Polynomials: Exercise 2.5

Q.1 Use suitable identities to find the following products :
(i) $(x + 4)(x + 10)$ (ii) $(x + 8)(x - 10)$

(iii) $(3x + 4)(3x - 5)$ (iv) $(y^2 + \frac{3}{2})(y^2 - \frac{3}{2})$ (v) $(3 - 2x)(3 + 2x)$

Sol.

(i) $(x + 4)(x + 10) = x^2+(4+10)x + 4\times10$
$= x^2 + 14x + 40$

(ii) $(x + 8)(x - 10) = x^2 + (8-10)x + 8\times(-10)$
$= x^2 - 2x - 80$

(iii) $(3x + 4)(3x - 5) = 3x(3x-5) + 4(3x-5)$
$= 3x\times3x - 3x\times5 + 4\times3x - 4\times5$
$= 9x^2 - 15x + 12x - 20$
$= 9x^2 - 3x - 20$

(iv) $(y^2 + \frac{3}{2})(y^2 - \frac{3}{2}) = (y^2)^2 - (\frac{3}{2})^2$

$= y^4 - \frac{9}{4}$

(v) $(3 - 2x)(3 + 2x) = (3)^2 - (2x)^2$
$= 9 - 4x^2$

Q.2 Evaluate the following products without multiplying directly:
(i) 103×107 (ii) 95×96 (iii) 104×96
Sol.
(i) $103 \times 107 = (100+3)(100+7)$
$= (100)^2 + (3+7)(100) + 3\times7$
$= 100\times100 + (10)(100) + 21$
$= 10000 + 1000 + 21$
$= 11021$

(ii) $95 \times 96 = (100-5)(100-4)$
$= (100)^2 + (-5-4)(100) + (-5)(-4)$
$= 100\times100 + (-9)(100) + 20$
$= 10000 - 900 + 20$
$= 9120$

(iii) $104 \times 96 = (100+4)(100-4)$
$= (100)^2 - (4)^2$
$= 10000-16$
$= 9984$

Q.3 Factorise the following using appropriate identifies:
(i) $9x^2 + 6xy + y^2$
(ii) $4y^2 - 4y + 1$
(iii) $x^2 - \frac{y^2}{100}$

Sol.

(i) $9x^2 + 6xy + y^2 = (3x)^2 + 2(3x)(y) + (y)^2$ (Since, $(a+b)^2 = a^2 + 2ab + b^2$)

$= (3x+y)^2$

$= (3x+y)\,(3x+y)$

(ii) $4y^2 - 4y + 1 = (2y)^2 - 2(2y)(1) + (1)^2$ [Since, $(a-b)^2 = a^2 - 2ab + b^2$]

$= (2y-1)^2$

$= (2y-1)\,(2y-1)$

(iii) $x^2 - \frac{y^2}{100} = (x)^2 - (\frac{y}{10})^2$ [Since, $(x + y)\,(x-y) = x^2 - y^2$]

$= (x - \frac{y}{10})\,(x + \frac{y}{10})$

Q.4 Expand each of the following using suitable identifies :

(i) $(x + 2y + 4z)^2$

(ii) $(2x - y + z)^2$

(iii) $(-2x + 3y + 2z)^2$

(iv) $(3a - 7b - c)^2$

(v) $(-2x + 5y - 3z)^2$

(vi) $[\frac{1}{4}a - \frac{1}{2}b + 1]^2$

Sol.

(i) $(x + 2y + 4z)^2 = x^2 + (2y)^2 + (4z)^2 + 2(x)(2y) + 2(2y)(4z) + 2(4z)(x)$

$= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8zx$

(ii) $(2x - y + z)^2 = [2x + (-y) + z]^2$

$= (2x)^2 + (-y)^2 + z^2 + 2(2x)(-y) + 2(-y)(z) + 2(z)(2x)$

$= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4zx$

(iii) $(-2x + 3y + 2z)^2 = [(-2x) + 3y + 2z]^2$

$= (-2x)^2 + (3y)^2 + (2z)^2 + 2(-2x)(3y) + 2(3y)(2z) + 2(2z)(-2x)$

$= 4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8zx$

(iv) $(3a - 7b - c)^2 = [3a + (-7b) + (-c)]^2$

$= (3a)^2 + (-7b)^2 + (-c)^2 + 2(3a)(-7b) + 2(-7b)(-c) + 2(-c)(3a)$

$= 9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ca$

(v) $(-2x + 5y - 3z)^2 = [(-2x)^2 + 5y + (-3z)]^2$

$= (-2x)^2 + (5y)^2 + (-3z)^2 + 2(-2x)(5y) + 2(5y)(-3z) + 2(-3z)(-2x)$

$= 4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12zx$

(vi) $[\frac{1}{4}a - \frac{1}{2}b + 1]^2 = [\frac{1}{4}a + (-\frac{1}{2}b) + 1]^2$

$= (\frac{1}{4}a)^2 + (-\frac{1}{2}b)^2 + (1)^2 + 2(\frac{1}{4}a)(-\frac{1}{2}b) + 2(-\frac{1}{2}b)(1) + 2(1)(\frac{1}{4}a)$

$= \frac{1}{16}a^2 + \frac{1}{4}b^2 + 1 - \frac{1}{4}ab - b + \frac{1}{2}a$

Q.5 Factorise:
(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$
(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy - 4\sqrt{2}yz - 8xz$

Sol.

