Real Numbers: Exercise 1.4

Q.1 Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non - terminating repeating decimal expansion :

(i) $\frac{13}{3125}$ **(ii)** $\frac{17}{8}$

(iii) $\frac{64}{455}$ **(iv)** $\frac{15}{1600}$

(v) $\frac{29}{343}$ **(iv)** $\frac{23}{2^3 5^2}$

(vii) $\frac{129}{2^2 5^7 7^5}$ **(viii)** $\frac{6}{15}$

(ix) $\frac{35}{50}$ **(x)** $\frac{77}{210}$

Sol. *I*f the denominator of a rational number has no prime factors other than 2 or 5, then it has terminating decimal expansion, otherwise it has non - terminating decimal expansion. Thus, we will have to check the prime factors of the denominators of the given rational numbers.

(i) In $\frac{13}{3125}$, the denominator is 3125.

5	3125
5	625
5	125
5	25
5	5
	1

Since, $3125 = 5 \times 5 \times 5 \times 5 \times 5 = 5^5$

Thus, 3125 has 5 as the only prime factor.

Hence, $\frac{13}{3125}$ must have a terminating decimal expansion.

(ii) In $\frac{17}{8}$, the denominator is 8.

2	8
2	4
2	2
	1

Since, $8 = 2 \times 2 \times 2 = 2^3$

therefore, 8 has 2 as the only prime factor. Hence, $\frac{17}{8}$ must have a terminating decimal expansion.

(iii) In $\frac{64}{455}$, denominator is 455.

$455 = 5 \times 7 \times 13$

Clearly, we can see that 455 has prime factors other than 2 and 5. So, it will not have a terminating decimal expansion.

(iv) In $\frac{15}{1600}$, the denominator is 1600.

2	1600
2	800
2	400
2	200
2	100
2	50
5	25
5	5
	1

$1600 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 = 2^6 5^2$

1600 has only 2 and 5 as prime factors. Therefore, $\frac{15}{1600}$ must have a terminating decimal expansion.

(v) In $\frac{29}{343}$, the denominator is 343.

7	343
7	49
7	7
	1

$343 = 7 \times 7 \times 7$

clearly, we can see that 343 has prime factors other than 2 and 5.

So, it will not have terminating decimal expansion.

(vi) In $\frac{23}{2^3 5^2}$ clearly, we can see in denominator $2^3.5^2$ has only 2 and 5 as prime factors.

Hence, $\frac{23}{2^3 5^2}$ will have a terminating decimal expansion.

(vii) In $\frac{129}{2^2 5^7 7^5}$ clearly, we can see denominator $2^2.5^7.7^5$ has prime factors other than 2 and 5. So, it will not have terminating decimal Expansion.

(viii) In $\frac{6}{15}$, the denominator is 15

$15 = 3 \times 5$

clearly, we can see that denominator 15 has prime factors other than 2 and 5. So, it will not have terminating decimal expansion.

(ix) In $\frac{35}{50}$, the denominator is 50

$50 = 2 \times 5 \times 5$, clearly we can see that denominator has only 2 and 5 as prime factors. Hence, $\frac{35}{50}$ will have a terminating decimal expansion.

(x) In $\frac{77}{210}$, the denominator is 210.

2	210
3	105
5	35
7	7
	1

$210 = 2 \times 3 \times 5 \times 7$

clearly, we can see that denominator 210 has prime factors other than 2 and 5. So, it will not have terminating decimal expansion.

Q.2 Write down the decimal expansion of those rational numbers in Question 1 above which have terminating decimal expansions.

Sol. **(i)** $\frac{13}{3125} = \frac{13}{5x5x5x5x5}$

$= \frac{13x2x2x2x2x2}{5x2x5x2x5x2x5x2x5x2} = \frac{416}{100000} = 0.00416$

(ii) $\frac{17}{8} = \frac{17x5^3}{2^3x5^3} = \frac{17x125}{10^3} = \frac{2125}{1000}$

(iii) $\frac{64}{455}$ has a Non – terminating decimal expansion.

(iv) $\frac{15}{1600} = \frac{15}{2^6x5^2} = \frac{15x5^4}{2^4x5^4x10^2} = \frac{15x625}{10^4x10^2} = \frac{9375}{1000000} = 0.009375$

(v) $\frac{29}{343}$ has a Non – terminating decimal expansion.

(vi) $\frac{23}{2^35^2} = \frac{23x5}{2x5x10^2} = \frac{115}{1000} = 0.115$

(vii) $\frac{129}{2^25^77^5}$ has a Non – terminating decimal expansion.

(viii) $\frac{6}{15} = \frac{6}{15} = \frac{2}{5} = 0.4$

(ix) $\frac{35}{50} = 0.70$

(x) $\frac{77}{210}$ has a Non – terminating decimal expansion.

Q.3 The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If they are rational, and of the form $\frac{p}{q}$, what can you say about the prime factors of q?
(i) 43.123456789 (ii) 0.120120012000120000..... (iii) 43 $\overline{123456789}$

Sol. **(i)** 43.123456789 is terminating number. So, it is a rational number.

Thus, 43.123456789 = $\frac{p}{q}$, where $q=10^9$. q has the factor 2 and 5 only.

(ii) 0.12012001200012000... is non - terminating and non-repeating number. So, it is irrational number.

(iii) 43.$\overline{123456789}$ is non - terminating but repeating number. So, it is rational number.

Therefore, 43.$\overline{123456789}$ = $\frac{p}{q}$, where q has the factor other than 2 and 5.