Quadrilaterals: Exercise 8.1

Q.1 The angles of quadrilateral are in the ratio 3 : 5 : 9 : 13. Find all the angles of the quadrilateral.

Sol. Suppose the angles are $(3x)^o$, $(5x)^o$, $(9x)^o$ and $(13x)^o$.

So, $3x + 5x + 9x + 13x = 360$ [Since, sum of the angles of a quadrilateral is 360^o]

$\Rightarrow 30x = 360$

$\Rightarrow x = \frac{360}{30}$

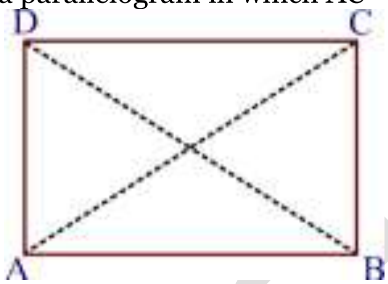
$= 12$

Hence, the angles $= (3 \times 12)^o, (5\times 12)^o, (9 \times 12)^o$ and $(13 \times 12)^o$

$= 36^o, 60^o, 108^o$ and 156^o.

Q.2 If the diagonals of a parallelogram are equal then show that it is a rectangle.

Sol. Let ABCD be a parallelogram in which AC = BD.



To prove: Parallelogram ABCD is a rectangle.

Proof: Firstly, In ΔABC and ΔDCB,

AB = DC (Opposite sides of a ||gm)

BC = BC (Common Side)

and AC = DB (Equal Diagonals)

So, from SSS criterion of congruence

$\Delta ABC \cong \Delta DCB$

$\Rightarrow \angle ABC = \angle DCB$ ... (i) (Corresponding parts of congruent triangles)

Since, AB || DC and BC cuts them.

So, $\angle ACB + \angle DCB = 180^o$

From eq. (i),

$\Rightarrow 2\angle ABC = 180^o$ ... (ii) (Since, sum of consecutive interior angles = 180^o)

$\Rightarrow \angle ABC = 90^o$

Hence, $\angle ABC = \angle DCB = 90^o$

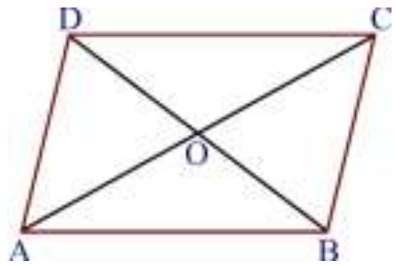
Therefore, ABCD is a parallelogram whose two angles are 90^o .

Thus, ABCD is a rectangle.

Hence Proved.

Q.3 Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

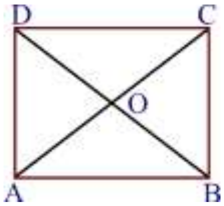
Sol. Let ABCD be a quadrilateral in which the diagonals AC and BD bisect each other at right angle at point O i.e. AO = OC , BO = OD and AC ⊥ BD.



To prove: Quadrilateral ABCD is a rhombus.
Proof: Here, diagonals AC and BD of quadrilateral ABCD bisect each other at right angles. So, diagonal AC is the perpendicular bisector of BD.
⇒ Therefore, A and C both are equidistant from B and D.
⇒ AB = AD and CB = CD ... (i)
Also, diagonal BD is the perpendicular bisector of AC.
⇒ Therefore, B and D both are equidistant from A and C.
⇒ AB = BC and AD = DC ... (ii)
From (i) & (ii),
AB = BC = CD = AD
Therefore, ABCD is a quadrilateral in which diagonals bisect each other at right angles and all four sides are equal.
Thus, ABCD is a rhombus.
Hence Proved.

