Number Systems: Exercise 1.3

Q.1 Write the following in decimal form and say what kind of decimal expansion each has:

(i) $\frac{36}{100}$ **(ii)** $\frac{1}{11}$ **(iii)** $4\frac{1}{8}$ **(iv)** $\frac{3}{13}$ **(v)** $\frac{2}{11}$ **(vi)** $\frac{329}{400}$

***Sol.* (i)** $\frac{36}{100}$ in decimal form:

$\frac{36}{100} = 0.36$ (Terminating Decimal)

(ii) $\frac{1}{11}$ in decimal form:

From long division,

```
      0.090909
11 ) 1.000000
      99
      ---
       100
        99
       ---
        100
         99
        ---
          1
```

Thus, $\frac{1}{11} = 0.090909..... = 0.\overline{09}$ (Non- terminating and repeating decimal)

(iii) $4\frac{1}{8}$ in decimal form:

$4\frac{1}{8} = \frac{4\times 8+1}{8} = \frac{33}{8}$

From long division,

```
     4.125
8 ) 33.000
    32
    ---
     10
      8
    ---
      20
      16
     ---
       40
       40
      ---
        0
      ---
```

Thus, $\frac{33}{8} = 4.125$(Terminating decimal)

(iv) $\frac{3}{13}$, in decimal form

From the long division,

$$
\begin{array}{r}
0.23076923\ldots \\
13\overline{)\,3.0000000} \\
\underline{2\;6} \\
40 \\
\underline{39} \\
100 \\
\underline{91} \\
90 \\
\underline{78} \\
120 \\
\underline{117} \\
30 \\
\underline{26} \\
40 \\
\underline{39} \\
\underline{1}
\end{array}
$$

Thus, $\frac{3}{13}$ = 0.23076923…. = $0.\overline{230769}$ (Non- terminating and repeating decimal)

(v) $\frac{2}{11}$ in decimal form

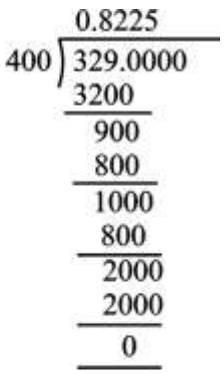
From long division,

$$
\begin{array}{r}
0.181818\ldots \\
11\overline{)\,2.00000} \\
\underline{1\;1} \\
90 \\
\underline{88} \\
20 \\
\underline{11} \\
90 \\
\underline{88} \\
20 \\
\underline{11} \\
90 \\
\underline{88} \\
\underline{2}
\end{array}
$$

Thus, $\frac{2}{11}$ = 0.181818…..= $0.\overline{18}$ (Non- terminating and repeating decimal)

(vi) $\frac{329}{400}$ in decimal form:

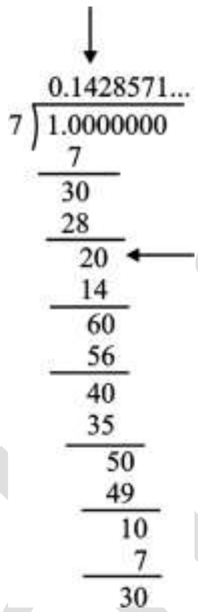
From the long division,



Thus, $\frac{329}{400}$ = 0.8225 (terminating decimal)

Q.2 You know that $\frac{1}{7} = 0.\overline{142857}$. Can you predict what the decimal expansions of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ are , without actually doing the long division? If so, how?

Sol. Yes. All of the above will have repeating decimals which are permutations of 1, 4, 2, 8, 5, 7.
Example, here



Therefore $\frac{2}{7} = 2 \times \frac{1}{7} = 2\times 0.\overline{\mathbf{142857}} = 0.\overline{285714}$

Similarly, $\frac{3}{7} = 3 \times \frac{1}{7} = 3\times 0.\overline{\mathbf{142857}} = 0.\overline{428571}$

$\frac{4}{7} = 4 \times \frac{1}{7} = 4\times 0.\overline{\mathbf{142857}} = 0.\overline{571428}$

$\frac{5}{7} = 5 \times \frac{1}{7} = 5\times 0.\overline{\mathbf{142857}} = 0.\overline{714285}$,

$\frac{6}{7} = 6 \times \frac{1}{7} = 6\times 0.\overline{\mathbf{142857}} = 0.\overline{857142}$

Q.3 Express the following in the form $\frac{p}{q}$, where p and q are integers and q ≠ 0.

