Polynomials: Exercise - 2.3

Q.1 Divide the polynomial p(x) by the polynomial g (x) and find the quotient and remainder in each of the following:

(i) $p(x) = x^3-3x^2+5x-3$, $g(x) = x^2-2$

(ii) $p(x) = x^4-3x^2+4x+5$, $g(x) = x^2+1-x$

(iii) $p(x) = x^4-5x+6$, $g(x) = 2-x^2$

***Sol.* (i)** Given, $p(x) = x^3-3x^2+5x-3$, $g(x) = x^2-2$

$$
\begin{array}{r|l}
 & x-3 \\
x^2-2 & x^3-3x^2+5x-3 \\
 & x^3 \qquad -2x \\
 & - \qquad\quad + \\
\hline
 & -3x^2+7x-3 \\
 & -3x^2 \qquad +6 \\
 & + \qquad\quad - \\
\hline
 & 7x-9
\end{array}
$$

So, the quotient is $(x - 3)$ and the remainder is $(7x - 9)$.

(ii) $p(x) = x^4-3x^2+4x+5$, $g(x) = x^2+1-x$

$$
\begin{array}{r|l}
 & x^2-x-3 \\
x^2-x+1 & x^4 \qquad -3x^2+4x+5 \\
 & x^4-x^3+x^2 \\
 & - \quad + \quad - \\
\hline
 & x^3-4x^2+4x \\
 & x^3-x^2+x \\
 & - \quad + \quad - \\
\hline
 & -3x^2+3x+5 \\
 & -3x^2+3x-3 \\
 & + \quad - \quad + \\
\hline
 & 8
\end{array}
$$

So, the quotient is $x^2 + x - 3$ and the remainder is 8.

(iii) Given: $p(x) = x^4-5x+6$, $g(x) = 2-x^2$

Firstly, we write divisor in the standard form. So, divisor $= -x^2+2$

$$
\begin{array}{r|l}
 & -x^2-2 \\
-x^2+2 & x^4 \qquad -5x+5 \\
 & x^4 -2x^2 \\
 & - \quad + \\
\hline
 & 2x^2 -5x \\
 & 2x^2 -4 \\
 & - \qquad + \\
\hline
 & -5x+10
\end{array}
$$

Therefore, the quotient is $-x^2+2$ and the remainder is $-5x + 10$.

Q.2 Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

(i) $t^2-3,\ 2t^4+3t^3-2t^2-9t-12$

(ii) $x^2+3x+1,\ 3x^4+5x^3-7x^2+2x+2$

(iii) $x^3-3x+1,\ x^5-4x^3+x^2+3x+1$

Sol. To check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial, the remainder should be zero.

(i) Divide $2t^4+3t^3-2t^2-9t-12$ by t^2-3.

$$
\begin{array}{r|l}
 & 2t^2+3t+4 \\
\hline
t^3-3 & 2t^4+3t^3-2t^2-9t-12 \\
 & 2t^4 \quad -t^2 \\
 & - \qquad + \\
\hline
 & 3t^3+4t^2-9t \\
 & 3t^3 \qquad -9t \\
 & - \qquad\quad + \\
\hline
 & +4t^2-12 \\
 & +4t^2-12 \\
 & - \qquad + \\
\hline
 & 0
\end{array}
$$

Since, after division, the remainder is zero, therefore, t^2-3 is a factor of polynomial $2t^4+3t^3-2t^2-9t-12$.

(ii) Divide $3x^4+5x^3-7x^2+2x+2$ by x^2+3x+1.

$$
\begin{array}{r|l}
 & 3x^2-4x+2 \\
\hline
x^2+3x+1 & 3x^4+5x^3-7x^2+2x+2 \\
 & 3x^4+9x^3+3x^2 \\
 & - \quad - \quad - \\
\hline
 & -4x^3-10x^2+2x \\
 & -4x^3-12x^2-4x \\
 & + \quad + \quad + \\
\hline
 & -2x^2+6x+2 \\
 & -2x^2+6x+2 \\
 & + \quad - \quad - \\
\hline
 & 0
\end{array}
$$

Since, after the division, remainder is zero, So x^2+3x+1 is a factor of the polynomial $3x^4+5x^3-7x^2+2x+2$.

(iii) Divide $x^5-4x^3+x^2+3x+1$ by x^3-3x+1

$$
\begin{array}{r|l}
 & x^2-1 \\
\hline
x^3-3x+1 & x^5-4x^3+x^2+3x+1 \\
 & x^5-3x^3+x^2 \\
 & - \quad + \quad - \\
\hline
 & -x^3 \qquad +3x+1 \\
 & -x^3 \qquad +4x-1 \\
 & + \qquad - \quad + \\
\hline
 & 2
\end{array}
$$

Since, after division, remainder is 2 which is not equal to zero. So, x^3-3x+1 is not a factor of the polynomial $x^5-4x^3+x^2+3x+1$.