Another way:
Proof: Firstly, we need to prove that ABCD is a || gm.
So, In ΔAOD and COB,
AO = OC (Given)
OD = OB (Given)
∠AOD = ∠COB (Vertically opposite angles)
So, from SAS criterion of congruence,
ΔAOD ≅ ΔCOB
⇒ ∠OAD = ∠OCB ... (i) (Corresponding parts of congruent triangles)
Since, line AC intersects AD and BC at A and C respectively such that
∠OAD = ∠OCB (From eq.(i)) (Alternate interior angles)
So, AD || BC
Similarly, AB || CD
Thus, ABCD is a parallelogram.
Now, we need to prove that ||gm ABCD is a rhombus.
In ΔAOD and ΔCOD,
OA = OC (Given)
∠AOD = ∠COD (Both diagonal intersect at right angle)
OD = OD (Common side)
So, from SAS criterion of congruence
ΔAOD ≅ ΔCOD
⇒ AD = CD ... (ii) (Corresponding parts of congruent triangles)
Now, ABCD is a || gm (Already Proved)
⇒ AB = CD and AD = BC (Opposite sides of a || gm are equal)
⇒ AB = CD = AD = BC (From eq. (ii))
Thus, quadrilateral ABCD is a rhombus.
Hence Proved.

Q.4 Show that the diagonals of a square are equal and bisect each other at right angles.
Sol. Let ABCD be a square.



To prove: AC = BD, AC $\perp$ BD and OA = OC, OB = OD.

Proof: Since, ABCD is a square.

So, AB || DC and AD || BC.

From figure, AB || DC and transversal AC intersects both at A and C respectively.

So, $\angle$BAC = $\angle$DCA (Alternate interior angles)

$\Rightarrow$ $\angle$BAO = $\angle$DCO ... (i)

Again AB || DC and BD intersects both at B and D respectively.

So, $\angle$ABD = $\angle$CDB (Alternate interior angles)

$\Rightarrow$ $\angle$ABO = $\angle$CDO ... (ii)

Now, in ΔAOB and ΔCOD,

$\angle$BAO = $\angle$DCO (From eq. (i))

AB = CD (Since, opposite sides of a ||gm are equal)

and $\angle$ABO = $\angle$CDO (From eq. (ii))

So, from ASA congruence criterion,

ΔAOB $\cong$ ΔCOD

$\Rightarrow$ OA = OC and OB = OD (Corresponding parts of congruent triangles)

Thus, the diagonals bisect each other.

Now, In ΔADB and ΔBCA,

AD = BC (Sides of a square)

$\angle$BAD = $\angle$ABC (Each 90^o)

and AB = BA (Common side)

So, from SAS criterion of congruence,

ΔADB $\cong$ ΔBCA

$\Rightarrow$AC = BD (Corresponding parts of congruent triangles)

Thus, the diagonals are equal .

Now, in ΔAOB and ΔAOD,

OB = OD (Diagonals of || gm bisect each other)

AB = AD (Sides of a square)

and, AO = AO (Common Side)

So, from SSS criterion of congruence,

ΔAOB $\cong$ ΔAOD

$\Rightarrow$ $\angle$AOB = $\angle$AOD...........(iii) (Corresponding parts of congruent triangles)

But $\angle$AOB + $\angle$AOD = 180^o

$\Rightarrow$$\angle$AOB + $\angle$AOB = 180^o

$\Rightarrow$ 2 $\angle$AOB = 180^o

$\Rightarrow$$\angle$AOB = 90^o

So, $\angle$AOB = $\angle$AOD = 90^o

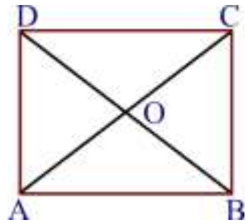
$\Rightarrow$ AO $\perp$ BD and AC $\perp$ BD

Thus, diagonals intersect at right angles.

Hence Proved.

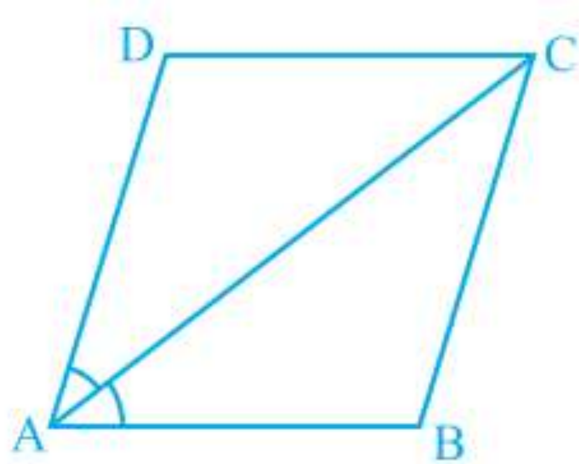
Q.5 Show that if the diagonals of a quadrilateral are equal and bisect each other at right, angles then it is a square.