(i) $0.\overline{6}$ **(ii)** $0.4\overline{7}$ **(iii)** $0.\overline{001}$

Sol.

(i) Suppose, $x = 0.\overline{6}$

Then, $x = 0.666 \ldots \ldots$ (i)

Since, we have only one repeating digit. So, multiply both sides of eq. (i) by 10,

$10x = 6.66\ldots. \ldots$ (ii)

Subtract (i) from (ii)

$10x - x = (6.66\ldots) - (0.66\ldots)$

$\Rightarrow 9x = 6$

$\Rightarrow x = \frac{2}{3}$

Thus, $0.\overline{6} = \frac{2}{3}$

(ii) Let $x = 0.4\overline{7}$

Since, there is just one digit after the decimal point which is without bar. So, we multiply both sides by 10. So that only the repeating decimal is left after the decimal point.

So, $10x = 4.\overline{7}$

$\Rightarrow 10x = 4 + 0.\overline{7}$

$\Rightarrow 10x = 4 + \frac{7}{9}$

$\Rightarrow 10x = \frac{4 \times 9 + 7}{9}$

$\Rightarrow 10x = \frac{43}{9}$

$\Rightarrow x = \frac{43}{90}$

Thus, $0.4\overline{7} = \frac{43}{90}$

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Let $x = 0.4\overline{7} = 0.4777\ldots$

So, $10x = 4.777\ldots \ldots$ (i)

and $100x = 47.777\ldots \ldots$ (ii)

Subtract (i) from (ii)

$100\,x - 10\,x = (47.777 \ldots) - (4.777\ldots)$

$\Rightarrow 90x = 43 \Rightarrow x = \frac{43}{90}$

Thus, $0.4\overline{7} = \frac{43}{90}$

(iii) Let $x = 0.\overline{001}$

$\Rightarrow x = 0.001001001 \ldots\ldots.$ (i)

Since , three repeating digits after the decimal point. So multiply (i) by1000,

$1000x = 1.001001\ldots \ldots$ (ii)

Subtract (i) from (ii)

$1000\,x - x = (1.001001\ldots) - (0.001001\ldots)$

$\Rightarrow \quad 999x = 1$

$\Rightarrow \quad x = \frac{1}{999}$

Thus, $0.\overline{001} = \frac{1}{999}$

Q.4 Express 0.99999... in the form $\frac{p}{q}$. Are you surprised by your answer? With your teacher and classmates discuss why the answer makes sense.

Sol. Suppose, x = 0.9999 … … (1)

Since, here only one repeating digit. So, multiply both sides of (i) by 10.

$10x = 9.999\ldots \ldots$ (ii)

Subtract (i) from (ii)

$10x - x = (9.999\ldots) - (0.999\ldots)$

$\Rightarrow \quad 9x = 9$

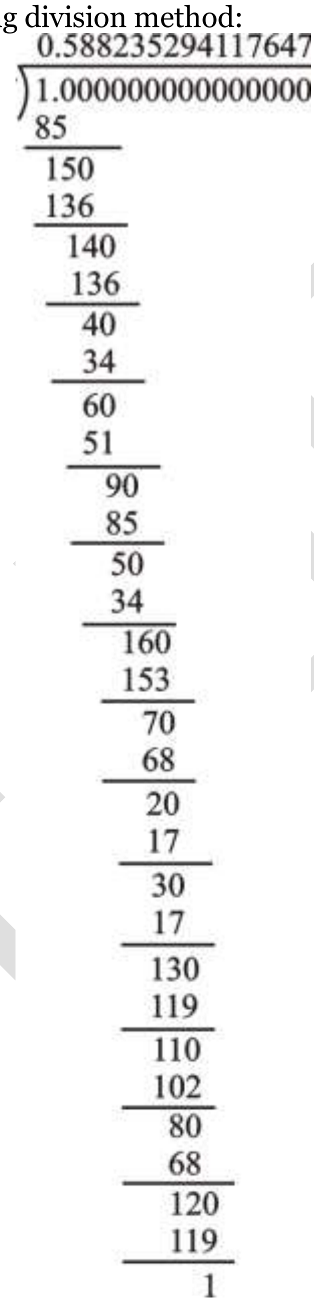
$\Rightarrow \quad x = 1$

Thus, 0.9999 … = 1

Since, 0.9999... goes to infinity . So, there is no gap between 1 and 0.9999... and Thus, they are equal.

