Quadratic Equations: Exercise 4.3

Q.1 Find the roots of the following quadratic equations, if they exist by the method of completing the square:

(i) $2x^2 - 7x + 3 = 0$

(ii) $2x^2 + x - 4 = 0$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

(iv) $2x^2 + x + 4 = 0$

Sol. **(i) The given equation 2x2−7x+3=0 can be written as** $x^2 - \frac{7}{2}x + \frac{3}{2} = 0$

Now, $x^2 - \frac{7}{2}x + \frac{3}{2}$

$= (x - \frac{7}{4})^2 - (\frac{7}{4})^2 + \frac{3}{2}$

$= (x - \frac{7}{4})^2 - \frac{49}{16} + \frac{3}{2}$

$= (x - \frac{7}{4})^2 - \frac{25}{16}$

For finding the roots, we need to equate the equation to zero.

Therefore, $2x^2 - 7x + 3 = 0$

$\Rightarrow (x - \frac{7}{4})^2 - \frac{25}{16}$

$\Rightarrow (x - \frac{7}{4})^2 = \frac{25}{16}$

By taking square root:

$\Rightarrow x - \frac{7}{4} = \pm \frac{5}{4}$

$\Rightarrow x = \frac{7}{4} \pm \frac{5}{4}$

$\Rightarrow x = \frac{7}{4} + \frac{5}{4} = 3$

$\Rightarrow x = \frac{7}{4} - \frac{5}{4} = \frac{1}{2}$

Therefore, The roots of the equation are 3 and $\frac{1}{2}$.

(ii) Given: $2x^2 + x - 4 = 0$

The given equation can be written as:

$x^2 + \frac{x}{2} - 2 = 0$

$\Rightarrow \quad (x + \frac{1}{4})^2 - (\frac{x}{4})^2 - 2 = 0$

$\Rightarrow \quad (x + \frac{1}{4})^2 - \frac{1}{16} - 2 = 0$

$\Rightarrow \quad (x + \frac{1}{4})^2 - \frac{33}{16} = 0$

$\Rightarrow \quad (x + \frac{1}{4})^2 = \frac{33}{16}$

$\Rightarrow \quad x + \frac{1}{4} = \pm\frac{\sqrt{33}}{4}$

$\Rightarrow \quad x = -\frac{1}{4} \pm \frac{\sqrt{33}}{4}$

$\Rightarrow \quad x = \frac{-1+\sqrt{33}}{4}$

$\Rightarrow \quad x = \frac{-1-\sqrt{33}}{4}$

Thus, the roots of the equation are $\frac{-1+\sqrt{33}}{4}$ and $\frac{-1-\sqrt{33}}{4}$.

(iii) Given: $4x^2 + 4\sqrt{3}x + 3 = 0$

$\Rightarrow (2x)^2 + 2\times(2x)\times\sqrt{3} + (\sqrt{3})^2 - (\sqrt{3})^2 + 3 = 0$

$\Rightarrow (2x + \sqrt{3})^2 - 3 + 3 = 0$

$\Rightarrow (2x+\sqrt{3})^2 = 0$

$\Rightarrow x = -\frac{\sqrt{3}}{2}$

$\Rightarrow x = -\frac{\sqrt{3}}{2}$

Thus, the roots of the equation are $-\frac{\sqrt{3}}{2}$ and $-\frac{\sqrt{3}}{2}$.

(iv) Given: $2x^2 + x + 4 = 0$

$\Rightarrow \quad x^2 + \frac{1}{2}x + 2 = 0$

$\Rightarrow (x+\frac{1}{4})^2 - \frac{1}{16} + 2 = 0$

$\Rightarrow (x + \frac{1}{4})^2 + \frac{31}{16} = 0$

$\Rightarrow (x+\frac{1}{4})^2 = -\frac{31}{16} < 0$

Since, the value of $(x + \frac{1}{4})^2$ cannot be negative for any real value of x. So, the given equation has no real roots.

Q.2 Find the roots of the quadratic equations given in Q.1 above by applying the quadratic formula.

Sol. **(i) Given: $2x^2 - 7x + 3 = 0$**

Here, a = 2, b = – 7 and c = 3.

