Polynomials: Exercise 2.4

Q.1 Determine which of the following polynomials has (x + 1) a factor :

(i) $x^3 + x^2 + x + 1$ **(ii) $x^4 + x^3 + x^2 + x + 1$**

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$ **(iv) $x^3 + x^2 - (2+\sqrt{2})x + \sqrt{2}$**

Sol.

(i) To prove that (x + 1) is a factor of $p(x) = x^3 + x^2 + x + 1$, We need to show that $p(-1) = 0$

So, $p(-1) = (-1)^3 + (-1)^2 + (-1) + 1$

$= -1 + 1 - 1 + 1$

$= 0$

Thus, (x + 1) is a factor of polynomial, $p(x) = x^3 + x^2 + x + 1$

(ii) To prove that (x + 1) is a factor of $p(x) = x^4 + x^3 + x^2 + x + 1$, we need to to show that $p(-1) = 0$.

So, $p(-1) = (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1$

$= 1 - 1 + 1 - 1 + 1$

$= 1 \neq 0$

Thus, (x + 1) is not a factor of $x^4 + x^3 + x^2 + x + 1$.

(iii) To prove that (x + 1) is a factor of $p(x) = x^4 + 3x^3 + 3x^2+x+1$, we need to show that $p(-1) = 0$.

So, $p(-1) = (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1$

$= 1 - 3 + 3 - 1 + 1$

$= 1 \neq 0$

Thus, (x + 1) is not a factor of $x^4 + 3x^3 + 3x^2 + x + 1$.

(iv) To prove that (x + 1) is a factor of $p(x) = x^3 - x^2 - (2+\sqrt{2})x + \sqrt{2}$, we need to show that $p(-1) = 0$

$p(-1) = (-1)^3 - (-1)^2 - (2+\sqrt{2})(-1) + \sqrt{2}$

$= -1 - 1 + 2 + \sqrt{2} + \sqrt{2}$

$= 2\sqrt{2} \neq 0$

Thus, (x + 1) is not a factor of $x^3 - x^2 - (2+\sqrt{2})x + \sqrt{2}$

Q.2 Use the factor theorem to determine whether g(x) is a factor of p (x) in each of the following cases :

(i) $p(x) = 2x^3 + x^2 - 2x - 1$, $g(x) = x + 1$

(ii) $p(x) = x^3 + 3x^2 + 3x + 1$, $g(x) = x + 2$

(iii) $p(x) = x^3 - 4x^2 + x + 6$, $g(x) = x - 3$

Sol.

(i) To prove that $g(x) = x + 1$ is a factor of $p(x) = 2x^3 + x^2 - 2x - 1$, we need to show that $p(-1) = 0$.

So, $p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$

$= -2 + 1 + 2 - 1$

$=0$

Thus, $g(x) = x+1$, is a factor of $p(x)= 2x^3 + x^2 - 2x - 1$.

(ii) To prove that $g(x) = x + 2$ is a factor of $p(x) = x^3 + 3x^2 + 3x + 1$, we need to show that $p(-2) = 0$

So, $p(-2) = (-2)^3 + 3(-2)^2 + 3(-2) + 1$

$= -8 + 12 - 6 + 1$

$= -1 \neq 0$

Thus, $g(x) = x + 2$ is not a factor of $p(x) = x^3 + 3x^2 + 3x + 1$.

(iii) To prove that $g(x) = x - 3$ is a factor of $p(x) = x^3 - 4x^2 + x + 6$. We need to show that $p(+3) = 0$

So, $p(3) = (3)^3 - 4(3)^2 + 3 + 6$

$= 27 - 36 + 3 + 6$

$= 36 - 36 = 0$

Thus, $g(x) = x - 3$ is a factor of $p(x) = x^3 - 4x^2 + x + 6$.