Sol. Let ABCD be a quadrilateral in which the diagonals AC = BD, AO= OC, BO = OD and AC $\perp$ BD.



To prove: Quadrilateral ABCD is a square.
Proof: Firstly, we need to prove that ABCD is a parallelogram.
So, In ΔAOD and ΔCOB,
AO = OC (Both diagonals bisect each other)
OD = OB (Both diagonals bisect each other)
∠AOD = ∠COB (Vertically opposite Angles)
So, from SAS criterion of congruence,
ΔAOD ≅ ΔCOB
⇒ ∠OAD = ∠OCB ……(i) (Corresponding parts of congruent triangles)
Line AC intersects AD and BC at A and C respectively,
∠OAD = ∠OCB (From eq. (i)) (Alternate interior angles)
So, AD || BC
And similarly, AB || CD
Therefore, ABCD is a parallelogram.
Now, we need to prove that it is a square.
So, In ΔAOB and ΔAOD,
AO = AO (Common side)
∠AOB = ∠AOD (Both diagonals bisect each other at right angle)
and OB = OD (Both diagonals of a || gm bisect each other)
SO, from SAS criterion of congruence,
ΔAOB ≅ ΔAOD
⇒ AB = AD (Corresponding parts of congruent triangles)
But AB = CD and AD = BC (Opposite sides of a || gm)
So, AB = BC = CD = AD … (ii)
Now, in ΔABD and ΔBAC,
AB = BA
AD = BC (Opposite sides of a ||gm)
and BD = AC (Equal diagonals)
So, from SSS criterion of congruence,
ΔABD ≅ ΔBAC
⇒ Thus, ∠DAB = ∠CBA (Corresponding parts of congruent triangles)
Hence Proved.

Q.6 Diagonal AC of parallelogram ABCD bisects ∠A (see figure). Show that
(i) it bisects ∠C also (ii) ABCD is a rhombus.

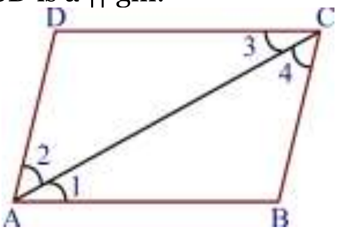


Sol.

(i) Given: A parallelogram ABCD and diagonal AC bisects $\angle A$.

To prove: AC bisects $\angle C$.

Proof: Here, ABCD is a || gm.



So, AB || DC and AC intersects them,

So, $\angle 1 = \angle 3$....(i) (Alternate interior angles)

Now, again AD || BC and AC intersects them

So, $\angle 2 = \angle 4$....(ii) (Alternate interior angles)

Since, AC is the bisector of $\angle A$

So, $\angle 1 = \angle 2$ .... (iii)

So, from (i), (ii) and (ii),

$\angle 3 = \angle 4$

Thus, AC bisects $\angle C$.

Hence Proved.

(ii) To prove: ABCD is a rhombus.

From part (i): Equation (i), (ii) and (iii)

Give $\angle 1 = \angle 2 = \angle 3 = \angle 4$

Now, in ΔABC,

$\angle 1 = \angle 4$

$\Rightarrow$ BC = AB (Sides opposite to equal angles in a triangle)

Similarly, in ΔADC,

AD = DC

Also, ABCD is a||gm

So, AB = CD, AD = BC (Opposite sides of a ||gm)

By combining,

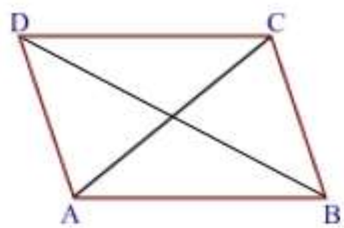
AB = BC = CD = DA

Thus, ABCD is a rhombus.

Hence Proved.

Q.7 ABCD is a rhombus. Show that diagonal AC bisects $\angle A$ as well as $\angle C$ and diagonal BD bisects $\angle B$ as well as $\angle D$.

Sol. **Given:** Quadrilateral ABCD is a rhombus.



