Linear Equations in Two Variables: Exercise - 3.2

Q.1 From the pair of linear equations in the following problems, and find their solution graphically.
(i) 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boy and girls who took part in the quiz.
(ii) 5 pencils and 7 pens together cost Rs. 50, whereas 7 pencils and 5 pens together cost Rs. 46. Find the cost of one pencil and that of one pen.

Sol. (i) Let X be the number of boys and y be the number of girls.
Total number of students:

$x + y = 10$ ..... (1)

The number of girls is 4 more than the number of boys:

$y = x + 4$

$\Rightarrow -x + y = 4$ ..... (2)

For drawing the graphs of equations (1) and (2) by putting the different values of x, we get different value of y.

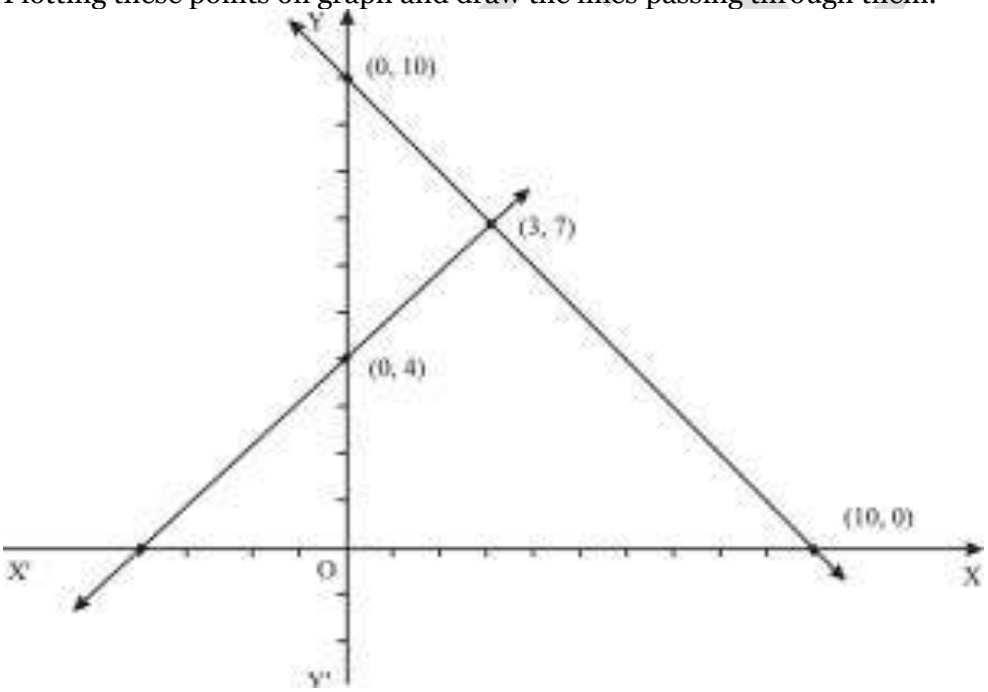
For $x + y = 10$

x	0	10
y	10	0

For $-x + y = 4$

x	0	−4
y	4	0

Plotting these points on graph and draw the lines passing through them.



Since, these two lines intersect at point (3, 7). So, x = 3 and y = 7 is the solution.
So, the number of boys are 3 and girls are 7.
Verification: Put x = 3 and y = 7 in both the equations (1) and (2), we find that both the equations are satisfied.

(ii) Let x be the cost of one pencil and y be the cost the one pen.
5 pencils and 7 pens together cost Rs. 50:

$5x + 7y = 50$ ... (1)

7 pencils and 5 pens together cost Rs. 46:

$7x + 5y = 46$ ... (2)

For drawing the graphs of equations (1) and (2) by putting the different values of x, we get different value of y.