Q.3 Obtain all the zeroes of $3x^4+6x^3-2x^2-10x-5$ if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

Sol. Since, here two zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$, so $(x-\sqrt{\frac{5}{3}})$ and $(x+\sqrt{\frac{5}{3}})$ are the factors of polynomial $3x^4+6x^3-2x^2-10x-5$.

Now, $(x-\sqrt{\frac{5}{3}})(x+\sqrt{\frac{5}{3}}) \Rightarrow x^2-\frac{5}{3}=0$

$\Rightarrow (3x^2-5)$ is a factor of the given polynomial.

Now, applying the division algorithm to the given polynomial and $3x^2-5$:

$$
\begin{array}{r|l}
 & x^2+2x-1 \\
3x^2-5 & 3x^4+6x^3-2x^2-10x-5 \\
 & 3x^4 \qquad\quad -5x^2 \\
 & - \qquad\qquad + \\
\hline
 & \qquad 6x^3+3x^2-10x \\
 & \qquad 6x^3 \qquad\; -10x \\
 & \qquad - \qquad\quad + \\
\hline
 & \qquad\quad +3x^2 \qquad -5 \\
 & \qquad\quad +3x^2 \qquad -5 \\
 & \qquad\quad - \qquad\quad + \\
\hline
 & \qquad\qquad\quad 0
\end{array}
$$

Here Quotient = $x^2 + 2x + 1$, and Remainder = 0
So, $3x^4+6x^3-2x^2-10x-5 = (3x^2-5)(x^2+2x+1)$
Now, $x^2 + 2x + 1 = x^2 + x + x + 1$
$= x(x + 1) + 1(x + 1)$
$= (x+1)(x+1)$
So, Two other zeroes are -1 and -1.

Thus, all the zeroes of the given polynomial are $\sqrt{\frac{5}{3}}$, $-\sqrt{\frac{5}{3}}$, -1 and -1.

Q.4 On dividing x^3-3x^2+x+2 by a polynomial g(x), the quotient and remainder were x – 2 and – 2x + 4 respectively. Find g(x).

Sol. Since on dividing x^3-3x^2+x+2 by a polynomial g(x), the quotient and remainder were (x – 2) and (– 2x + 4) respectively.
So, by applying division algorithm:

Quotient × Divisor + Remainder = Dividend

$\Rightarrow (x-2) \times g(x) + (-2x+4) = x^3-3x^2+x-2$

$\Rightarrow (x-2) \times g(x) = x^3-3x^2 + x - 2 + 2x - 4$

$$\Rightarrow g(x) = \frac{x^3-3x^2+3x-2}{x-2} \qquad \ldots (1)$$

Now, we have to divide $x^3-3x^2 + 3x - 2$ by $x-2$.

$$
\begin{array}{r|l}
 & x^2 - x + 1 \\
\hline
x - 2 & x^3 - 3x^2 + 3x - 2 \\
 & x^3 - 2x^2 \\
 & -\quad + \\
\hline
 & -x^2 + 3x \\
 & -x^2 + 2x \\
 & +\quad - \\
\hline
 & x - 2 \\
 & x - 2 \\
 & -\quad + \\
\hline
 & 0
\end{array}
$$

So, g (x) = $x^2 - x + 1$

Q.5 Give examples of polynomials p(x), g(x) , q (x) and r (x), which satisfy the division lgorithm and

(i) deg p(x) = deg q(x)

(ii) deg q(x) = deg r (x)

(iii) deg r (x) = 0

Sol. Given: p(x) = Dividend; g(x) = Divisor; q(x) = Quotient; r(x) = Remainder.

According to Division algorithm:

Quotient × Divisor + Remainder = Dividend

q(x) x g(x) + r(x) = P(x)

(i) deg p(x) = deg q(x)

Degree of dividend is equal to degree of quotient, when divisor is a constant term.

$p(x) = 2x^2 - 2x + 14$, g(x) = 2,

$q(x) = x^2 - x + 7$, r(x) = 0

(ii) deg q(x) = deg r (x)

Degree of quotient is equal to degree of remainder.

$p(x) = x^3 + x^2 + x + 1$, $g(x) = x^2 - 1$,

q(x) = x+1, r(x) = 2x+2

(iii) deg r (x) = 0

Degree of remainder is equal to zero.

$p(x) = x^3 + 2x^2 - x + 2$,

$g(x) = x^2 - 1$, q(x) = x+2, r(x) = 4