Therefore, $D = b^2 - 4ac = (-7)^2 - 4\times2\times3$

$= 49-24 = 25 > 0$

So, the given equation has real roots:

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{-(-7) \pm \sqrt{25}}{2x2}$$

$= \frac{7 \pm 5}{4}$

$= 12/4$ or $2/4$

$= 3$ or $\frac{1}{2}$

Thus the roots of the given equation are: 3 or ½

(ii) Given: $2x^2 + x - 4 = 0$

Here, a = 2, b = 1 and c = − 4

Since, $D = b^2 - 4ac = (1)^2 - 4 \times 2 \times (-4)$

$= 1+32$

$= 33 > 0$

So, the given equation has real roots:

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{-(1) \pm \sqrt{33}}{2x2} = \frac{-(1) \pm \sqrt{33}}{4}$$

Thus the roots of the given equation are: $\frac{-1 \pm \sqrt{33}}{4}$

(iii) Given equation: $4x^2 + 4\sqrt{3}x + 3 = 0$

Here a = 4, $b = 4\sqrt{3}$ and c = 3

Since, $D = b^2 - 4ac = (4\sqrt{3})^2 - 4\times4\times3$

$= 48-48 = 0$

So, the given equation has real equal roots:

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-b}{2a} = \frac{-4\sqrt{3}}{2x4} = \frac{-\sqrt{3}}{2}$$

(iv) Given equation: $2x^2 + x + 4 = 0$

Here, a = 2, b = 1 and c = 4

Since, $D = b^2 - 4ac = (1)^2 - 4 \times 2 \times 4$

$= 1-32 = -31 < 0$

So, there is no real value of x satisfying the given equation. Therefore, the given equation has no real roots.

Q.3 Find the roots of the following equations:

(i) $x - \frac{1}{x} = 3, x \neq 0$

(ii) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq -4, 7$

Sol. **(i) Given equation: $x - \frac{1}{x} = 3, x \neq 0$**

$\Rightarrow \quad x^2 - 3x - 1 = 0$

From the above equation, here, a = 1, b = − 3 and c = − 1

Since, $D = b^2 - 4ac = (-3)^2 - 4(1)(-1)$

$= 9 + 4 = 13 > 0$

So, the equation has real roots:

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-3) \pm \sqrt{13}}{2} = \frac{3 \pm \sqrt{13}}{2}$$

Thus, the roots of the given equation: $\frac{3 \pm \sqrt{13}}{2}$

(ii) The given equation is $\frac{1}{x+4}-\frac{1}{x-7}=\frac{11}{30}$**, x ≠ −4, 7**

$$\frac{(x-7)-(x+4)}{(x-7)(x+4)}=\frac{11}{30}$$

$$\frac{-11}{(x-7)(x+4)}=\frac{11}{30}$$

$$\Rightarrow \frac{-1}{x^2-7x+4x-28}=\frac{1}{30}$$

$$\Rightarrow \frac{-1}{x^2-3x-28}=\frac{1}{30}$$

⇒ $-30 = x^2 - 3x - 28$
⇒ $x^2-3x+2=0$
⇒ $x^2 -2x - x + 2 = 0$
⇒ $x(x-2)-1(x-2)=0$
⇒ $(x-1)(x-2)=0$
⇒ x=1or x=2
Thus, the roots of the given equation are x = 1 and x = 2.

Q.4 The sum of the reciprocals of Rehman's ages, (in years) 3 years ago and 5 years from now is $\frac{1}{3}$**. Find his present age.**

Sol. Let x be the present age of Rehman's.

The sum of the reciprocals of Rehman's ages, 3 years ago and 5 years from now is $\frac{1}{3}$:

$$\frac{1}{x-3}+\frac{1}{x+5}=\frac{1}{3}$$

⇒ $3(x+5) + 3(x-3) = (x-3)(x+5)$
⇒ $3x + 15 + 3x - 9 = x^2 + 2x - 15$
⇒ $x^2 - 4x - 21 = 0$
⇒ $x^2 -7x + 3x - 21 = 0$
⇒ $(x-7)(x+3) = 0$
⇒ $x(x-7) + 3(x-7) = 0$
⇒ $(x+3) (x-7) = 0$
⇒ x = 7or x= −3
Thus, x=7 [Since, age cannot be negative]
So, Rehman's present age is 7 years.