Q.3 Find the value of k, if x -1 is a factor of p (x) in each of the following cases:
(i) $p(x) = x^2 + x + k$
(ii) $p(x) = 2x^2 + kx + \sqrt{2}$
(iii) $p(x)=kx^2 - \sqrt{2}x + 1$
(iv) $p(x) = kx^2 - 3x + k$
Sol.
(i) Since, (x -1) is a factor of $p(x) = x^2 + x + k$, So
$p(1) = 0$
$\Rightarrow (1)^2 + 1 + k = 0$
$\Rightarrow 1 + 1 + k = 0$
$\Rightarrow k = -2$
Thus, k = -2

(ii) Since, (x - 1) is a factor of $p(x) = 2x^2 + kx + \sqrt{2}$, So
$p(1) = 0$
$\Rightarrow 2(1)^2 + k(1) + \sqrt{2} = 0$
$\Rightarrow 2 + k + \sqrt{2} = 0$
$\Rightarrow k = -(2+\sqrt{2})$

(iii) Since, (x -1) is a factor of $p(x) = kx^2 - \sqrt{2}x + 1$, So
$p(1) = 0$
$\Rightarrow k(1)^2 - \sqrt{2}(1) + 1 = 0$
$\Rightarrow k - \sqrt{2} + 1 = 0$
$\Rightarrow k = \sqrt{2} - 1$

(iv) Since, (x -1) is a factor of $p(x) = kx^2 - 3x + k$, So
$p(1) = 0$
$\Rightarrow k(1)^2 - 3(1) + k = 0$
$\Rightarrow k - 3 + k = 0$
$\Rightarrow 2k = 3$
$\Rightarrow k = \frac{3}{2}$

Q.4 Factorise:
(i) $12x^2 - 7x + 1$
(ii) $2x^2 + 7x + 3$
(iii) $6x^2 + 5x - 6$
(iv) $3x^2 - x - 4$
Sol.
(i) Given: $p(x) = 12x^2 - 7x + 1$,
By using splitting middle term method,
Coefficient of x = - 7
and coefficient of x^2 × constant term = 12 × 1 = 12
So, $-7 = -4 - 3$ and $12 = (-4)(-3)$
Therefore, $12x^2 - 7x + 1 = 12x^2 - 4x - 3x + 1$
$= 4x(3x-1) - 1(3x-1)$
$= (3x-1)(4x-1)$
Thus, factor of polynomial $p(x) = 12x^2 - 7x + 1$ are $(3x-1)(4x-1)$.

(ii) Given: $p(x) = 2x^2 + 7x + 3$
By using splitting middle term method,

Coefficient of x = 7
and coefficient of x^2 × constant term = 2 × 3 = 6
Therefore p + q = 7 = 1 + 6 and 6 = 1 × 6
Therefore, $2x^2 + 7x + 3 = 2x^2 + x + 6x + 3$
$= x(2x+1) + 3(2x+1)$
$= (2x+1)(x+3)$
Thus, factor of polynomial $p(x) = 2x^2 + 7x + 3$ are $(2x+1)(x+3)$.

(iii) Given: $p(x) = 6x^2 + 5x - 6$
By using splitting middle term method,
Coefficient of x = 5
and Coefficient of x^2 × constant term = 6 × (– 6) = – 36
Therefore, 5 = 9 + (– 4) and – 36 = 9 × (– 4)
Therefore, $6x^2 + 5x - 6 = 6x^2 + 9x - 4x - 6$
$= 3x(2x+3) - 2(2x+3)$
$= (2x+3)(3x-2)$
Thus, factor of polynomial $p(x) = 6x^2 + 5x - 6$ are $(2x+3)(3x-2)$.

(iv) Given: $p(x) = 3x^2 - x - 4$
By using splitting middle term method,
Coefficient of x = –1
and coefficient of x^2× constant term= 3 × (– 4) = – 12
Therefore, –1 = 3 + (– 4) and – 12 = 3 × (–4)
Therefore $3x^2 - x - 4 = 3x^2 + 3x - 4x - 4$
$= 3x(x + 1) - 4(x + 1)$
$= (x+1)(3x-4)$
Thus, factor of polynomial $p(x) = 3x^2 - x - 4$ are $(x+1)(3x-4)$.

Q.5 Factorise:
(i) $x^3 - 2x^2 - x + 2$
(ii) $x^3 - 3x^2 - 9x - 5$
(iii) $x^3 + 13x^2 + 32x + 20$
(iv) $2y^3 + y^2 - 2y - 1$
Sol.
(i) Suppose, $p(x) = x^3 - 2x^2 - x + 2$
Since, constant term in p(x) is + 2 and factors of + 2 are ±1 and ±2.
Putting x = 1 in p(x),
$p(1) = (1)^3 - 2(1)^2 - 1 + 2$
$= 1 - 2 - 1 + 2$
$= 0$
So, (x–1) is a factor of p(x).
Now, putting x = – 1 in p(x),
$p(-1) = (-1)^3 - 2(-1)^2 - (-1) + 2$
$= -1 - 2 + 1 + 2$
$= 0$
So, (x + 1) is a factor of p (x).
Putting x = 2 in p(x),
$p(2) = (2)^3 - 2(2)^2 - (2) + 2$
$= 8 - 8 - 2 + 2$
$= 0$
So, (x - 2) is a factor of p(x)
Now, Putting x = – 2 in p(x),
$p(-2) = (-2)^3 - 2(-2)^2 - (-2) + 2$
$= -8 - 8 + 2 + 2$
$= -12 \neq 0$
So, (x + 2) is not a factor of f (x).
Thus, factors of p(x) are (x – 1), (x + 1) and (x – 2).