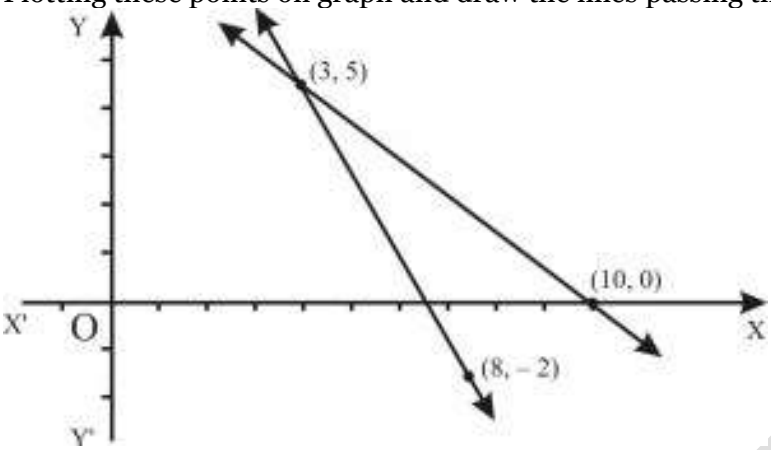
For $5x + 7y = 50$

x	10	3
y	0	5

For $7x + 5y = 46$

x	8	3
y	-2	5

Plotting these points on graph and draw the lines passing through them.



These two lines intersect at point (3, 5). So x = 3 and y = 5 is the required solution.
So, the cost of one pencil is Rs. 3 and one pen is Rs. 5.
Verification: By putting x = 3 and y = 5 in both the equations (1) and (2), we find that both the equations are satisfied.

Q.2 On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$ find out whether the representing the following pairs of linear equations intersect at a point, parallel or coincide.

(i) $5x - 4y + 8 = 0$; $7x + 6y - 9 = 0$
(ii) $9x + 3y + 12 = 0$; $18x + 6y + 24 = 0$
(iii) $6x - 3y + 10 = 0$; $2x - y + 9 = 0$

Sol. (i) Given pair of linear equations are:

$5x - 4y + 8 = 0$ ..... (i)

and, $7x + 6y - 9 = 0$.....(ii)

Here, $\frac{5}{7} \neq \frac{-4}{6}$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, So, the pairs of equations given in the question have a unique solution. So, equation (i) and (ii) are intersecting lines.

(ii) Given pair of linear equations are

$9x + 3y + 12 = 0$ .....(i)

and, $18x + 6y + 24 = 0$ .....(ii)

Here, $\frac{9}{18} = \frac{3}{6} = \frac{12}{24} = \frac{1}{2}$

Since, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2}$ so, the pairs of equations given in the question have infinite possible solutions.

Therefore, (i) and (ii) are coincident lines.

(iii) Given pair of linear equations are

$6x - 3y + 10 = 0$ .....(i)

and, $2x - y + 9 = 0$ .....(ii)

Here, $\frac{6}{2} = \frac{-3}{-1} \neq \frac{10}{9}$

Since, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, So, (i) and (ii) are parallel lines

Q.3 On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the following pairs of linear equations are consistent, or inconsistent.

(i) $3x + 2y = 5$; $2x - 3y = 7$

(ii) $2x - 3y = 8$; $4x - 6y = 9$

(iii) $32x+53y=7$; $9x - 10y = 14$

(iv) $5x - 3y = 11$; $-10x + 6y = -22$

(v) $43x+2y=8$; $2x + 3y = 12$.

Sol. (i) Given pair of linear equations:

$3x + 2y = 5$; $2x - 3y = 7$

$3x + 2y - 5 = 0$; $2x - 3y - 7 = 0$

From both the equations:

$a_1 = 3, b_1 = 2, c_1 = -5$

$a_2 = 2, b_2 = -3, c_2 = -7$

$\frac{a_1}{a_2} = \frac{3}{2}, \frac{b_1}{b_2} = \frac{2}{-3}$

Since, $\frac{3}{2} \neq \frac{2}{3}$, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Therefore, the given pair of linear equations intersect each other at one point and they have only one possible solution. So, the pair of linear equations is consistent.