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$
$= (2x)^2 + (3y)^2 + (-4z)^2 + 2(2x)(3y) + 2(3y)(-4z) + 2(2x)(-4z)$
$= [2x + 3y + (-4z)]^2$
$= (2x + 3y - 4z)^2$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy - 4\sqrt{2}yz - 8xz$
$= (\sqrt{2}x)^2 + (-y)^2 + (-2\sqrt{2}z)^2 + 2(\sqrt{2}x)(-y) + 2(-y)(-2\sqrt{2}z) + 2(\sqrt{2}x)(-2\sqrt{2}z)$
$= [\sqrt{2}x + (-y) + (-2\sqrt{2}z)]^2$
$=(\sqrt{2}x - y - 2\sqrt{2}z)^2$

Q.6 Write the following cubes in expanded form :
(i) $(2x+1)^3$
(ii) $(2a-3b)^3$
(iii) $[\frac{3}{2}x + 1]^3$
(iv) $[x - \frac{2}{3}y]^3$

Sol.

(i) $(2x+1)^3 = (2x)^3 + 3(2x)^2(1) + 3(2x)(1)^2 + (1)^3$
$= 8x^3 + 12x^2 + 6x + 1$

(ii) $(2a-3b)^3 = (2a)^3 - 3(2a)^2(3b) + 3(2a)(3b)^2 - (3b)^3$
$= 8a^3 - 36a^2b + 54ab^2 - 27b^3$

(iii) $[\frac{3}{2}x + 1]^3 = (\frac{3}{2}x)^3 + 3(\frac{3}{2}x)^2(1) + 3(\frac{3}{2}x)(1)^2 + 1^3$
$= \frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1$

(iv) $[x - \frac{2}{3}y]^3 = x^3 - 3(x)^2(\frac{2}{3}y) + 3(x)(\frac{2}{3}y)^2 - (\frac{2}{3}y)^3$
$= x^3 - 2x^2y + \frac{4}{3}xy^2 - \frac{8}{27}y^3$

Q.7 Evaluate the following using suitable identities:
(i) $(99)^3$
(ii) $(102)^3$
(iii) $(998)^3$

Sol.

(i) $(99)^3 = (100 - 1)^3$
$= (100)^3 - 1^3 - 3(100)(1)(100-1)$
$= 1000000 - 1 - 29700$
$= 970299$

(ii) $(102)^3 = (100 + 2)^3$
$= (100)^3 + (2)^3 + 3(100)(2)(100+2)$
$= 1000000 + 8 + 61200$
$= 1061208$

(iii) $(998)^3 = (1000-2)^3$
$= (1000)^3 - (2)^3 - 3(1000)(2)(1000-2)$
$= 1000000000 - 8 - 5988000$
$= 994011992$

Q.8 Factorise each of the following:
(i) $8a^3 + b^3 + 12a^2b + 6ab^2$
(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$
(iii) $27 - 125a^3 - 135a + 225a^2$
(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$
(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

Sol.
(i) $8a^3 + b^3 + 12a^2b + 6ab^2 = (2a)^3 + (b)^3 + 3(2a)(b)(2a+b)$
$= (2a+b)3$
$= (2a+b)(2a+b)(2a+b)$

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2 = (2a)^3 - b^3 - 3(2a)(b)(2a-b)$
$= (2a-b)^3$
$= (2a-b)(2a-b)(2a-b)$

(iii) $27 - 125a^3 - 135a + 225a^2 = (3)^3 - (5a)^3 - 3(3)(5a)(3-5a)$
$= (3-5a)^3$
$= (3-5a)(3-5a)(3-5a)$

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2 = (4a)^3 - (3b)^3 - 3(4a)(3b)(4a-3b)$
$= (4a-3b)^3$
$= (4a-3b)(4a-3b)(4a-3b)$

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p = (3p)^3 - (\frac{1}{6})^3 - 3(3p)(\frac{1}{6})(3p - \frac{1}{6})$
$= (3p - \frac{1}{6})^3$
$= (3p - \frac{1}{6})(3p - \frac{1}{6})(3p - \frac{1}{6})$

Q.9 Verify:
(i) $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$
(ii) $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$
Sol.
(i) By taking R.H.S $= (x + y)(x^2 - xy + y^2)$
$= x(x^2 - xy + y^2) + y(x^2 - xy + y^2)$
$= x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3$
$= x^3 + y^3$
= L.H.S.
Hence verified

(ii) By taking R.H.S. $= (x-y)(x^2 + xy + y^2)$
$= x(x^2 + xy + y^2) - y(x^2 + xy + y^2)$

$= x^3 + x^2y + xy^2 - x^2y - xy^2 - y^3$
$= x^3 - y^3$
$=$ L.H.S.

