Heron's Formula: Exercise 12.2

Q.1 A park, in the shape of a quadrilateral ABCD, has ∠C = 90°, AB = 9 m, BC = 12 m, CD = 5 m and AD = 8 m. How much area does it occupy?

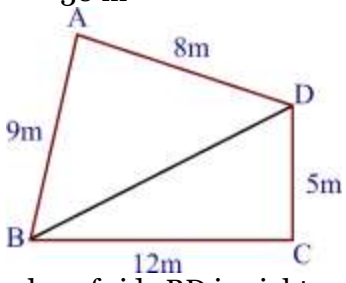
***Sol.* Given:** A quadrilateral ABCD has ∠C = 90°, AB = 9 m, BC = 12 m, CD = 5 m and AD = 8 m.

Construction: Join BD

Firstly, Area of ΔBCD = $\frac{1}{2}$ × BC × CD

= ($\frac{1}{2}$ × 12 × 5) m²

= 30 m²



Now, we find the value of side BD in right angled ΔBCD,

So, from Pythagoras theorem,

$BD^2 = BC^2 + CD^2$

$\Rightarrow BD^2 = 12^2 + 5^2$

$\Rightarrow BD^2 = 144+25$

$\Rightarrow BD^2 = 169$

$\Rightarrow BD = 13m$

Now, Let a = 13 m, b = 8 m and c = 9 m be the sides of ΔABD,

Now, s = $\frac{1}{2}$ (a + b + c)

s = $\frac{1}{2}$ (13 + 8 + 9) m

s = $\frac{1}{2}$ × 30 m = 15m

s−a = (15−13) m = 2m; s−b = (15−8)m = 7m and s−c = (15−9)m = 6m

So, Area of ΔABD = $\sqrt{15\times2\times7\times6}$ m²

= $2\times 3\sqrt{35}$ m²

= 6 × 5.9 m² (approx.)

= 35.4 m² (approx.)

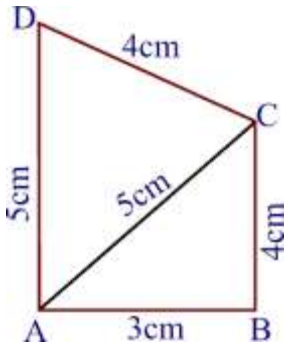
Thus, Required area of quadrilateral ABCD = ΔABD + ΔBCD

= 35.4 m² + 30 m²

= 65.4 m²

Q.2 Find the area of a quadrilateral ABCD in which AB = 3 cm, BC = 4 cm, CD = 4 cm, DA = 5 cm and AC = 5 cm.

***Sol.* Given:** A quadrilateral ABCD in which AB = 3 cm, BC = 4 cm, CD = 4 cm, DA = 5 cm and AC = 5 cm.



Firstly, In ΔABC,

Since, $AC^2 = AB^2 + BC^2$

$5^2 = 3^2 + 4^2$

$25 = 25$

So, ∠ABC = 90°

Therefore, ΔABC is a right angled triangle.

Now, Area of right angled ΔABC = $\frac{1}{2} \times AB \times BC$

$= (\frac{1}{2} \times 3 \times 4)\ cm^2$

$= 6\ cm^2$

Now, for area of ΔACD, Let a = 5 cm, b = 4cm and c = 5 cm.

Half of perimeter, s = $\frac{1}{2}$ (a + b + c)

$= \frac{1}{2}$ (5 + 4 + 5) cm

= 7cm

Now, s−a = (7−5)cm = 2cm; s−b = (7−4)cm = 3cm and s−c = (7−5)cm = 2cm

So, Area of ΔACD = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{7 \times 2 \times 3 \times 2}$

$= 2\sqrt{21}\ cm^2$

$= 2 \times 4.6\ cm^2$ (approx.)

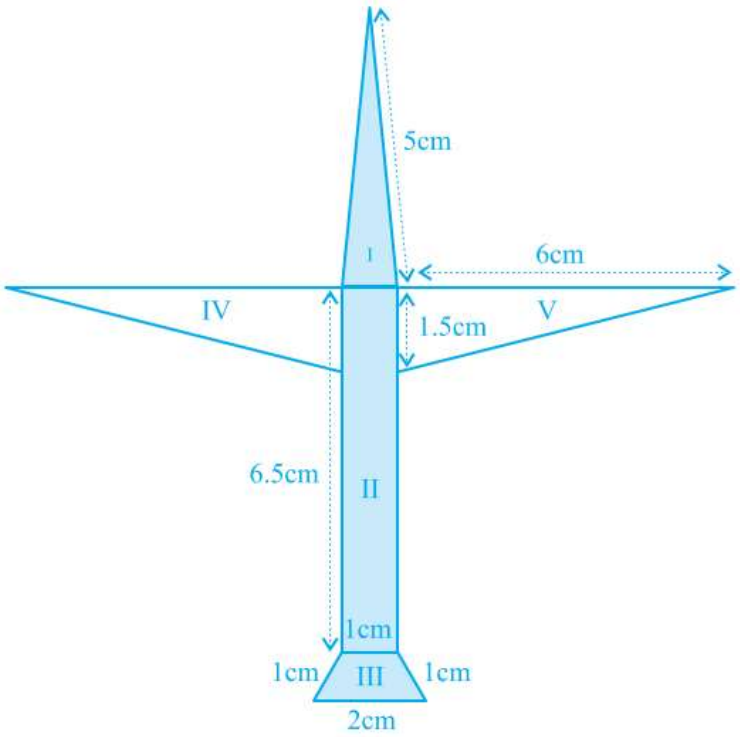
$= 9.2\ cm^2$ (approx.)