Q.5 What can the maximum number of digits be in the repeating block of digits in the decimal expansion of $\frac{1}{17}$? Perform the division to check your answer.

Sol. From the long division method:



Thus, $\frac{1}{17} = 0.\overline{5882352941176470}$

Thus, the maximum number of digits in the quotient while computing $\frac{1}{17}$ are 15.

Q.6 Look at several examples of rational numbers in the form $\frac{P}{q}$ (q ≠ 0), where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy?

Sol. Consider some rational numbers in the form $\frac{P}{q}$ (q ≠ 0), where p and q are integers with common factors other than 1 and having terminating decimal representation.

Let the various rational numbers be $\frac{1}{2}, \frac{1}{4}, \frac{7}{8}, \frac{37}{25}, \frac{8}{125}$, etc.

In all cases, some natural number which when multiplied by their respective denominators gives 10 or a power of 10.

$\frac{1}{2} = \frac{1 \times 5}{2 \times 5} = \frac{5}{10} = 0.5$

$\frac{1}{4} = \frac{1 \times 25}{4 \times 25} = \frac{25}{100} = 0.25$

$\frac{7}{8} = \frac{7x125}{8x125} = \frac{875}{1000} = 0.875$

$\frac{37}{25} = \frac{37 \times 4}{25 \times 4} = \frac{128}{100} = 1.28$

$\frac{8}{125} = \frac{8 \times 8}{125 \times 8} = \frac{64}{1000} = 0.064$

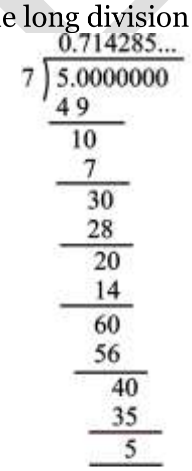
From the above examples, we have seen that those rational numbers whose denominators when multiplied by a suitable integer convert into a power of 10 are expressible in the finite decimal form.
But this can always be done only when the denominator of the given rational number has either 2 or 5 or both of them as the only prime factors. Therefore, we obtain the following properties:
If the denominator of a rational number in standard form has no prime factors other than 2 or 5, then and only then it can be represented as a terminating decimal.

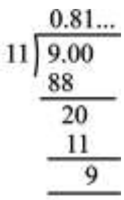
Q.7 Write three numbers whose decimal expansions are non- terminating non- recurring.

Sol. The three numbers whose decimal representations are non- terminating non- recurring are: $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{5}$ or 0.100100010001..., 0.20200200020002... and 0.003000300003.

Q.8 Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.

Sol. From the long division method.





So, $\frac{5}{7} = 0.\overline{714285}$ and $\frac{9}{11} = 0.\overline{81}$

Therefore, three different irrational numbers between $\frac{5}{7}$ and $\frac{9}{11}$ are:

0.75075007500075000075.... , 0.767076700767000... and 0.808008000800000...

Q.9 Classify the following numbers as rational or irrational :

(i) $\sqrt{23}$

(ii) $\sqrt{225}$

(iii) 0.3796

(iv) 7.478478 ...

(v) 1.101001000100001...

Sol.

(i) $\sqrt{23}$ is an irrational number. Since 23 is not a perfect square.

(ii) $\sqrt{225} = \sqrt{3\times3\times5\times5}$

$= 3\times5$

$= 15$

3	225
3	75
5	25
5	5
	1

Therefore, 15 is a rational number.

(iii) 0.3796 is a rational number. Since, it is terminating decimal.

(iv) 7.478478 .... is non- terminating and repeating. So, it is a rational number.

(v) 1.101001000100001 ... is non - terminating and non- repeating. So, it is an irrational number.