

Real Numbers: Exercise 1.2

Q.1 Express each number as product of its prime factors:

(i) 140 (ii) 156 (iii) 3825 (iv) 5005 (v) 7429

Sol. Number as product of its prime factors:

(i) We use the division method:

$$\begin{array}{r} 2 \overline{)140} \\ 2 \overline{)70} \\ 5 \overline{)35} \\ 7 \overline{)7} \\ 1 \end{array}$$

$$\text{Thus, } 140 = 2 \times 2 \times 5 \times 7 = 2^2 \times 5 \times 7$$

(ii) We use the division method:

$$\begin{array}{r} 2 \overline{)156} \\ 2 \overline{)78} \\ 3 \overline{)39} \\ 13 \overline{)13} \\ 1 \end{array}$$

$$\text{Thus, } 156 = 2 \times 2 \times 3 \times 13 = 2^2 \times 3 \times 13$$

(iii) We use the division method:

$$\begin{array}{r} 3 \overline{)3825} \\ 3 \overline{)1275} \\ 5 \overline{)425} \\ 5 \overline{)85} \\ 17 \overline{)17} \\ 1 \end{array}$$

$$\text{Thus, } 3825 = 3 \times 3 \times 5 \times 5 \times 17 = 3^2 \times 5^2 \times 17$$

(iv) We use the division method:

$$\begin{array}{r} 5 \overline{)5005} \\ 7 \overline{)1001} \\ 11 \overline{)143} \\ 13 \overline{)13} \\ 1 \end{array}$$

$$\text{Thus, } 5005 = 5 \times 7 \times 11 \times 13$$

(v) We use the division method:

$$\begin{array}{r} 17 \overline{)7429} \\ 19 \overline{)437} \\ 23 \overline{)23} \\ 1 \end{array}$$

$$\text{Thus, } 7429 = 17 \times 19 \times 23$$

Q.2 Find the LCM and HCF of the following pairs of integers and verify that $\text{LCM} \times \text{HCF} = \text{product of the two numbers}$.

(i) 26 and 91 (ii) 510 and 92 (iii) 336 and 54

Sol. (i) 26 and 91

$$\begin{array}{r|l} 2 & 26 \\ 13 & 13 \\ \hline & 1 \end{array} \quad \begin{array}{r|l} 7 & 91 \\ 13 & 13 \\ \hline & 1 \end{array}$$

The number as product of its prime factors:

$$26 = 2 \times 13 \quad \text{and} \quad 91 = 7 \times 13$$

Therefore, $\text{LCM of } 26 \text{ and } 91 = 2 \times 7 \times 13 = 182$

and $\text{HCF of } 26 \text{ and } 91 = 13$

Now, $\text{Product of LCM and HCF} = 182 \times 13 = 2366$

And $\text{Product of number} = 26 \times 91 = 2366$

from the above result, $\text{LCM} \times \text{HCF} = \text{product of the two numbers}$

Hence verified.

(ii) 510 and 92

$$\begin{array}{r|l} 2 & 510 \\ 3 & 255 \\ 5 & 85 \\ 17 & 17 \\ \hline & 1 \end{array} \quad \begin{array}{r|l} 2 & 92 \\ 2 & 46 \\ 23 & 23 \\ \hline & 1 \end{array}$$

The number as product of its prime factors:

$$510 = 2 \times 3 \times 5 \times 17 \quad \text{and} \quad 92 = 2 \times 2 \times 23$$

Therefore, $\text{LCM of } 510 \text{ and } 92 = 2 \times 2 \times 3 \times 5 \times 17 \times 23 = 23460$

and $\text{HCF of } 510 \text{ and } 92 = 2$

Now, $\text{Product of LCM and HCF} = 23460 \times 2 = 46920$

And $\text{Product of number} = 510 \times 92 = 46920$

from the above result, $\text{LCM} \times \text{HCF} = \text{product of the two numbers}$

Hence verified.

(iii) 336 and 54

$$\begin{array}{r|l} 2 & 336 \\ 2 & 168 \\ 2 & 84 \\ 2 & 42 \\ 3 & 21 \\ 7 & 7 \\ \hline & 1 \end{array} \quad \begin{array}{r|l} 2 & 54 \\ 3 & 27 \\ 3 & 9 \\ \hline & 1 \end{array}$$

The number as product of its prime factors:

$$336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7 \quad \text{and} \quad 54 = 2 \times 3 \times 3$$

Therefore, $\text{LCM of } 336 \text{ and } 54 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 7 = 3024$

and $\text{HCF of } 336 \text{ and } 54 = 2 \times 3 = 6$

Now, $\text{Product of LCM and HCF} = 3024 \times 6 = 18144$

And $\text{Product of number} = 336 \times 54 = 18144$

from the above result, $\text{LCM} \times \text{HCF} = \text{product of the two numbers}$

Hence verified.