Thus, Area of quadrilateral ABCD = Area of ΔABC + Area of ΔACD

$= (6 + 9.2)\ cm^2$

$= 15.2\ cm^2$ approx

Q.3 Radha made a picture of an aero plane with coloured paper as shown in figure. Find the total area of the paper used.



Sol. For, find the total area of paper used, we need to calculate the area of each region.

(i) Region I: Since, this region is enclosed by a triangle of sides a = 5 cm, b = 5 cm and c = 1 cm.

Now, half perimeter of triangle, $s = \frac{1}{2}(a + b + c)$

$$= \frac{1}{2}(5 + 5 + 1)\ m$$

$\Rightarrow$ $= 112$ cm

Then, Area of region-I = $\sqrt{s(s-a)(s-b)(s-c)}$ cm²

$$= \sqrt{\frac{11}{2}\times\left(\frac{11}{2}-5\right)\times\left(\frac{11}{2}-5\right)\times\left(\frac{11}{2}-1\right)}\ \text{cm}^2$$

$$= \sqrt{\frac{11}{2}\times\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)}\ \text{cm}^2$$

$$= \frac{1}{4}\sqrt{11}\ \text{cm}^2$$

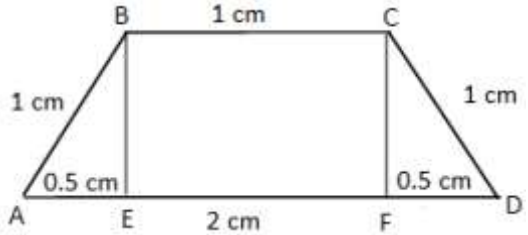
$$= 2.49\ \text{cm}^2$$

(ii) Region II: Since, region II is a rectangle of length 6.5 cm and breadth 1 cm.

Area of region II = 6.5×1

$= 6.5$ cm²

(iii) Region III: Since, region III is an isosceles trapezium.



Firstly, in ΔABE, angle E is right angled,
So, from the Pythagoras theorem,

$AB^2 = AE^2 + BE^2$

$\Rightarrow \quad 1 = 0.25 + BE^2$

$\Rightarrow \quad BE = \sqrt{0.75} = \sqrt{\frac{3}{4}}$

Thus, Area of region III = $\frac{1}{2}$ (AD + BC) × BE

$= \frac{1}{2} (2+1) \times \sqrt{\frac{3}{4}}$ cm²

⇒Area of region III = $\frac{3\sqrt{3}}{8}$ cm² = 1.3cm²

(iv) Region IV: It forms a right triangle whose two sides are of lengths 6 cm and 1.5 cm.

So, Area of region IV = $\frac{1}{2} \times 6 \times 1.5$ cm²

= 4.5 cm²

(v) Region V: Since, region IV and V are congruent.
Therefore, area of region V = 4.5 cm²
Thus, total area of the paper used = (2.49 + 6.5 + 1.3 + 4.5 + 4.5) cm²

= 19.29 cm²

Q.4 A triangle and a parallelogram have the same base and the same area. If the sides of the triangle are 26 cm, 28 cm and 30 cm, and the parallelogram stands on the base 28 cm, find the height of the parallelogram.

***Sol.* Given:** Sides of the triangle let a = 26 cm, b = 28 cm and c = 30cm and base of parallelogram = 28 cm.
For the triangle:

So, half of perimeter, s = $\frac{1}{2}$ (26 + 28 + 30) cm

$= \frac{1}{2} \times 84$ cm

= 42 cm

Now, s–a = (42–26) cm = 16cm; s–b = (42–28) cm = 14 cm
and s–c = (42–30) cm =12cm

Thus, Area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{42 \times 16 \times 14 \times 12}$ cm²

$= \sqrt{2 \times 3 \times 7 \times 4 \times 4 \times 2 \times 7 \times 2 \times 2 \times 3}$ cm²