Hence verified

Q.10 Factorise each of the following:
(i) $27y^3 + 125z^3$
(ii) $64m^3 - 343n^3$

Sol.

(i) $27y^3 + 125z^3 = (3y)^3 + (5z)^3$
$= (3y+5z)[(3y)^2 - (3y)5z + (5z)^2]$
$= (3y+5z)(9y^2 - 15yz + 25z^2)$

(ii) $64m^3 - 343n^3 = (4m)^3 - (7n)^3$
$= (4m-7n)[(4m)^2 + (4m)(7n) + (7n)^2]$
$= (4m-7n)(16m^2 + 28mn + 49n^2)$

Q.11 Factorise: $27x^3 + y^3 + z^3 - 9xyz$

Sol. $27x^3 + y^3 + z^3 - 9xyz = (3x)^3 + y^3 + z^3 - 3(3x)(y)(z)$
$= (3x + y + z)[(3x)^2 + y^2 + z^2 - (3x)y - yz - z(3x)]$
$= (3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3zx)$

Q.12 Verify that $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)[(x-y)^2 + (y-z)^2 + (z-x)^2]$

Sol.

By taking R.H.S $= \frac{1}{2}(x + y + z)[(x-y)^2 + (y-z)^2 + (z-x)^2]$

$= \frac{1}{2}(x + y + z)(x^2 - 2xy + y^2 + y^2 - 2yz + z^2 + z^2 - 2zx + x^2)$

$= \frac{1}{2}(x + y + z)(2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx)$

$= \frac{2}{2}(x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$= (x + y + z)(x^2 + y^2 + z^2 - yz - zx - xy - xy - yz - zx)$
$= x^3 + y^3 + z^2 - 3xyz$
$=$ L.H.S.

Hence verified

Q.13 If $x + y + z = 0$, show that $x^3 + y^3 + z^3 = 3xyz$.

Sol.

Given: $x + y + z = 0$
$\Rightarrow x + y = -z$
By cubing both sides,
$(x + y)^3 = (-z)^3$
$\Rightarrow x^3 + y^3 + 3xy(x + y) = -z^3$
$\Rightarrow x^3 + y^3 - 3xyz = -z^3$ (Since, $x + y = -z$)
$\Rightarrow x^3 + y^3 + z^3 = 3xyz$
Hence Proved

Q.14 Without actually calculating the cubes, find the value of each of the following:
(i) $(-12)^3 + (7)^3 + (5)^3$
(ii) $(28)^3 + (-15)^3 + (-13)^3$

Sol.

(i) Suppose, $x = -12$, $y = 7$ and $z = 5$
Here $x + y + z = -12 + 7 + 5 = 0$
$\Rightarrow x^3 + y^3 + z^3 = 3xyz$
$\Rightarrow (-12)^3 + (7)^3 + (5)^3 = 3 \times (-12) \times 7 \times 5$
$= -1260$

(ii) Suppose, $x = 28$, $y = -15$ and $z = -13$
Here, $x + y + z = 28 - 15 - 13$
$= 0$
$\Rightarrow x^3 + y^3 + z^3 = 3xyz$
$\Rightarrow (28)^3 + (-15)^3 + (-13)^3 = 3(28)(-15)(-13)$
$= 16380$

Q.15 Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:
(i) Area: $25a^2 - 35a + 12$ (ii) Area: $35y^2 + 13y - 12$

Sol.

Since, possible length and breadth of the rectangle are the factors of its given area.
So, Area = length × breadth
(i) Area: $25a^2 + 35a + 12 = 25a^2 - 15a - 20a + 12$
$= 5a(5a - 3) - 4(5a - 3)$
$= (5a - 3)(5a - 4)$
Thus, possible length and breadth are $(5a - 3)$ and $(5a - 4)$ units respectively.

(ii) Area: $35y^2 + 13y - 12 = 35y^2 + 28y - 15y - 12$
$= 7y(5y + 4) - 3(5y + 4)$
$= (5y + 4)(7y - 3)$
Thus, possible length and breadth are $(5y + 4)$ and $(7y - 3)$ units respectively.

Q.16 What are the possible expressions for the dimensions of the cuboids whose volumes are given below:
(i) Volume: $3x^2 - 12x$
(ii) Volume: $12ky^2 + 8ky - 20k$

Sol.

Since, possible expressions for the dimensions of the cuboids are the factors of their volumes.
(i) Volume: $3x^2 - 12x = 3x(x - 4)$
Thus, the possible dimensions of cuboid are 3, x and $(x - 4)$ units.

(ii) Volume: $12ky^2 + 8ky - 20k$
$= 4k(3y^2 + 2y - 5)$
$= 4k[3y(y - 1) + 5(y - 1)]$
$= 4k(y - 1)(3y + 5)$
Thus, the possible dimensions of cuboid are $4k$, $(y - 1)$ and $(3y + 5)$ units.