(ii) Given pair of linear equations:

$2x - 3y = 8$ and $4x - 6y = 9$

$2x - 3y - 8 = 0$ and $4x - 6y - 3 = 0$

$a_1 = 2, b_1 = -3, c_1 = -8$ and $a_2 = 4, b_2 = -6, c_2 = -9$

$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-8}{-9} = \frac{8}{9}$

Since, $\frac{1}{2} = \frac{1}{2} \neq \frac{8}{9}$, i.e., $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Therefore, So, the equations are parallel to each other and they have no possible solution. The pair of linear equations is inconsistent.

(iii) Given pair of linear equations:

$\frac{3}{2}x + \frac{5}{3}y = 7$ and $9x - 10y = 14$

$\frac{3}{2}x + \frac{5}{3}y - 7 = 0$ and $9x - 10y - 14 = 0$

$a_1 = \frac{3}{2}, b_1 = \frac{5}{3}, c_1 = -7$ and $a_2 = 9, b_1 = -10, c_2 = -14$

$\frac{a_1}{a_2} = \frac{3/2}{9} = \frac{1}{6}, \frac{b_1}{b_2} = \frac{5/3}{-10} = -\frac{1}{6}$

Since, $\frac{1}{6} \neq -\frac{1}{6}$, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Therefore, the equations are intersecting each other at one point and they have only one possible solution. The pair of linear equations is consistent.

(iv) Given pair of linear equations:

$5x - 3y = 11$ and $-10x + 6y = -22$

$5x - 3y - 11 = 0$ and $-10x + 6y + 22 = 0$

$a_1 = 5, b_1 = -3, c_1 = -11$ and $a_2 = -10, b_2 = 6, c_2 = 22$

$$\frac{a_1}{a_2} = \frac{5}{10} = -\frac{1}{2}, \frac{b_1}{b_2} = \frac{-3}{6} = -\frac{1}{2}, \frac{c_1}{c_2} = \frac{-11}{22} = -\frac{1}{2}$$

Since, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

These linear equations are coincident lines and have infinite number of possible solutions. The pair of linear equation is consistent.

(v) Given pair of linear equations:

$\frac{4}{3}x + 2y = 8$ and $2x + 3y = 12$

$\frac{4}{3}x + 2y - 8 = 0$ and $2x + 3y - 12 = 0$

$a_1 = \frac{4}{3}, b_1 = 2, c_1 = -8; a_2 = 2, b_2 = 3, c_2 = -12$

$$\frac{a_1}{a_2} = \frac{4/3}{2} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{2}{3}, \frac{c_1}{c_2} = \frac{-8}{-12} = \frac{2}{3}$$

Since, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Therefore, These linear equations are coincident lines and have infinite number of possible solutions. The pair of linear equation is consistent.

Q.4 Which of the following pairs of linear equations are consistent? Obtain the solution in such cases graphically.

(i) $x + y = 5, 2x + 2y = 10$

(ii) $x - y = 8, 3x - 3y = 16$

(iii) $2x + y - 6 = 0, 4x - 2y - 4 = 0$

(iv) $2x - 2y - 2 = 0, 4x - 4y - 5 = 0$

Sol. (i) Graph of $x + y = 5$:

$x + y = 5$

$\Rightarrow y = 5 - x$

When $x = 0, y = 5$ and when $x = 5, y = 0$

x	0	5
y	5	0

Graph of $2x + 2y = 10$:

