Quadratic Equations: Exercise 4.2

Q1. Find the roots of the following quadratic equations by factorisation:
(i) $x^2 - 3x - 10 = 0$
(ii) $2x^2 + x - 6 = 0$
(iii) $\sqrt{2}\,x^2 + 7x + 5\sqrt{2} = 0$
(iv) $2x^2 - x + 1/8 = 0$
(v) $100x^2 - 20x + 1 = 0$
Sol:
(i) Given: $x^2 - 3x - 10 = 0$

$x^2 - 5x + 2x - 10 = 0$
$x(x - 5) + 2(x - 5)$
=> $(x - 5)(x + 2)$
$x-5=0$ or $x+2=0$
=> Therefore, root of the given equation is $x = 5$ or $x = -2$.

(ii) Given: $2x^2 + x - 6 = 0$

$2x^2 + 4x - 3x - 6 = 0$
$2x(x + 2) - 3(x + 2) = 0$
$(x + 2)(2x - 3) = 0$
$x + 2 = 0$ or $2x - 3 = 0$
$x = -2$ or $x = 3/2$
=> Therefore, root of the given equation is $x = -2$ or $x = 3/2$.

(iii) Given: $\sqrt{2}\,x^2 + 7x + 5\sqrt{2} = 0$

=> $\sqrt{2}\,x^2 + 5x + 2x + 5\sqrt{2} = 0$
=> $x(\sqrt{2}\,x + 5) + \sqrt{2}\,(\sqrt{2}\,x + 5) = (\sqrt{2}\,x + 5)(x + \sqrt{2}) = 0$
$\sqrt{2}\,x + 5 = 0$ or $x + \sqrt{2} = 0$
=> $x = -5/\sqrt{2}$ or $x = \sqrt{2}$
=> Therefore, root of the given equation is $x = -5/\sqrt{2}$ or $x = \sqrt{2}$

(iv) Given: $2x^2 - x + 1/8 = 0$

$=1/8\,(16x^2 - 8x + 1) = 0$
$= 1/8\,(16x^2 - 4x - 4x + 1) = 0$
$= 1/8\,(4x\,(4x - 1) - 1(4x - 1)) = 0$
$(4x - 1) = 0$ or $(4x - 1) = 0$
$\Rightarrow x = 1/4$ or $x = 1/4$
=> Therefore, root of the given equation is $x = 1/4$ or $x = 1/4$

(v) Given: $100x^2 - 20x + 1 = 0$

$= 100x^2 - 10x - 10x + 1 = 0$
$= 10x\,(10x - 1) - 1(10x - 1) = 0$
$= (10x - 1)^2 = 0$
$\therefore (10x - 1) = 0$ or $(10x - 1) = 0$
$\Rightarrow x = 1/10$ or $x = 1/10$
=> Therefore, root of the given equation is $x = 1/10$ or $x = 1/10$.

Q2. Solve the problems given in Example 1.Represent the following situations mathematically:

(i) John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. We would like to find out how many marbles they had to start with.

(ii) A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was ` 750. We would like to find out the number of toys produced on that day.

Sol:

(i) Let x be the number of marbles John have.

Therefore, number of marble Jivanti have = $45 - x$

After losing 5 marbles each,

Number of John's marbles = $x - 5$

Number of Jivanti's marble = $45 - x - 5 = 40 - x$

Then their product:

$\therefore (x - 5)(40 - x) = 124$

$\Rightarrow x^2 - 45x + 324 = 0$

$\Rightarrow x^2 - 36x - 9x + 324 = 0$

$\Rightarrow x(x - 36) - 9(x - 36) = 0$

$\Rightarrow (x - 36)(x - 9) = 0$

Thus, we can say,

$x - 36 = 0$ or $x - 9 = 0$

$\Rightarrow x = 36$ or $x = 9$

Thus x = 9 and x = 36 are two roots of the equation $x^2 - 45x + 324 = 0$.

If, John has marbles = 36,

Then, Jivanti has marbles = 45 – 36 = 9

And if John has marbles = 9,

Then, Jivanti has marbles = 45 – 9 = 36

(ii) Let x be the number of toys produced in a day.