To prove: (i) Diagonal AC bisects $\angle A$ as well $\angle C$

(ii) Diagonal BD bisects $\angle B$ as well as $\angle D$

Proof: Firstly, In ΔADC,

AD = DC (Sides of a rhombus)

$\Rightarrow \angle DAC = \angle DCA$ ... (i) (Angles opposite to equal sides of a triangle)

Side AB || DC and AC intersects them

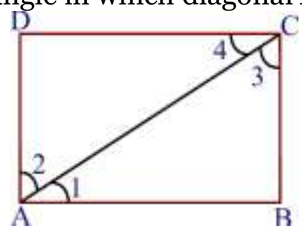
So, $\angle BCA = \angle DAC$ ... (ii) (Alternate interior angles)

So, from (i) and (ii),

∠DCA = ∠BCA
⇒ Thus, AC bisects ∠C
Now, In ΔABC,
AB = BC (Sides of a rhombus)
⇒ ∠BCA = ∠BAC ...... (iii) (Angles opposite to equal sides of a triangle)
So, from (ii) and (iii),
∠BAC = ∠DAC
⇒ Thus, AC bisects ∠A
Thus, diagonal AC bisects ∠A as well as ∠C.
In the same way, diagonal BD bisects ∠B as well as ∠D
Hence Proved.

Q.8 ABCD is a rectangle in which diagonal AC bisects ∠A as well as ∠C. Show that:
(i) ABCD is a square
(ii) diagonal BD bisects ∠B as well as ∠D

Sol. **Given:** ABCD is a rectangle in which diagonal AC bisects ∠A as well as ∠C.



To prove: (i) ABCD is a square.
(ii) Diagonal BD bisects ∠B as well as ∠D.
Proof : (i) Here, AC bisects ∠A as well as ∠C in the rectangle ABCD.
So, ∠A = ∠C (All angles of a rectangle are 90°)

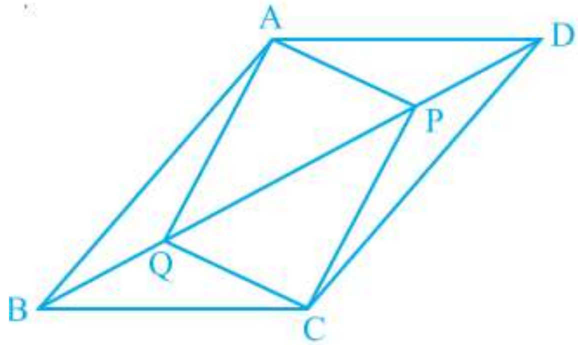
And, ∠1 = ∠2 = ∠3 = ∠4 (Since, AC bisects the ∠A and ∠C, Each = $\frac{90^o}{2}$ = 45°)

So, In ΔADC,
∠2 = ∠4
⇒ Therefore, AD = CD (Since, sides opposite to equal angles)
Hence, the rectangle ABCD is a square. Hence Proved.

(ii) Since, In square ABCD, diagonals bisect the angles.
So, BD bisects ∠B as well as ∠D.
Hence Proved.

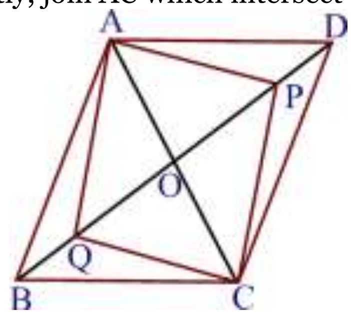
Q.9 In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ (see figure). Show that:

(i) ΔAPD ≅ ΔCQB **(ii) AP = CQ** **(iii) ΔAQB ≅ ΔCPD**
(iv) AQ = CP **(v) APCQ is a parallelogram**



Sol. Given: ABCD is a parallelogram. P and Q are the points on the diagonal BD and DP = BQ.

To prove: (i) ΔAPD ≅ ΔCQB
(ii) AP = CQ
(iii) ΔAQB ≅ ΔCPD
(iv) AQ = CP
(v) APCQ is a parallelogram.
Construction: Firstly, join AC which intersect BD at O.