$2x + 2y = 10$

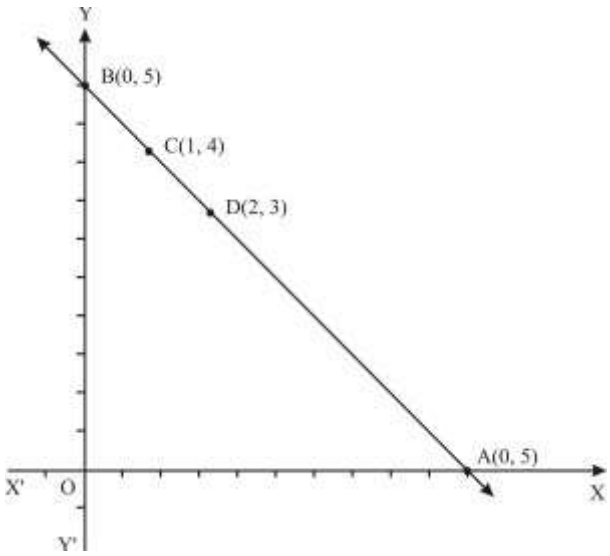
$\Rightarrow 2y = 10 - 2x$

$\Rightarrow y = 5 - x$

When $x = 1, y = 5 - 1 = 4$ and when $x = 2, y = 5 - 2 = 3$

x	1	2
y	4	3

Plotting the points A (0, 5) and (5, 0) and the points C(1, 4) and D(2, 3) on the graph paper. Join the line AB and CD by drawing a line passing through these points on the same graph paper.



We find that both the line AB and CD are coincident. Hence, the system of equations has infinitely many solutions, i.e., consistent.

(ii) Graph of x – y = 8:

When x = 0, y = – 8; when x = 8, y = 0

x	0	8
y	– 8	0

For graph of 3x – 3y = 16:

We have, 3x – 3y = 16

⇒ 3y = 3x – 16

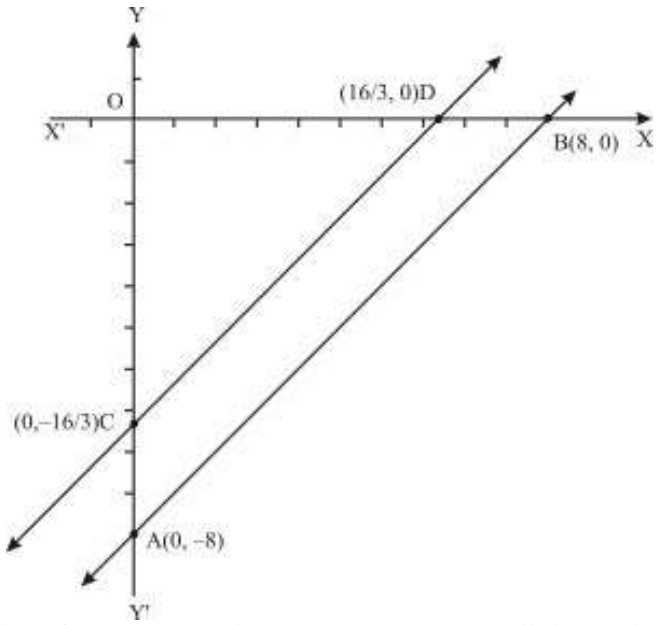
$\Rightarrow y = x - \frac{16}{3}$

When x = 0, $y = -\frac{16}{3} = -5\frac{1}{3}$;

When $x = \frac{16}{3} = 5\frac{1}{3}$, y = 0

x	0	16 / 3
y	– 16 / 3	0

Plot the points A(0, – 8), B(8, 0), C(0, $-\frac{16}{3}$) and D($\frac{16}{3}$, 0) on the same graph paper. Join the pint A to B and C to D by drawing a line and extend them on both sides.



