Polynomials: Exercise - 2.4

Q.1 Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

(i) $2x^3+x^2-5x+2;\ \frac{1}{2}, 1, -2$

(ii) $x^3-4x^2+5x-2;\ 2, 1, 1$

Sol. **(i)** Given: Polynomial $p(x) = 2x^3+x^2-5x+2$

Compare the given polynomial with cubic polynomial $ax^3 + bx^2 + cx + d$, then

$a = 2, b = 1, c = -5$ and $d = 2$.

Numbers $= \frac{1}{2}, 1, -2$

$$p(\tfrac{1}{2}) = 2(\tfrac{1}{2})^3+(\tfrac{1}{2})^2 - 5(\tfrac{1}{2}) + 2$$
$$= \frac{1}{4}+\frac{1}{4}-\frac{5}{2}+2$$
$$= \frac{1+1-10+8}{4} = \frac{0}{4} = 0$$

$$p(1) = 2(1)^3 + (1)^2 - 5(1) + 2$$
$$= 2 + 1 - 5 + 2$$
$$= 0$$

$$p(-2) = 2(-2)^3 + (-2)^2 - 5(-2) + 2$$
$$= 2(-8) + 4 + 10 + 2$$
$$= -16 + 16 = 0$$

Since, given values the satisfy the given polynomial. So, therefore, $\frac{1}{2}$, 1 and -2 are the zeroes of $2x^3+x^2-5x+2$.

So, $\alpha = \frac{1}{2}$, $\beta = 1$ and $\gamma = -2$.

Therefore, $\alpha+\beta+\gamma = \frac{1}{2} + 1 + (-2) = \frac{1+2-4}{2}$

$$= -\frac{1}{2} = -\frac{b}{a}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = (\tfrac{1}{2})(1) + (1)(-2) + (-2)(\tfrac{1}{2})$$
$$= \frac{1}{2}-2-1$$
$$= \frac{1-4-2}{2} = \frac{-5}{2} = \frac{c}{a}$$

and $\alpha\beta\gamma = \frac{1}{2}\times 1\times -2 = 1 = -2/2 = -\frac{d}{a}$

From the above, we have verified the relationship between the zeroes and the coefficients.

(ii) Given: Polynomial $p(x) = x^3-4x^2+5x-2$

Compare the given polynomial with cubic polynomial ax^3+bx^2+cx+d, then

$a = 1, b = -4, c = 5$ and $d = -2$.

Numbers = 2, 1, 1

$$p(2)=(2)^3-4(2)^2+5(2)-2 = 8-16+10-2 = 0$$

$p(1) = (1)^3 - 4(1)^2 + 5(1) - 2 = 1-4+5-2 = 0$

Given numbers satisfy the given polynomial. So 2, 1 and 1 are the zeros of x^3-4x^2+5x-2.

Thus, $\alpha=2$, $\beta=1$ and $\gamma=1$.

Now $\alpha+\beta+\gamma = 2+1+1= 4 = -(-4)/1= -\frac{b}{a}$

$\alpha\beta + \beta\gamma + \gamma\alpha = (2)(1) + (1)(1) + (1)(2)$

$= 2+1+2 = 5 = 5/1= \frac{c}{a}$

and $\alpha\beta\gamma = (2)(1)(1) = 2 = -(-2) / 1= -\frac{d}{a}$

From the above, we have verified the relationship between the zeroes and the coefficients.

Q.2 Find a cubic polynomial with the sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, – 7, – 14 respectively.

Sol. Let α, β and γ be the zeroes of the cubic polynomial.

Given: $\alpha + \beta + \gamma = 2$

$\alpha\beta + \beta\gamma + \gamma\alpha = -7$

and $\alpha\beta\gamma = -14$

Cubic polynomial is given by $(x-\alpha)(x-\beta)(x-\gamma) = 0$

$\Rightarrow x^3-x^2(\alpha+\beta+\gamma) + x(\alpha\beta + \beta\gamma + \gamma\alpha) - \alpha\beta\gamma = 0$

$\Rightarrow x^3 - 2x^2 - 7x + 14 = 0$

So, The polynomials is $x^3 - 2x^2 - 7x + 14$.