Proof: Since, diagonals of a parallelogram bisect each other. So, AC and BD bisect each other at O.
So, OB = OD
But BQ = DP (Given)
⇒ OB – BQ = OD – DP
⇒ OQ = OP
In quadrilateral APCQ,
OQ = OP and OA = OC.
So, diagonals AC and PQ bisects each other.
(v) Thus, APCQ is a parallelogram............. Hence Proved.

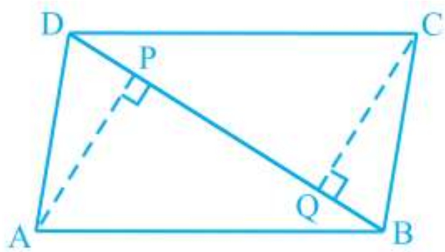
(i) Now, In ΔAPD and ΔCQB,
AD = CB (Opposite sides of a ||gm ABCD)
AP = CQ (Opposite sides of a ||gm APCQ)
DP = BQ (Given)
So, from SSS criterion of congruence,
ΔAPD ≅ ΔCQB.............Hence Proved.

(ii) AP = CQ (Opposite sides of a ||gm APCQ)......... Hence Proved.

(iii) Now, In ΔAQB and ΔCPD,
AB = CD (Opposite sides of a ||gm ABCD)
AQ = CP (Opposite sides of a ||gm APCQ)
BQ = DP (Given)
So, from SSS criterion of congruence,
ΔAQB ≅ ΔCPD............. Hence Proved.

(iv) AQ = CP (Opposite sides of a ||gm APCQ)........Hence Proved.

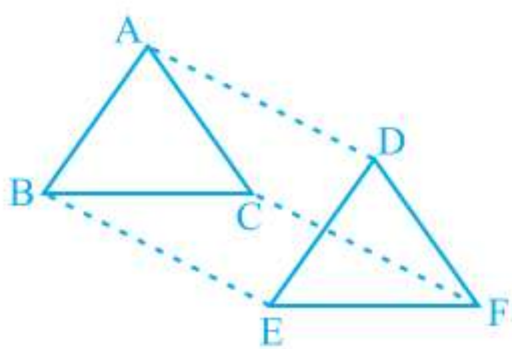
Q.10 ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD respectively (See figure). Show that
(i) ΔAPB ≅ ΔCQD (ii) AP = CQ



Sol.
(i) Given: ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD respectively.
So, DC || AB
Now, since DC || AB and transversal BD intersects them at B and D.
So, ∠ABD = ∠BDC .... (i) (Alternate interior angles)
Now in ΔAPB and ΔCQD,
∠ABP = ∠QDC (from eq. (i))
∠APB = ∠CQD (Since, AP ⊥ BD and QC ⊥ BD, Each = 90°)
and AB = CD (Opposite sides of a || gm)
So, from AAS criterion of congruence,
ΔAPB ≅ ΔCQD.............Hence Proved.

(ii) Since, we have proved that ΔAPB≅ΔCQD
So, AP = CQ (Corresponding parts of congruent triangles)
Hence Proved.

Q.11 In ΔABC and Δ DEF, AB = DE, AB || DE, BC = EF and BC || EF. Vertices A, B and C are joined to vertices D, E and F respectively (see figure). Show that
(i) Quadrilateral ABED is a parallelogram
(ii) Quadrilateral BEFC is a parallelogram
(iii) AD || CF and AD = CF
(iv) Quadrilateral ACFD is a parallelogram.
(v) AC = DF
(vi) ΔABC ≅ ΔDEF



***Sol.* Given:** ΔABC and ΔDEF such that AB = DE and AB || DE. Also BC = EF and BC || EF.
To prove: (i) Quadrilateral ABED is parallelogram.
(ii) Quadrilateral BEFC is a parallelogram.
(iii) AD || CF and AD = CF.
(iv) Quadrilateral ACFD is a parallelogram.
(v) AC|| DF and AC = DF
(vi) ΔABC ≅ ΔDEF

Proof: (i) Firstly, consider the quadrilateral ABED
Since, AB = DE and AB || DE (Given)

Here, One pair of opposite sides are equal and parallel.
Thus, ABED is a parallelogram...............Hence Proved.

(ii) Now, consider quadrilateral BEFC,
Since, BC = EF and BC || EF (Given)
Here, One pair of opposite sides are equal and parallel.
Thus, BEFC is a parallelogram..................Hence Proved.

