Number Systems: Exercise 1.5

Q.1 Classify the following numbers as rational or irrational:

(i) $2-\sqrt{5}$ **(ii)** $(3+\sqrt{23})-\sqrt{23}$ **(iii)** $\frac{2\sqrt{7}}{7\sqrt{7}}$ **(iv)** $\frac{1}{\sqrt{2}}$ **(v)** 2π

Sol. **(i)** $2-\sqrt{5}$ is an irrational number because it is a difference between a rational and an irrational.

(ii) $(3+\sqrt{23})-\sqrt{23} = 3+\sqrt{23}-\sqrt{23} = 3$, it is a rational number.

(iii) $\frac{2\sqrt{7}}{7\sqrt{7}} = \frac{2}{7}$, It is a rational number.

(iv) $\frac{1}{\sqrt{2}}$ is an irrational number because the quotient of a rational and an irrational number.

(v) 2π is irrational because the product of rational and irrational number.

Q.2 Simplify each of the following expressions:

(i) $(3+\sqrt{3})(2+\sqrt{2})$ **(ii)** $(3+\sqrt{3})(3-\sqrt{3})$

(iii) $(\sqrt{5}+\sqrt{2})^2$ **(iv)** $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$

Sol.

(i) $(3+\sqrt{3})(2+\sqrt{2}) = 3\times2+3\sqrt{2}+2\sqrt{3}+\sqrt{2}\times\sqrt{3}$

$= 6+3\sqrt{2}+2\sqrt{3}+\sqrt{6}$

(ii) $(3+\sqrt{3})(3-\sqrt{3}) = (3)^2-(\sqrt{3})^2$

$= 9-3 = 6$

(iii) $(\sqrt{5}+\sqrt{2})^2 = (\sqrt{5})^2+2\sqrt{5}\sqrt{2}+(\sqrt{2})^2$

$= 5+2\sqrt{10}+2$

$= 7+2\sqrt{10}$

(iv) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2}) = (\sqrt{5})^2-(\sqrt{2})^2$

$= 5-2$

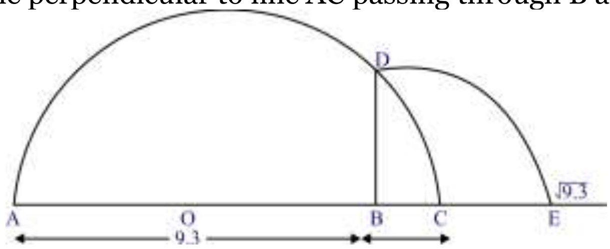
$=3$

Q.3 Recall, π is defined as the ratio of the circumference (say c) of a circle to its diameter (say d). That is $\pi = \frac{c}{d}$. This seems to contradict the fact that π is irrational. How will you resolve this contradiction?

Sol. There is no contradiction because either c or d irrational and thus, π is also irrational number.

Q.4 Represent $\sqrt{9.3}$ on the number line.

Sol. Draw the line segment of length 9.3 units from a fixed point A to obtain a point B such that AB = 9.3 units. From point B, mark a distance of 1 unit and mark the new point as C.
Now, find the mid- point of AC and mark that point as O.
Draw a semi-circle with centre O and take as radius OC.
Now, draw a line perpendicular to line AC passing through B and intersecting the semicircle at point D.



Then BD = $\sqrt{9.3}$.

For representing $\sqrt{9.3}$ on the number line, let the line BC consider as the number line, with B as Origin, C as 1 and so on.
Now, draw an arc with centre B and radius BD, which intersects the number line in E. Then point E represents $\sqrt{9.3}$.

Q.5 Rationalise the denominators of the following:

(i) $\frac{1}{\sqrt{7}}$

(ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$

(iv) $\frac{1}{\sqrt{7}-2}$

Sol.

(i) $\frac{1}{\sqrt{7}} = \frac{1 x \sqrt{7}}{\sqrt{7} x \sqrt{7}} = \frac{\sqrt{7}}{7}$

(ii) $\frac{1}{\sqrt{7}-\sqrt{6}} = \frac{1}{\sqrt{7}-\sqrt{6}} x \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}} = \frac{\sqrt{7}+\sqrt{6}}{(\sqrt{7})^2-(\sqrt{6})^2}$

$= \frac{\sqrt{7}+\sqrt{6}}{7-6} = \sqrt{7}+\sqrt{6}$

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}} = \frac{1}{\sqrt{5}+\sqrt{2}} x \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}} = \frac{\sqrt{5}-\sqrt{2}}{(\sqrt{5})^2-(\sqrt{2})^2}$

$= \frac{\sqrt{5}-\sqrt{2}}{5-2} = \frac{\sqrt{5}-\sqrt{2}}{3}$

(iv) $\frac{1}{\sqrt{7}-2} = \frac{1}{\sqrt{7}-2} x \frac{\sqrt{7}+2}{\sqrt{7}+2} = \frac{\sqrt{7}+2}{(\sqrt{7})^2-(2)^2}$

$= \frac{\sqrt{7}+2}{7-4} = \frac{\sqrt{7}+2}{3}$