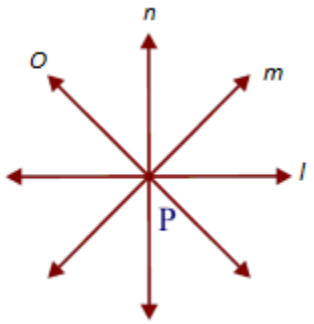
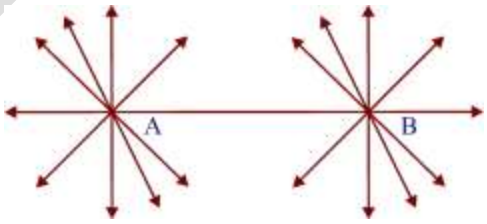
Introduction to Euclid's Geometry: Exercise 5.1

Q.1 Which of the following statements are true and which are false? Give reasons for your answers?
(i) Only one line can pass through a single point.
(ii) There are an infinite number of lines which pass through two distinct points.
(iii)A terminated line can be produced indefinitely on both the sides.
(iv)If two circles are equal, then their radii are equal.
(v) In figure if AB = PQ and PQ = XY, then AB = XY.

Sol.
(i) This statement is False.
Since, an infinite number of lines can be drawn passing through a given point.
Firstly, mark a point P on the plane of paper and by using a sharp pencil and a ruler, draw a line *l* passing through it. Now, draw another line *m* passing through P as shown in the figure. If we continue this process, we can draw as many lines.



(ii) This statement is False.
Since, only one line can be drawn through any two points in a plane. For this, firstly mark two points A and B on the plane of paper. Now, fold the paper so that a crease passes through point A. we observe that infinite number of creases (lines) can pass through A.



Similarly, fold the paper so that a crease passes through B. Again, we observe that infinite number of creases (lines) can pass through B. Now, fold the paper in such a way that a crease (line) passes through both points A and B. Here, we observe that only one crease (line) passes through both the points A and B.

(iii) This statement is True.
Since, a line segment is called a terminated line. Since, a line can be extended indefinitely in both the directions.

So, it cannot be drawn wholly on paper.

(iv) This statement is True.
By supperimposing the region bounded by one circle on the other circle, if the circles coincide. Then, their centers and boundaries coincide. So, their radii will be equal.

(v) This statement is True.
Since, we know that things which are equal to the same thing are equal to one another.

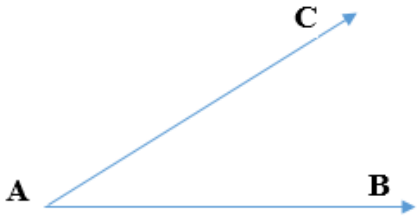
Q.2 Give a definition for each of the following terms. Are there other terms that need to be defined first? What are they, and how might you define them?
(i) Parallel lines (ii) Perpendicular lines
(iii) Line segment (iv) Radius of a circle
(v) Square
Sol.
For the desired definition, we need to define the following terms:
(a) point (b) line (c) plane (d) ray (e) angle (f) circle (g) quadrilateral.
(a) **Point:** A point has no length and width, it has only a position. It is a made by a sharp pencil on a sheet paper gives an idea about a point.

(b) **Line:** A line should be straight and extend indefinitely in both the directions.
A straight crease obtained by folding a paper, a straight string pulled at its two ends etc.

(c) **Plane:** The surface of a sheet of paper or smooth wall are close examples of a plane.

(d) **Ray:** This is a part of line ℓ which has only one end- point A and contains the point B is called a ray AB.

(e) **Angle:** An angle is defined as the two non - collinear rays with common initial point.



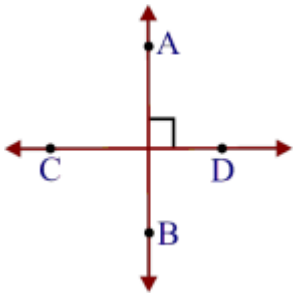
(f) **Circle:** A circle is defined as the set of all those points in a plane whose distance from a fixed point remains constant. This fixed point is called the centre of the circle.

