Polynomials: Exercise 2.3

Q.1 Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by

(i) $x + 1$ **(ii) $x - \frac{1}{2}$**

(iii) x **(iv) $x + \pi$** **(v) $5 + 2x$**

Sol. **(i)** From the remainder theorem, the required remainder is equal to p (-1).
Given: $p(x) = x^3 + 3x^2 + 3x + 1$
So, $p(-1) = (-1)^3 + 3(-1)^2 + 3(-1) + 1$
$= -1 + 3 - 3 + 1$
$= 0$
Thus, required remainder $= p(-1) = 0$

(ii) From the remainder theorem, the required remainder is equal to $p(\frac{1}{2})$.

Given: $p(\frac{1}{2}) = x^3 + 3x^2 + 3x + 1$

$$= (\frac{1}{2})^3 + 3(\frac{1}{2})^2 + 3(\frac{1}{2}) + 1$$

$$= \frac{1}{8} + \frac{3}{4} + \frac{3}{2} + 1 \quad = \frac{1+6+12+8}{8} \quad = \frac{27}{8}$$

Thus, the required remainder $= p\,(\frac{1}{2}) = \frac{27}{8}$

(iii) From the remainder theorem, the required remainder is equal to p (0).
Given: $p(x) = x^3 + 3x^2 + 3x + 1$
So, $p(0) = 0 + 0 + 0 + 1$
$= 1$
Thus, the required remainder $= p(0) = 1$

(iv) From the remainder theorem, the required remainder is $p(-\pi)$
Given: $p(x) = x^3 + 3x^2 + 3x + 1$
So, $p(-\pi) = (-\pi)^3 + 3(-\pi)^2 + 3(-\pi) + 1$
$= -\pi^3 + 3\pi^2 - 3\pi + 1$
Thus, the required remainder $= p(x) = -\pi^3 + 3\pi^2 - 3\pi + 1$

(v) From the remainder theorem, the required remainder is $p(-\frac{5}{2})$

Given: $p(x) = x^3 + 3x^2 + 3x + 1$

So, $p(-\frac{5}{2}) = (-\frac{5}{2})^3 + 3(-\frac{5}{2})^2 + 3(-\frac{5}{2}) + 1$

$$= -\frac{125}{8} + \frac{75}{4} - \frac{15}{2} + 1$$

$$= \frac{-125+150-60+8}{8} = \frac{-27}{8}$$

Thus, required remainder $= p(-\frac{5}{2}) = \frac{-27}{8}$

Q.2 Find the remainder when $x^3 - ax^2 + 6x - a$ is divided by $x - a$.

Sol. Let $p(x) = x^3 - ax^2 + 6x - a$

From the remainder theorem, when p(x) is divided by (x – a).

Then remainder = p (a)

So, $p(a) = a^3 - a.a^2 + 6a - a$

$= a^3 - a^3 + 6a - a$

$= 5a$

Thus, required remainder = p(x) = 5a

Q.3 Check whether $7 + 3x$ is a factor of $3x^3 + 7x$

Sol. If (7 + 3x) will be a factor of $p(x)=3x^3 + 7x$, then $p(\frac{-7}{3}) = 0$

So, $p(\frac{-7}{3}) = 3(\frac{-7}{3})^3 + 7(\frac{-7}{3})$

$= 3\times(-\frac{343}{27}) - \frac{49}{3}$

$= -\frac{343}{9} - \frac{49}{3} \neq 0$

Thus, (7 + 3 x) is not a factor of $(3x^3+7x)$.