Real Numbers: Exercise 1.1

Q.1. Use Euclid's division algorithm to find the HCF of:
(i) 135 and 225 (ii) 196 and 38220 (iii) 867 and 255

Sol. **(i) 135 and 225**

Since, $225 > 135$

By applying Euclid's division lemma on 135 and 225.

We get

$225 = 135 \times + 90$

Here, remainder $90 \neq 0$, so we again apply EDL on divisor 135 and remainder 90.

$135 = 90 \times 1 + 45$

Again, remainder $45 \neq 0$, so we apply Euclid's again division lemma on divisor 90 and remainder 45.

We get,

$90 = 45 \times 2 + 0$

Here, remainder = 0

So the divisor at this stage is 45, then HCF (135, 225) = 45

(ii) 196 and 38220

Since, $38220 > 196$ so applying Euclid's division lemma,

We get,

$38220 = 196 \times 195 + 0$

At this stage remainder is zero. So, the divisor of this stage (196) is HCF of 38220 and 196.

So, HCF (196, 38220) = 196

(iii) 867 and 255

Since, $867 > 255$ so, applying Euclid's division lemma

We get,

$867 = 255 \times 3 + 102$

Here, remainder $102 \neq 0$ so, we again apply Euclid's division algorithm on divisor 255 and remainder 102.

We get,

$225 = 102 \times 2 + 51$

Again remainder $51 \neq 0$, so we again apply Euclid's division algorithm on divisor 102 and remainder 51.

We get,

$102 = 51 \times 2 + 0$

Here, at this stage remainder is zero. So, the divisor of this stage (51) is HCF of 867 and 225.

So, HCF (867, 225) = 51

Q.2 Show that any positive odd integer is of the form 6q + 1 or 6q + 3 or 6q + 5, where q is some integer.

Sol. Let a be any positive integer and b = 6.

Then, by Euclid's algorithm $a = 6q + r$, for some integer $q \geq 0$

Where $0 \leq r < 6$ the possible remainders are 0, 1, 2, 3, 4, 5

When r = 0, then a = 6q

When r = 1, then a = 6q + 1

When r =2, then a = 6q +2

When r = 3, then a = 6q + 3

When r = 4, then a = 6q + 4

When r = 5, then a = 6q + 5

Where q is quotient.

If a = 6q or 6q + 2 or 6q + 4, then a is an even integer.

Therefore, any odd integer is of the form 6q + 1or 6q + 3 or 6q + 5, where q is some integer.

Q.3 An army contingent of 616 members is to march behind an army band of 32 members in parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Sol. Given: Number of contingent members = 616

Number of army band members =32

To find the maximum number of columns, we need to find the HCF of 616 and 32 by using Euclid's division lemma.

Since 616 > 32

$$\begin{array}{r|l} & 19 \\ 32 & 616 \\ & 32 \\ \hline & 296 \\ & 288 \\ \hline & 008 \end{array}$$

$\Rightarrow 616 = 32 \times 19 + 8$

Here, remainder $8 \neq 0$, so we again apply Euclid's division lemma on divisor 32 and remainder 8.

$$\begin{array}{r|l} & 4 \\ 8 & 32 \\ & 32 \\ \hline & 00 \end{array}$$

$\Rightarrow 32 = 8 \times 4 + 0$

At this stage reminder is zero. So, the divisor of this stage (8) is HCF of 616 and 32.

Hence, maximum number of columns is 8.

Q.4 Use Euclid's division lemma to show that the square of any positive integer is either of the form 3m or 3m +1 for some integer m.

Sol. Let x be any positive integer and y=3,

By applying EDA, then $x = 3q + r$, Where q is integer $q \geq 0$ and $r = 0, 1, 2$

Now, When $r = 0$

$x = 3q$

$x^2 = (3q)^2 = 9q^2 = 3(3q^2) = 3m$, where $m=3q^2$

When $r = 1$

$x = 3q + 1$

$x^2 = (3q+1)^2 = 9q^2 + 6q + 1$

$x^2 = 3(3q^2+2q) + 1$

$x^2 = 3m + 1$, where $m=3q^2+2q$

When $r = 2$

$x = 3q + 2$

$x^2 = (3q+2q)^2 = 9q^2 + 12q + 4$

$x^2 = 3(3q^2+4q+1)+1$

$x^2 = 3m + 1$, where $m = 3q^2+4q+1$

Hence, from the result. It is proved that the square of each of these can be written in the form 3m or 3m +1.

Q.5 Use Euclid's division lemma to show that cube of any positive integer is either of the form 9q, 9q + 1 or 9q + 8.

Sol. Let x be any positive integer, and y = 3

By applying EDA, we get

$x = 3q + r$, Where q is integer $q \geq 0$ and $r = 0, 1, 2$

Now, When $r = 0$

$x = 3q$

$x^3 = (3q)^3 = 27q^2 = 9(3q^3) = 9m$, where $m = 3q^3$

When $r = 1$

$x = 3q + 1$

$x^3 = (3q+1)^3 = (3q)^3 + 1^3 + 3 \times 3q \times 1(3q+1)$

$x^3 = 27q^3 + 1 + 27q^2 + 9q$

$x^3 = 9(3q^3 + 3q^2 + q) + 1$

$x^3 = 9m + 1$, where $m = 3q^3 + 3q^2 + q$

When $r = 2$

$x = 3q + 2$

$x^3 = (3q+2q)^3 = (3q)^3 + 2^3 + 3 \times 3q \times 2(3q+2)$

$x^3 = 27q^3 + 54q^2 + 36q + 8$

$x^2 = 9(3q^3 + 6q^2 + 4q) + 8$

$x^2 = 9m + 8$, where $m = 3q^3 + 6q^2 + 4q$

Hence, from the result. It is proved that the cube of each of these can be rewritten in the form 9q + 1 or 9q + 8.