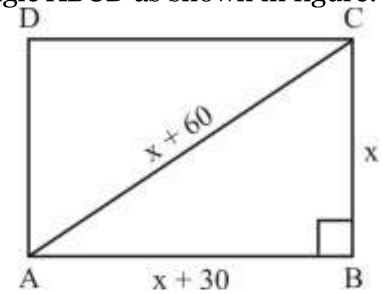
Q.5 In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

Sol. Let Shefali's marks in Mathematics be x and marks in English will be (30 – x).
She got 2 marks more in Mathematics and 3 marks less in English, then product of their marks = 210:
$(x+2)\times[(30-x) -3] = 210$
⇒ $(x+2)(27-x) = 210$
⇒ $27x - x^2+54-2x = 210$
⇒ $x^2 - 25x + 156 = 0$
⇒ $x^2 - 12x - 13x + 156 = 0$
⇒ $x(x-12) - 13(x-12) = 0$
⇒ $(x-12)(x-13) = 0$
⇒ x = 12 or x=13

Therefore, Shafali's marks in Mathematics is 12 and in English is 18 or in Mathematics is 13 and in English is 17.

Q.6 The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.

Sol. Let the rectangle ABCD as shown in figure:



In figure, shorter side, BC = x m.
Then Diagonal, AC = (x + 60)
Longer side, AB = (x + 30) m.
From the Pythagoras Theorem:

$AC^2 = BC^2 + AB^2$
$\Rightarrow (x+60)^2 = x^2 + (x+30)^2$
$\Rightarrow x^2 + 120x + 3600 = x^2 + x^2 + 60x + 900$
$\Rightarrow x^2 - 60x - 2700 = 0$
$\Rightarrow x^2 - 90x + 30x - 2700 = 0$
$\Rightarrow x(x - 90) + 30(x - 900) = 0$
$\Rightarrow (x+30)(x-90)=0$
$\Rightarrow x = -30 \text{ or } x = 90$
$\Rightarrow x=90$ (Since length can never be negative)

Thus, the sides of the field are 120 m and 90 m.

Q.7 The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

Sol. Let x be the larger number. Then,
Square of the smaller number = 8x
Also, square of the larger number = x^2
Since, the difference of the squares of the number is 180

$x^2 - 8x = 180$
$\Rightarrow x^2 - 8x - 180 = 0$
$\Rightarrow x^2 - 18x + 10x - 180 = 0$
$\Rightarrow x(x - 18) + 10(x - 18) = 0$
$\Rightarrow (x-18)(x+10) = 0$
$\Rightarrow x-18=0 \text{ or } x + 10 = 0$
$\Rightarrow x = 18 \text{ or } x = -10$

Case 1: When the value of x = 18.
Square of the smaller number = $8x = 8 \times 18 = 144$
Then, Smaller number = $\sqrt{144} = \pm 12$
Therefore, the number are 18, 12 or 18, – 12.

Case 2: When the value of x = – 10
Square of the smaller number = $8x = 8 \times -10 = -80$
But, square of a number cannot be negative.
Therefore, x = – 10 is not possible.

Therefore, the numbers are 18, 12 or 18, – 12.

Q.8 A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Sol. Let x km/hr be the uniform speed of the train.

Then time taken to cover 360km = 360 / x hr.

When the speed is increased by 5km/hr, time taken = $\frac{360}{x+5}$ hr

since, time to cover 360 km is reduced by 1 hour.

Therefore, $\frac{360}{x} - \frac{360}{x+5} = 1$

$\Rightarrow 360x + 1800 - 360x = x^2 + 5x$
$\Rightarrow x^2 + 5x - 1800 = 0$
$\Rightarrow x^2 + 45x - 40x - 1800 = 0$
$\Rightarrow x(x + 45) - 40(x + 45) = 0$
$\Rightarrow (x-40)(x+45) = 0$
$\Rightarrow x-40 = 0$ or $x+45 = 0$
$\Rightarrow x = 40$ or $x = -45$

But Speed x cannot be negative. Therefore, we take the value of x = 40.