(g) **Quadrilateral:** It is a closed figure enclosed by four line segments is called a quadrilateral.

(i) Parallel Lines: Two lines are said to be parallel, If they are not intersecting and they are co- planar.

In fig., the two lines ℓ_1 and ℓ_2 are parallel.

(ii) Perpendicular Lines: Two lines CD and AB lying in the same plane are said to be perpendicular when they form a right angle.

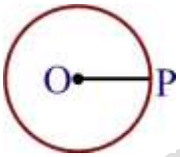


From figure, AB⊥CD

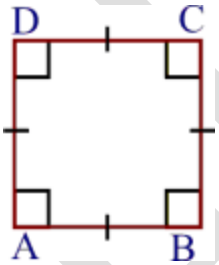
(iii) Line segment: It is the part of line with two end- points i.e. A and B is called the line segment.

From the above figure. It is line segment $\overline{AB}$. AB and BA denote the same line segment.

(iv) Radius: It is the distance between from the centre to a point on the circle is called the radius of the circle. From the figure, OP is the radius.



(v) Square: It is a quadrilateral in which all the four angles are right angles and four sides are equal is called a square. From the figure, ABCD is a square.



Q.3 Consider two 'postulates' given below:
(i) Given any two distinct points A and B, there exists a third point C which is in between A and B.
(ii) There exist at least three points that are not on the same line.
Do these postulates contain any undefined terms are these postulates consistent?
Do they follow from Euclid's postulates? Explain.
Sol.
There are several undefined terms. They are consistent, because these terms deal with two different situations:
(i) The first postulate says that the given two points A and B, there is a point C lying on the line in between them.
(ii) Second postulate says that given two points A and B, we can take another point C not lying on the line through points A and B.

These 'postulates' do not follow from Euclid's postulate. Since, these postulates follow from axiom stated that given two distinct points, there is a unique line that passes through them.

Q.4 If a point C lies between two points A and B such that AC = BC, then prove that AC = $\frac{1}{2}$ AB. Explain by drawing the figure.

Sol. Given: a point C lying between two points A and B such that
AC = BC.
Now, adding AC on both sides,

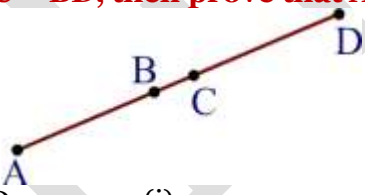
AC + AC = AC + BC
(Since, AC + CB coincides with AB)
⇒ 2AC = AB
So, AC = $\frac{1}{2}$ AB.....Hence Proved.

Q.5 In question 4, point C is called a mid- point of line segment AB. Prove that every line segment has one and only one mid-point.

Sol. Firstly, let us assume D is another mid- point of AB.

So, AD = DB ..... (i)
But given, C is the mid- point of AB.
So, AC = CB ..... (ii)
Now, subtracting (i) from (ii),
AC – AD = CB – DB
⇒ DC = – DC
⇒ 2DC = 0
⇒ DC = 0
So, C and D coincides.
Hence, It is proved that every line segment has one and only one mid- point.

Q.6 In figure if AC = BD, then prove that AB = CD.



Sol. Given: AC = BD..... (i)
Also AC = AB + BC ... (ii) (Since, point B lies between points A and C)
and, BD = BC + CD ... (iii) (Since, point C lies between points B and D)
Now, substituting for AC and BD from (ii) and (iii) in (i),
AB + BC = BC + CD
⇒ AB = CD.
Thus, hence proved.

Q.7 Why is Axiom 5, in the list of Euclid's axioms, considered a 'universal truth'? (Note that the question is not about the 5th postulate).

Sol. Since, axiom 5 in the list of Euclid's axioms, is true for anything in any part of universe. So, this axiom is a universal truth.