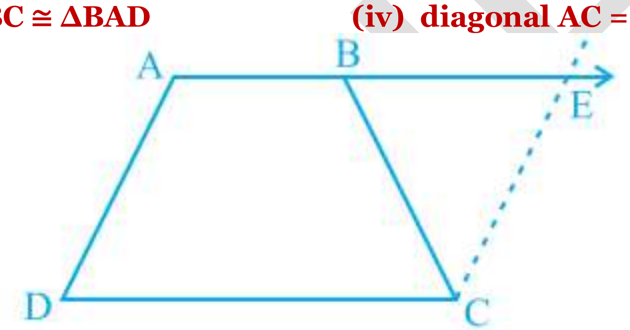
(iii) AD = BE and AD || BE (ABED is a ||gm (already Proved))... (i)
and CF = BE and CF|| BE (BEFC is a ||gm (Already Proved))... (ii)
So, from (i) & (ii),
Thus, AD = CF and AD|| CF....................Hence Proved.

(iv) Since, AD = CF and AD || CF (Already Proved)
Here, one pair of opposite sides are equal and parallel.
So, ACFD is a parallelogram............Hence Proved.

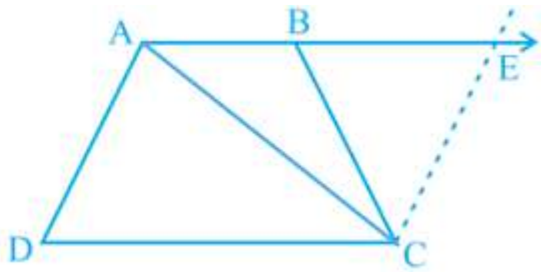
(v) Since, ACFD is parallelogram (Already Proved)
So, AC = DF (Since, Opposite sides of a|| gm ACFD)
Hence Proved.

(vi) Now, In ΔABC and ΔDEF,
AB = DE (Opposite sides of a|| gm ABED)
BC = EF (Opposite sides of a|| gm BEFC)
and CA = FD (Opposite sides of || gm ACFD)
So, from SSS criterion of congruence,
ΔABC ≅ ΔDEF.............Hence Proved.

Q.12 ABCD is a trapezium in which AB || CD and AD = BC (see figure) Show that
(i) ∠A = ∠B **(ii) ∠C = ∠D**
(iii) ΔABC ≅ ΔBAD **(iv) diagonal AC = diagonal BD**



Sol.
Given: Here, ABCD is a trapezium in which AB || CD and AD = BC
To prove: (i) ∠A = ∠B
(ii) ∠C = ∠D
(iii) ΔABC ≅ ΔBAD
(iv) Diagonal AC = diagonal BD.
Construction: Firstly, produce AB to point E and also draw a line CE which is parallel to AD. Also join AC in figure.



Proof:

(i) From figure, AD || CE and transversal AE intersects at A and E respectively.

So, ∠A + ∠E = 180° ... (i) (Since, consecutive interior angles are supplementary.)

Again, AB || CD and AD || CE.

Thus, AECD is parallelogram.

⇒ AD = CE

⇒ BC = CE ....(i) (Since, given that AD = BC)

Now, in ΔBCE,

BC = CE (From eq. (i))

⇒ ∠CBE = ∠CEB (Angles opposite to equal sides are equal)

⇒ 180 − ∠B = ∠E

⇒ 180 − ∠E = ∠B ... (ii)

From (i) and (ii),

∠A = ∠B....................Hence Proved.

(ii) Since, ∠A = ∠B (Already Proved)

⇒ ∠BAD = ∠ABC

⇒ 180° − ∠BAD = 180° −∠ABC

⇒ ∠ADC = ∠BCD

⇒ ∠D = ∠C

Hence Proved.

(iii) Now, In ΔABC and ΔBAD,

BC = AD (Given)

AB = BA (Common Side)

∠A = ∠B (Already Proved)

So, from SAS criterion of congruence,

ΔABC ≅ ΔBAD..........Hence Proved.

(iv) Since, ΔABC ≅ ΔBAD (Already Proved)

So, AC = BD (Corresponding parts of congruent triangles)

Hence Proved.