Therefore, the speed of the train is 40 km/hr.

Q.9 Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Sol. Let the smaller tap takes x hr, then larger tap will take (x – 10) hr to fill the tank.

Then, Portion of the tank filled by the larger tap in one hours = $\frac{1}{x-10}$

Portion of the tank filled by the larger tap $9\frac{3}{8}$ hr or 75/8 hr = $\frac{1}{x-10} \times \frac{75}{8}$

Similarly, portion of the tank filled by the smaller tap in 75/8 hr = $\frac{1}{x} \times \frac{75}{8}$

Since the tank is filled in 75/8 hr

$\frac{75}{8(x-10)} - \frac{75}{8x} = 1$

$\Rightarrow \frac{1}{(x-10)} - \frac{1}{x} = \frac{8}{75}$

$\Rightarrow \frac{x+x-10}{x(x-10)} = \frac{8}{75}$

$\Rightarrow 75(2x-10) = 8x(x-10)$
$\Rightarrow 150x - 750 = 8x^2 - 80x$
$\Rightarrow 8x^2 - 230x + 750 = 0$
$\Rightarrow 4x^2 - 115x + 375 = 0$
$\Rightarrow 4x^2 - 100x - 15x + 375 = 0$
$\Rightarrow 4x(x-25) - 15(x-25) = 0$
$\Rightarrow (x-25)(4x-15) = 0$
$\Rightarrow x-25 = 0$ or $4x-15 = 0$
$\Rightarrow x=25$ or $x = 15/4$

Therefore, x=25 as x=154 is inadmissible

Thus, the smaller tap fills the tank in 25 hr and the larger tap takes 15 hr to fill the tank.

Q.10 An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speed of the express train is 11 km/h more than that of the passenger train. Find the average speed of the two trains.

Sol. Let x km/h be the speed of the express train.

Then, the speed of the passenger train = (x – 11) km/hr

Distance to be covered = 132

Time taken by express train = $\frac{132}{x}$ hr

time taken by the passenger train = $\frac{132}{x-11}$

Since, $\frac{132}{x-11} - \frac{132}{x} = 1$

$132x - 132x + 1452 = x(x-11)$

$\Rightarrow 132x - 132x + 1452 = x^2 - 11x$

$\Rightarrow x^2 - 11x - 1452 = 0$

$\Rightarrow x^2 - 44x + 33x - 1452 = 0$

$\Rightarrow x(x - 44) + 33(x - 44) = 0$

$\Rightarrow (x-44)(x+33) = 0$

$\Rightarrow x = 44$ or $x = -33$

But x cannot be negative. So, x = 44

Therefore, Speed of express train = 44 km/hr

and, speed of passenger train will be = (44 – 11) = 33 km/hr.

Q.11 Sum of the areas of two squares is 468m². If the difference of their perimeters is 24 m, find the sides of the two squares.

Sol. Let x and y be the sides of the two squares (where x > y).

Sum of the areas of two squares is $468m^2$

$x^2 + y^2 = 468$ .... (i)

The difference of their perimeters is 24 m:

$4x - 4y = 24$

$\Rightarrow x - y = 6$........... (ii)

Now, put x = y + 6 in (i), we get $(y+6)^2 + y^2 = 468$

$\Rightarrow y^2 + 12y + 36 + y^2 = 468$

$\Rightarrow 2y^2 + 12y + 36 - 468 = 0$

$\Rightarrow 2y^2 + 12y - 432 = 0$

$\Rightarrow y^2 + 6y - 216 = 0$

$\Rightarrow y^2 + 18y - 12y - 216 = 0$

$\Rightarrow y(y + 18) - 12(y + 18) = 0$

$\Rightarrow (y+18)(y-12) = 0$

$\Rightarrow y = -18$ or $y = 12$

Since, y cannot be negative. Thus, y = 12

Therefore, x = y + 6 = 12 + 6 = 18

Hence, The sides of the squares are 18 m and 12 m.