Quadratic Equations: Exercise 4.4

Q.1 Find the nature of the roots of the following quadratic equations. If the real roots exist, find them:

(i) $2x^2 - 3x + 5 = 0$

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

(iii) $2x^2 - 6x + 3 = 0$

Sol. **(i) Given equation: $2x^2 - 3x + 5 = 0$**

From above equation, $a = 2$, $b = -3$ and $c = 5$

Therefore, $D = b^2 - 4ac$

$= (-3)^2 - 4 \times 2 \times 5$

$= 9 - 40 = -31 < 0$

Therefore, the equation has no real roots.

(ii) Given equation: $3x^2 - 4\sqrt{3}x + 4 = 0$

From above, $a = 3$, $b = -4\sqrt{3}$ and $c = 4$

So, $D = b^2 - 4ac$

$= (-4\sqrt{3})^2 - 4 \times 3 \times 4$

$= 48 - 48 = 0$

So, the given equation has real equal roots.

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{(-4\sqrt{3})^2 \pm 10}{2x2} = \sqrt{3}.$$

Thus, the root of the given equation: $\sqrt{3}$

(iii) Given equation: $2x^2 - 6x + 3 = 0$

From above, $a = 2$, $b = -6$ and $c = 3$

Since, $D = b^2 - 4ac = (-6)^2 - 4 \times 2 \times 3$

$= 36 - 24 = 12 > 0$

So, the given equation has real roots.

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-6) \pm \sqrt{12}}{2x2}$$

$$= \frac{3 \pm \sqrt{3}}{2}$$

Q.2 Find the values of k for each of the following quadratic equations, so that they have two equal roots.

(i) $2x^2 + kx + 3 = 0$

(ii) $kx(x - 2) + 6 = 0$

Sol. **(i) Given equation: $2x^2 + kx + 3 = 0$**

From above, $a = 2$, $b = k$ and $c = 3$

Since, $D = b^2 - 4ac = k^2 - 4 \times 2 \times 3$

$= k^2 - 24$

Thus, the given equation will have real and equal roots.

For real and equal root: $D = 0$

$\Rightarrow k^2 - 24 = 0$

$\Rightarrow k = \pm\sqrt{24} = \pm 2\sqrt{6}$

(ii) Given equation: $kx(x - 2) + 6 = 0$

$\Rightarrow kx^2 - 2kx + 6 = 0$

From above, $a = k$, $b = -2k$ and $c = 6$

Since, $D = b^2 - 4ac = (-2k)^2 - 4 \times k \times 6$

$= 4k^2 - 24k$

the given equation will have real and equal roots.

For real and equal root D = 0

$\Rightarrow 4k^2-24k = 0$

$\Rightarrow 4k(k - 6) = 0$

$\Rightarrow k = 0$ or $k = 6$

Q.3 Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 m²? If so, find its length and breadth.

Sol. Suppose, 2x be the length and x be the breadth of a rectangular mango grove.

Then, Area = (2x) (x) = $800m^2$

$\Rightarrow x^2 = 800/2 = 400$

$\Rightarrow x = 20$ (Since length cannot be negative)

since, the value of x is real integer, so design of grove is possible.

Length = 40 m and breadth = 20 m.

Q.4 Is the following situation possible? If so, determine their present ages. The sum of the ages of two friend is 20 years. Four year ago, the product of their ages in years was 48.

Sol. Let x be the age of one of the friends. Then, age of the other friend is 20 – x.

Four years ago,

Age of one of the friend = (x – 4) years

and age of the other friend = (20 – x – 4) years

= (16 – x) years

The product of their ages in years was 48:

$(x-4)(16-x) = 48$

$\Rightarrow 16 - x^2 - 64 + x = 48$

$\Rightarrow x^2-20+112=0$

From above, a = 1, b = – 20 and c = 112

Since, $D = b^2-4ac = (-20)^2 - 4\times1\times112$

$= 400-448$

$= -48 < 0$ since $D < 0$

So, the given equation has no real roots.

Therefore, the given situation is not possible.

Q.5 Is it possible to design a rectangular park of perimeter 80 m and area 400 m²? If so, find its length and breadth.

Sol. Let x m be the length and y be the breadth of rectangle.

So, Perimeter = 80 m

$\Rightarrow 2(x + y) = 80$

$\Rightarrow x + y = 40$..... (i)

Also, Area = $400m^2$

$\Rightarrow xy = 400$

$\Rightarrow x(40-x)=400$ from (i)

$\Rightarrow 40x - x^2 = 400$

$\Rightarrow x^2-40x+400=0$

From above, a = 1, b = – 40 and c = 400

Since, $D = b^2 - 4ac$

$=(-40)^2-4\times1\times400$

$=1600-1600=0$

Since D = 0.

So, the equation has equal real roots.

Therefore, length and breadth of the rectangle is given by

$x^2-40x+400=0$

$\Rightarrow (x-20)^2=0$

$\Rightarrow x=20$ or $x =20$

So, Length = 20 m

and Breadth = 20 m

Thus, Design is possible.