We find the graphs of $x - y = 8$ and $3x - 3y = 16$ are parallel. So, the two lines have no common point. So, the given equations has no solution, i.e., **inconsistent.**

(iii) For graph of $2x + y - 6 = 0$:

$2x + y - 6 = 0$

$\Rightarrow y = 6 - 2x$

When $x = 0$, $y = 6 - 0 = 6$; when $x = 3$, $y = 6 - 6 = 0$

x	0	3
y	6	0

Now for Graph of $4x - 2y - 4 = 0$:

$4x - 2y - 4 = 0$

$\Rightarrow 2y = 4x - 4$

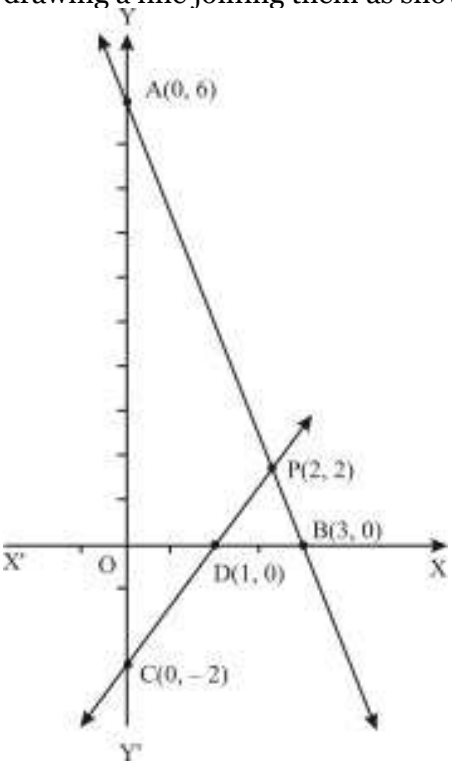
$\Rightarrow y = 2x - 2$

When $x = 0$, $y = -2$; when $x = 1$, $y = 0$

x	0	1
y	–2	0

Plot the points A(0, 6), B(3, 0), C(0, –2) and D(2, 0) on the graph and Join the points A to B and C To D by

drawing a line joining them as shown.



Clearly, the both the lines intersect at point P (2, 2). Hence, x = 2, y = 2 is the solution of the given equations, i.e., **consistent.**

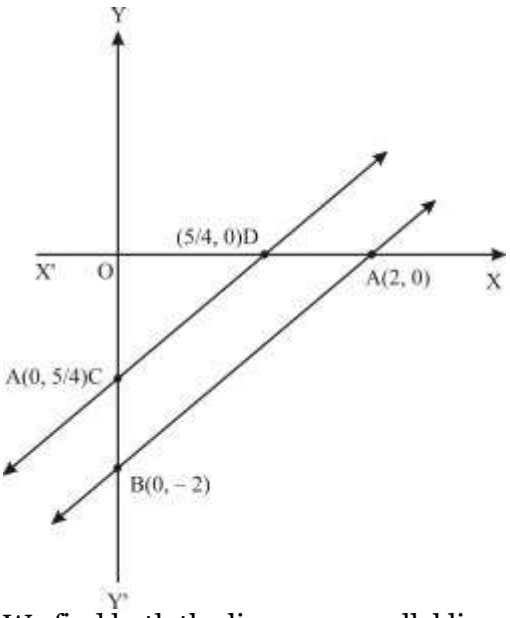
(iv) For graph of 2x – 2y – 2 = 0;
2x – 2y – 2 = 0
⇒ 2y = 2x – 2
⇒ y = x – 2
When x = 2, y = 0; when x = 0, y = – 2

x	2	0
y	0	– 2

For Graph of 4x – 4y – 5 = 0:
4x – 4y – 5 = 0
⇒ 4y = 5 – 4x
⇒ $y = \frac{5-4x}{4}$
When x = 0, y = −5/4; when x = 5/4, y = 0

x	0	5 / 4
y	– 5 / 4	0

Now, plot the points A (2, 0), B (0, −2), C (0, −5/4) and D (5/4, 0) on the same graph paper. Join the points A to B and C to D by drawing a line joining them as shown.



We find both the lines are parallel lines. So, the two lines have no common point. Hence, the given system of equations has no solution, i.e., **inconsistent.**

Q.5 Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.