So, cost of production of each toy = Rs. $(55 - x)$

The total cost of production of the toys = Rs. 750

$\therefore x(55 - x) = 750$

$\Rightarrow x^2 - 55x + 750 = 0$

$\Rightarrow x^2 - 25x - 30x + 750 = 0$

$\Rightarrow x(x - 25) - 30(x - 25) = 0$

$\Rightarrow (x - 25)(x - 30) = 0$

Thus, either $x - 25 = 0$ or $x - 30 = 0$

$\Rightarrow x = 25$ or $x = 30$

Thus, x = 30 and x = 25 are two roots of the equation $x^2 - 55x + 750 = 0$.

Hence, the number of toys produced in a day, will be either 25 or 30.

Q3. Find two numbers whose sum is 27 and product is 182.

Sol: Let x be first number and the second number is 27 – x.

Therefore, the product of two numbers

$x(27 - x) = 182$

$\Rightarrow x^2 - 27x - 182 = 0$

$\Rightarrow x^2 - 13x - 14x + 182 = 0$

$\Rightarrow x(x - 13) - 14(x - 13) = 0$

$\Rightarrow (x - 13)(x - 14) = 0$

Thus, $x - 13 = 0$ or $x - 14 = 0$

$\Rightarrow x = 13$ or $x = 14$

Thus, x = 13 and x = 14 are two roots of the equation $x^2 - 27x - 182 = 0$.

Thus, if first number = 13, then second number = 27 – 13 = 14

And if first number = 14, then second number = 27 – 14 = 13

Q4. Find two consecutive positive integers, sum of whose squares is 365.

Sol: Let x be the first and another consecutive positive integers be $x + 1$.

The sum of whose squares is 365.

$x^2 + (x + 1)^2 = 365$

$\Rightarrow x^2 + x^2 + 1 + 2x = 365$

$\Rightarrow 2x^2 + 2x - 364 = 0$

$\Rightarrow x^2 + x - 182 = 0$

$\Rightarrow x^2 + 14x - 13x - 182 = 0$

$\Rightarrow x(x + 14) - 13(x + 14) = 0$

$\Rightarrow (x + 14)(x - 13) = 0$

$x + 14 = 0$ or $x - 13 = 0$,
$\Rightarrow x = -14$ or $x = 13$
Thus, x = -14 and x = 13 are two roots of the equation $x^2 + x - 182 = 0$.
Since, the integers are positive, so x can be 13, only.
$\therefore x + 1 = 13 + 1 = 14$
Thus, two consecutive positive integers are 13 and 14.

Q5. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

Sol:

Let x cm be base of the right triangle.
The altitude of right triangle = (x – 7) cm
Pythagoras theorem: Base^2 + Altitude^2 = Hypotenuse^2
$\therefore \quad x^2 + (x - 7)^2 = 13^2$
$\Rightarrow x^2 + x^2 + 49 - 14x = 169$
$\Rightarrow 2x^2 - 14x - 120 = 0$
$\Rightarrow x^2 - 7x - 60 = 0$
$\Rightarrow x^2 - 12x + 5x - 60 = 0$
$\Rightarrow x(x - 12) + 5(x - 12) = 0$
$\Rightarrow (x - 12)(x + 5) = 0$
$x - 12 = 0$ or $x + 5 = 0$,
$\Rightarrow x = 12$ or $x = -5$
Thus, x = 12 and x = -5 are two roots of the equation $x^2 - 7x - 60 = 0$
Since, sides cannot be negative. So, x can only be 12.
Thus, the base of triangle is 12 cm and the altitude of triangle will be = (12 – 7) cm = 5 cm.

Q6. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was Rs.90, find the number of articles produced and the cost of each article.

Sol:

Let us say, the number of articles produced be x.
Therefore, cost of production of each article = Rs $(2x + 3)$
Given, total cost of production is Rs.90
$\therefore x(2x + 3) = 90$
$\Rightarrow 2x^2 + 3x - 90 = 0$
$\Rightarrow 2x^2 + 15x - 12x - 90 = 0$
$\Rightarrow x(2x + 15) - 6(2x + 15) = 0$
$\Rightarrow (2x + 15)(x - 6) = 0$
$2x + 15 = 0$ or $x - 6 = 0$
$\Rightarrow x = -15/2$ or $x = 6$
Thus, x = -15/2 and x = 6 are two roots of the equation $2x^2 + 3x - 90 = 0$.
Since, the number of articles produced can be a positive integer. So, x can only be 6.
Thus, number of articles produced = 6
And cost of each article = 2 × 6 + 3 = Rs. 15.