Q.3 Find the LCM and HCF of the following integers by applying the prime factorisation method.

(i) 12, 15 and 21 (ii) 17, 23 and 29 (iii) 8, 9 and 25

Sol. (i) Firstly we write the prime factorisation of the given numbers.

$$12 = 2 \times 2 \times 3 = 2^2 \times 3,$$

$$15 = 3 \times 5$$

$$21 = 3 \times 7$$

$$\text{So, LCM} = 2^2 \times 3 \times 5 \times 7 = 420$$

$$\text{and, HCF} = 3$$

(ii) Firstly, we write the prime factorisation of the given numbers.

$$17 = 17$$

$$23 = 23$$

$$29 = 29$$

$$\text{So, LCM} = 17 \times 23 \times 29 = 11339$$

$$\text{and HCF} = 1$$

(iii) Firstly, we write the prime factorisation of the given numbers.

$$8 = 2 \times 2 \times 2 = 2^3$$

$$9 = 3 \times 3 = 3^2$$

$$25 = 5 \times 5 = 5^2$$

$$\text{Therefore, LCM} = 2^3 \times 3^2 \times 5^2$$

$$= 8 \times 9 \times 25 = 1800$$

$$\text{and HCF} = 1$$

Q.4 Given that HCF (306, 657) = 9, find LCM (306, 657).

Sol. Since, the product of the HCF and the LCM of two numbers is equal to the product of the given numbers.

Product of the HCF and the LCM of the two number = The product of the two numbers

$$\text{So, HCF (306, 657)} \times \text{LCM (306, 657)} = 306 \times 657$$

$$\Rightarrow 9 \times \text{LCM (306, 657)} = 306 \times 657$$

$$\Rightarrow \text{LCM (306, 657)} = 306 \times 657 / 9 = 22338$$

$$\text{Thus, LCM (306, 657)} = 22338$$

Q.5. Check whether 6^n can end with the digit 0 for any natural number n .

Sol. If the number 6^n , for any n natural number ends with the digit zero, then it should be divisible by 5. The prime factorisation of 6^n should contain the prime number 5. That is not possible as the only prime in the factorisation of 6^n is 2 and 3.

$$6^n = (2 \times 3)^n$$

The uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes numbers in the factorisation of 6^n . So, there is no $n \in \mathbb{N}$ for which 6^n ends with the digit zero.

Q.6 Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

Sol. According to definition of composite number, if a number is composite, then it means it has factors other than 1 and itself.

Since, $7 \times 11 \times 13 + 13$

$$= 13 \times (7 \times 11 \times 1 + 1) \quad (\text{since by taking 13 common})$$

$$= 13 \times (77 + 1)$$

$$= 13 \times 78$$

So, It is a composite number.

Again, $7 \times 6 \times 5 \times 4 \times 3 \times 1 \times 1 \times 1 + 5$

$$= 5 \times (7 \times 6 \times 4 \times 3 \times 1 \times 1 + 1)$$

So, It is a composite number.

Q.7 There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the path, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they again at the starting point?

Sol. Sonia and Ravi move in the same direction and at the same time. The time when they will be meeting again at the starting point is the LCM of 18 and 12. To find the LCM of 18 and 12, we write the prime factorisation of the given numbers:

$$\begin{array}{r} 2 \overline{)18} \end{array}$$

$$\begin{array}{r} 2 \overline{)12} \end{array}$$

$$\begin{array}{r} 3 \overline{)9} \end{array}$$

$$\begin{array}{r} 2 \overline{)6} \end{array}$$

$$\begin{array}{r} 3 \overline{)3} \end{array}$$

$$\begin{array}{r} 3 \overline{)3} \end{array}$$

$$\begin{array}{r} \underline{1} \end{array}$$

$$\begin{array}{r} \underline{1} \end{array}$$

$$18 = 2 \times 3 \times 3 \text{ and}$$

$$12 = 2 \times 2 \times 3$$

$$\text{LCM of 18 and 12} = 2 \times 2 \times 3 \times 3 = 36$$

So, Sonia and Ravi will meet again at the starting point after 36 minutes.