Sol. Let x m be the length of the garden and y m be its width.

Then, perimeter = 2 (Length + Width)

= 2(x + y)

Therefore, 36 = 2(x + y)

⇒ x + y = 36 ..... (i)

Also, x = y + 4 ..... (ii)

Putting value of x into equation (i).

Therefore, y + 4 + y = 36

⇒ 2y = 36 – 4 = 32

⇒y = 32/2 = 16

Therefore, x = 16 + 4 = 20

Hence, Length x = 20 m and width y = 16m.

Q.6 Given the linear equation 2x + 3y – 8 = 0, write other linear equation in two variables such that the geometrical representation of the pair so formed

(i) intersecting lines

(ii) parallel lines

(iii) coincident lines

Sol. Given: linear equation 2x + 3y – 8 = 0 ... (1)

(i) For intersecting lines:

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Any line intersecting line may be taken as

$$3x + 7y - 9 = 0$$

$$\frac{2}{3} \neq \frac{3}{7}$$

So, we can see another equation satisfies the condition.

(ii) For parallel lines: $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Therefore, Any line parallel to (1) may be taken as

$$6x + 9y - 3 = 0$$

$$\frac{2}{6}=\frac{3}{9}\neq\frac{-8}{-9}$$
$$\frac{1}{3}=\frac{1}{3}\neq\frac{8}{9}$$

(iii) For coincident lines: $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

Any line coincident to (1) may be taken as

$6x + 9x - 24 = 0$

$$\frac{2}{6}=\frac{3}{9}=\frac{-8}{-24}$$
$$\frac{1}{3}=\frac{1}{3}=\frac{1}{3}$$

Q.7 Draw the graphs of the equations x – y + 1 = 0 and 3x + 2y – 12 = 0. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region.

Sol. For the graph of x – y + 1 = 0.

$x - y + 1 = 0$

$\Rightarrow y = x + 1$

When x = 0, y = 1 and when x = – 1, y = 0

x	0	– 1
y	1	0

Plot the points A(0, 1) and B(–1, 0) on a graph paper. Join A and B and extend it on both sides to obtain the graph of x – y + 1 = 0.

For the graph of 3x + 2y – 12 = 0:

$3x + 2y - 12 = 0$

$\Rightarrow 2y = 12 - 3x$

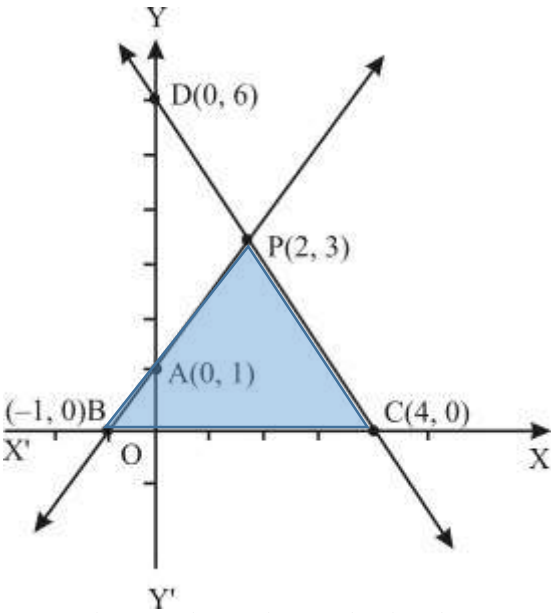
$\Rightarrow y=\frac{12-3x}{2}$

When x = 4, y = 0

When x = 0, y = 12/2 = 6

x	4	0
y	0	6

Plot the points A(0, 1), B(–1, 0), C(4, 0) and D(0, 6) on the graph paper and draw the line passing through these points



From the graph, we have obtained a ΔPBC formed by the given lines and the x - axis. The co-ordinates of the vertices are P (2, 3), B (–1, 0) and C (4, 0).