sqrt{3} - 29\sqrt{3})$ m

$= 58\sqrt{3}$ m

Hence, the balloon travels $58\sqrt{3}$ m.

Q.15 A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six minutes later, the angle of depression of the car is found to be 60°.

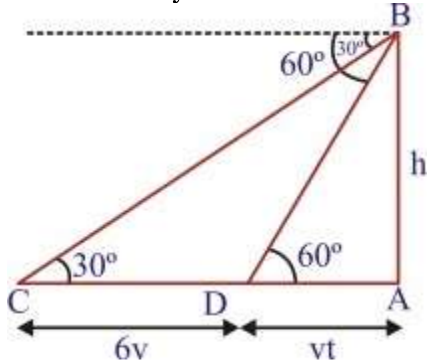
Find the time taken by the car to reach the foot of the tower.

Sol. Suppose, AB is the tower of height h m. C is the initial position of the car and after 6 min the car be at position D. It is given that the angles of depression at C and D are 30° and 60° respectively. CD will be distance travelled by the car in 6 min and Let v m/min be the speed of the car. Then,

Since, Distance = Speed × Time

$\Rightarrow$ CD = 6υ m

Let t min. be time taken by car to reach the tower from D. Then, DA = υt m



In right angled triangle Δ ABD,

$\tan 60^\circ = P/B = AB/AD$

$\Rightarrow h/\upsilon t = \sqrt{3}$

$\Rightarrow h = \sqrt{3}\,\upsilon t$ .... (i)

Now, in right angled triangle Δ ABC,

$\tan 30^\circ = P/B = AB/AC$

$\Rightarrow \dfrac{h}{vt+6v} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \sqrt{3}\,h = \upsilon t + 6\upsilon$ .... (ii)

By putting the value of h from (i) in (ii),

$\sqrt{3} \times \sqrt{3}\,\upsilon t = \upsilon t + 6\upsilon$

$\Rightarrow 3vt = vt + 6v$

$\Rightarrow 3vt - vt = 6$

$\Rightarrow 2vt = 6v$

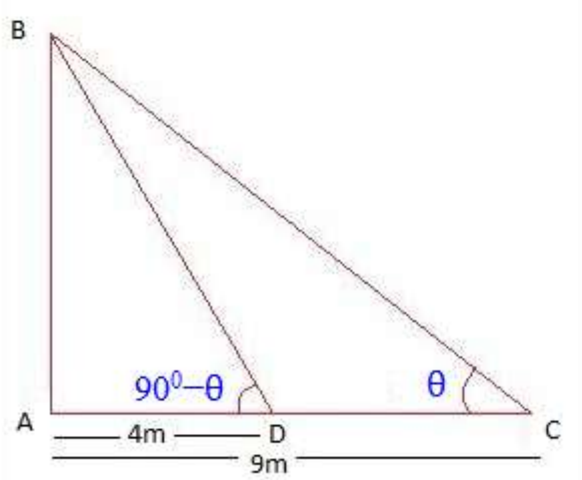
$\Rightarrow t = 3$

Therefore, the car will reach the tower from D in 3 minutes.

Q.16 The angle of elevation of the top of a tower from two points at a distance of 4 m and 9m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.

Sol. Suppose, AB is the tower of height h m. C and D are the two points at distances 9 m and 4 m respectively from the base of the tower. So, AC = 9 m and AD = 4m.

Let $\angle ACB = \theta$ and $\angle ADB = 90^\circ - \theta$



In right angled triangle Δ ACB,

$\tan\theta = P/B = AB/AC$

$\Rightarrow h/9 = \tan\theta$ .... (i)

Now, in right angled Δ ADB,

$\tan(90^\circ - \theta) = AB/AD$

$\Rightarrow h/4 = \cot\theta$ .... (ii)

From (i) and (ii),

$$\frac{h}{9} \times \frac{h}{4} = \tan\theta \times \cot\theta$$

Since, $\tan\theta = 1/\cot\theta$

$$\Rightarrow \frac{h^2}{36} = 1$$

$\Rightarrow h^2 = 36$

$\Rightarrow h = 6$

Thus, the height of the tower is 6 m.