Number Systems: Exercise 1.1

Is zero a rational number? Can you write it in the form $\frac{p}{a}$, where p and q are integers **Q.1 and q ≠ o**?

Sol. Yes, zero is a rational number. Since, 0 can be written as $\frac{p}{q}$, where p = 0 and q is any non-zero integer.

Zero can be written in any of the following forms: $\frac{0}{1}, \frac{0}{-1}, \frac{0}{2}$, and so on. Thus, o is a rational number.

Q.2 Find six rational numbers between 3 and 4.

Sol. Since, between two rational numbers x and y, such that x < y, there is a rational number $\frac{x+y}{2}$. i.e., x <

$$\frac{x+y}{2} < y$$

In question, two rational numbers 3 and 4 are given, such that 3 < 4 , a rational number between 3 and 4 $\frac{7}{2}$

$$15 - (3 + 4)$$
 i.e.,
So, $3 < \frac{7}{2} < 4$

Now, two rational numbers 3 and $\frac{7}{2}$ such that, a rational number between 3 and $\frac{7}{2}$ is $\frac{1}{2}(3+\frac{7}{2}) = \frac{1}{2} \times \frac{6+7}{2}$

is

$$=\frac{13}{4}$$
.

Two rational numbers $\frac{7}{2}$ and 4, a rational number between $\frac{7}{2}$ and 4 is

$$\frac{1}{2} \left(\frac{7}{2} + 4\right) = \frac{1}{2} \times \frac{7+8}{2} = \frac{15}{4}$$

So, $3 < \frac{13}{4} < \frac{7}{2} < \frac{15}{4} < 4$
Two rational numbers 3 and $\frac{13}{4}$, a rational number between 3 and $\frac{13}{4}$ is
 $\frac{1}{2} \left(3 + \frac{13}{4}\right) = \frac{1}{2} \times \frac{12+13}{4} = \frac{25}{8}$
Two rational numbers $\frac{15}{4}$ and 4, a rational number between $\frac{15}{4}$ and 4 is
 $\frac{1}{2} \left(\frac{15}{4} + 4\right) = \frac{1}{2} \times \frac{15+16}{4} = \frac{31}{8}$
Two rational numbers $\frac{31}{8}$ and 4, a rational number between $\frac{31}{8}$ and 4 is
 $\frac{1}{2} \left(\frac{31}{8} + 4\right) = \frac{1}{2} \times \frac{31+32}{8} = \frac{63}{16}$

So, $3 < \frac{25}{8} < \frac{13}{4} < \frac{7}{2} < \frac{15}{4} < \frac{31}{8} < \frac{63}{16} < 4$
Thus, six rational numbers between 3 and 4 are: $\frac{25}{8}$, $\frac{13}{4}$, $\frac{7}{2}$, $\frac{15}{4}$, $\frac{31}{8}$ and $\frac{63}{16}$
ALTERNATIVE METHOD
Since, six rational numbers between 3 and 4, so we can write
$3 = \frac{3x7}{1x7} = \frac{21}{7}$ and $4 = \frac{4x7}{1x7} = \frac{28}{7}$
As we know that 21 < 22 < 23 < 24 < 25 < 26 < 27 < 28
$\Rightarrow \frac{21}{7} < \frac{22}{7} < \frac{23}{7} < \frac{24}{7} < \frac{25}{7} < \frac{26}{7} < \frac{27}{7} < \frac{28}{7}$
Thus, six rational numbers between $3 = \frac{21}{7}$ and $4 = \frac{28}{7}$ are $\frac{22}{7}$, $\frac{23}{7}$, $\frac{24}{7}$, $\frac{25}{7}$, $\frac{26}{7}$ and $\frac{27}{7}$.
Q.3 Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.
Sol. Since, five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$, so we can write.
$\frac{3}{5} = \frac{3x6}{5x6} = \frac{18}{30}$ and $\frac{4}{5} = \frac{4x6}{5x6} = \frac{24}{30}$
As we know that 18 < 19 < 20 < 21 < 22 < 23 < 24
$\Rightarrow \frac{18}{30} < \frac{19}{30} < \frac{20}{30} < \frac{21}{30} < \frac{22}{30} < \frac{23}{30} < \frac{24}{30}$
Thus, five rational numbers between $\frac{3}{5} = \frac{18}{30}$ and $\frac{4}{5} = \frac{24}{30}$ are: $\frac{19}{30}$, $\frac{20}{30}$, $\frac{21}{30}$, $\frac{22}{30}$ and $\frac{23}{30}$.

Q.4 State whether the following statements are true or false. Give reasons for your answers. (i) Every natural number is a whole number. (ii) Every integer is a whole number.

(iii) Every rational number is a whole number. Sol. (i) This statement is true. Since, every natural number lies in whole numbers. (ii) This statement is false. Since, number -3 is not a whole number.

(iii) This statement is false. Since, $\frac{3}{5}$ is not a whole number.