Q.3 If the zeroes of the polynomial x^3-3x^2+x+1 are a – b, a, a + b, find a and b

Sol. Since, $(a-b)$, a $(a+b)$ are the three zeroes of the given polynomial x^3-3x^2+x+1,

Therefore, Sum of the zeros = -b/a

$(a-b) + a + (a+b) = -(-3)/1 = 3$

$\Rightarrow 3a = 3 \Rightarrow a = 1$ .... (i)

Sum of product of zeros = c/a

$(a-b)a + a(a+b) + (a+b)(a-b) = 1/1 = 1$

$\Rightarrow a^2 - ab + a^2 + ab + a^2 - b^2 = 1$

$\Rightarrow 3a^2 - b^2 = 1$

$\Rightarrow 3(1)^2 - b^2 = 1$ From the equation (i)

$\Rightarrow 3-b^2 = 1$

$\Rightarrow b^2 = 2$

$\Rightarrow b = \pm\sqrt{2}$

Thus, $a = 1$ and $b=\pm\sqrt{2}$.

Q.4 If two zeroes of the polynomial $x^4-6x^3-26x^2+138x-35$ are $2\pm\sqrt{3}$, find other zeroes.

***Sol.* Given:** $2\pm\sqrt{3}$ are two zeroes of the polynomial $p(x) = x^4 - 6x^3 - 26x^2 + 138x - 35$.

Suppose, $x = 2 \pm \sqrt{3} \Rightarrow x - 2 = \pm\sqrt{3}$

Squaring both the side, we get

$(x-2)^2 = (\pm\sqrt{3})^2$

$x^2-4x+4 = 3$

$\Rightarrow x^2-4x+1=0$

Let us divide polynomial $p(x) = \mathbf{x^4-6x^3-26x^2+138x-35}$ by x^2-4x+1 to obtain other zeroes.

$$\begin{array}{r} x^2-2x-35 \\ x^2-4x+1\overline{\big)\,x^4-6x^3-26x^2+138x-35} \\ x^4-4x^3+x^2 \qquad\qquad\qquad \\ -\quad+\quad- \qquad\qquad\qquad \\ \hline -2x^3-27x^2+138x \qquad \\ -2x^3+8x^2-2x \qquad \\ +\quad-\quad+ \qquad\qquad \\ \hline -35x^2+140x-35 \\ -35x^2+140x-35 \\ +\quad-\quad+ \\ \hline 0 \end{array}$$

$p(x) = x^4-6x^3-26x^2+138x-35$
$= (x^2-4x+1)(x^2-2x-35)$
$= (x^2-4x+1)(x^2-7x+5x-35)$
$= (x^2-4x+1)[x(x-7)+5(x-7)]$
$= (x^2-4x+1)(x+5)(x-7)$

$\Rightarrow$ $(x + 5)$ and $(x - 7)$ are other factors of the given [polynomial $p(x)$ = $\mathbf{x^4-6x^3-26x^2+138x-35}$.
Therefore,
$x = -5$ and $x = 7$ are other zeroes of the given polynomial.

Q.5 If the polynomial $x^4-6x^3+16x^2-25x+10$ is divided by another polynomial x^2-2x+k, the remainder comes out to be $x + a$ find k and a.

Sol. Given: Polynomial $p(x) = x^4-6x^3+16x^2-25x+10$ and $g(x) = x^2-2x+k$
Divide $x^4-6x^3+16x^2-25x+10$ by x^2-2x+k

$$\begin{array}{r} x^2-4x+(8-k) \\ x^2-2x+k\overline{\big)\,x^4-6x^3-16x^2-25x+10} \\ x^4-2x^3+kx^2 \qquad\qquad\qquad \\ -\quad+\quad- \qquad\qquad\qquad \\ \hline -4x^3+(16-k)x^2-25x \\ -4x^3+8x^2-4kx \\ +\quad-\quad+ \\ \hline (8-k)x^2+(4k-25)x \\ (8-k)x^2-2(8-k)x+(8-k)k \\ -\quad+\quad- \\ \hline (2k-9)x-(8-k)k+10 \end{array}$$

So, remainder = $(2k-9)x-(8-k)k+10$
given remainder = $x + a$.
$(2k-9)x-(8-k)k+10 = x + a$
By comparing their coefficients,
$2k-9 = 1$
$\Rightarrow 2k = 10$
$\Rightarrow k = 5$
and
$-(8-k)k+10 = a$
$\Rightarrow a = -(8-5)5+10$
$= -3\times 5+10$

$= -15 + 10$

$= -5$

Thus, the value of $k = 5$ and $a = -5$.