Let $p(x) = k(x-1)(x+1)(x-2)$
$\Rightarrow x^3 - 2x^2 - x + 2 = k(x-1)(x+1)(x-2)$
Now, putting $x = 0$ on both sides,
$2 = k(-1)(1)(-2)$
$\Rightarrow k = 1$
Thus, $x^3 - 2x^2 - x + 2 = (x-1)(x+1)(x-2)$

(ii) Suppose, $p(x) = x^3 - 3x^2 - 9x - 5$
Since, constant term in p(x) is -5, all factors of -5 are ±1 and ±5.
Now put $x = -1$in p(x)
$p(-1) = -1 - 3 + 9 - 5$
$= 0$.
So $(x + 1)$ is a factor of p(x).
Now, divide p(x) by $(x + 1)$

$$\begin{array}{r|l} & x^2 - 4x - 5 \\ \hline x+1 & x^3 - 3x^2 - 9x - 5 \\ & x^3 + x^2 \\ & - \quad - \\ \hline & -4x^2 - 9x \\ & -4x^2 - 4x \\ & + \quad + \\ \hline & -5x - 5 \\ & -5x - 5 \\ & + \quad + \\ \hline & 0 \end{array}$$

Therefore, $p(x) = (x+1)(x^2 - 4x - 5)$
$= (x+1)(x^2 + x - 5x - 5)$
$= (x+1)[x(x+1) - 5(x+1)]$
$= (x+1)(x+1)(x-5)$
Thus, $x^3 - 2x^2 - x + 2 = (x+1)(x+1)(x-5)$

(iii) Suppose, $p(x) = x^3 + 13x^2 + 32x + 20$
Since, constant term in p(x) is 20, all factors of + 20are $\pm1, \pm2, \pm4, \pm5, \pm10$ and ±20.
Now, put $x = -2$ in p(x),
$p(-2) = (-2)^3 + 13(2)^2 + 32(-2) + 20$
$p(-2) = -8 + 52 - 64 + 20$
$= 0$
Thus, $(x + 2)$ is a factor of p(x)
Now, divide p(x) by $(x + 2)$

$$\begin{array}{r|l} & x^2 + 11x + 10 \\ \hline x+2 & x^3 + 13x^2 + 32x + 20 \\ & x^3 + 2x^2 \\ & - \quad - \\ \hline & 11x^2 + 32x \\ & 11x^2 + 22x \\ & - \quad - \\ \hline & 10x + 20 \\ & 10x + 20 \\ & - \quad - \\ \hline & 0 \end{array}$$

Thus, $p(x) = (x+2)(x^2 + 11x + 10)$
$= (x+2)(x^2 + x + 10x + 10)$
$= (x+2)[x(x+1) + 10(x+1)]$
$= (x+2)(x+1)(x+10)$

(iv) Suppose, $p(y) = 2y^3 + y^2 - 2y - 1$
Since, constant term in p(x) is -1, So all the factor of -1 is ±1.
Now put $x = 1$ in p(x),
$p(1) = 2 + 1 - 2 - 1$

= 0

So, (y – 1) is a factor of p(y)

Now, divide p(y) by (y – 1)

$$
\begin{array}{r}
2y^3 + 3y + 1 \\
y-1 \overline{\big)\, 2y^3 + y^2 - 2y - 1} \\
2y^3 - 2y^2 \\
\underline{- \quad +} \\
3y^2 - 2y \\
3y^2 - 3y \\
\underline{- \quad +} \\
y - 1 \\
\underline{y - 1} \\
0
\end{array}
$$

Thus, p(y) = $(y-1)(2y^2 + 3y + 1)$

$= (y-1)(2y^2 + 2y + y + 1)$

$= (y-1)[2y(y+1)+1(y+1)]$

$= (y-1)(y+1)(2y+1)$