= 336 cm²

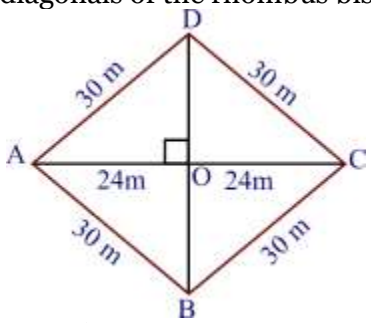
Now, for the parallelogram: Area = base × height
Since, Triangle and parallelogram, both have same base and same area.
Area of parallelogram = Area of triangle

Base × Height = 336 cm²
28 x Height = 336 cm²
Height = (336/28) cm
= 12cm

Thus height of parallelogram = 12 cm

Q.5 A rhombus shaped field has green grass for 18 cows to graze. If each side of the rhombus is 30 m and its longer diagonal is 48 m, how much area of grass field will each cow be getting?

Sol. Since, the diagonals of the rhombus bisect each other at right angles. So, in right angled triangle AOD,



By using Pythagoras theorem,

$OD^2 = AD^2 - AO^2$

$= 30^2 - 24^2$ m

$= (30+24)(30-24)$

$= 54 \times 6$ m

$= 9 \times 6 \times 6$m

$OD = (3 \times 6)$ m $= 18$ m

Therefore, area of one $\Delta AOD = (\frac{1}{2} \times 24 \times 18)$ m^2

$= 216$ m^2

Thus, Area of rhombus $= 4 \times \Delta$ AOD

$= (4 \times 216)$ m^2

$= 864$ m^2

Grass area for 18 cows $= 864$ m^2

Grass area for 1 cow $= (864/18)$ m^2

$= 48$ m^2

Thus, area of grass field will each cow get $= 48$ m^2

Q.6 An umbrella is made by stitching 10 triangular pieces of cloth of two different colours (see figure), each piece measuring 20 cm, 50 cm and 50 cm. How much cloth of each colour is required for the umbrella?

Sol. Since, an umbrella is made by stitching 10 triangular pieces of cloth of two different colours.

One triangular piece, let a = 20 cm, b = 50 cm and c = 50 cm

Now, half perimeter of the triangle, $s = \frac{1}{2}(a + b + c)$

$$= \frac{1}{2}(20 + 50 + 50)\ \text{cm}$$
$$= 60\text{cm}$$

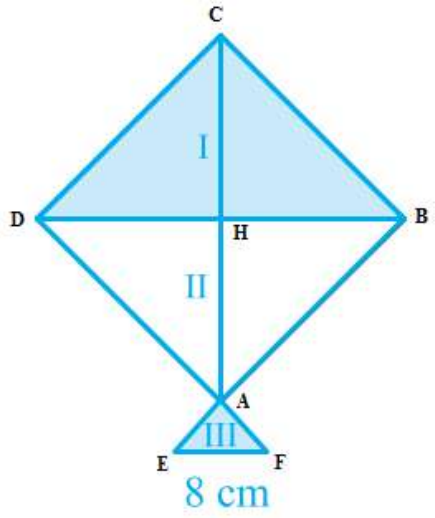
Then, s–a = (60–20) cm = 40cm; s–b = (60–50) cm = 10 cm
and s–c = (60–50) cm = 10 cm.

So, Area of one triangular piece = $\sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{60 \times 40 \times 10 \times 10}$$
$$= 200\sqrt{6}\ \text{cm}^2$$

Thus, Area of 10 two different triangles =10 x $200\sqrt{6}$ cm²

Q.7 A kite in the shape of a square with a diagonal 32 cm and an isosceles triangle of base 8 cm and sides 6 cm each is to be made of three different shades as shown in figure. How much paper of each shade has been used in it?



***Sol.* Given:** let ABCD be a square such that AC = BC = 32 cm and ΔAEF be an isosceles triangle in which AE = AF = 6 cm and its base EF =8 cm

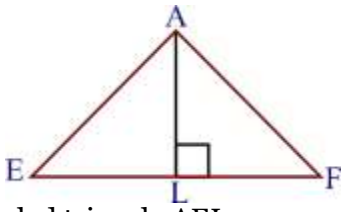
Now, the area of shades I and II:
So, from the figure, Area of shade I = Area of shade II (Area of ΔCDB)

$$= \frac{1}{2} \times \text{DB} \times \text{CM}$$
$$= \frac{1}{2} \times 32 \times 16\ \text{cm}^2$$
$$= 256\ \text{cm}^2$$

For the area of shade III (isosceles ΔAEF):

$$\text{EL} = \text{LF} = \frac{1}{2}\text{EF} = \frac{1}{2} \times 8 = 4\text{cm}$$

And AE = 6cm



Now, in right angled triangle AEL,
From Pythagoras theorem, $AL^2 = AE^2 - EL^2$

$= 36 - 16$

$= 20$

$AL = 2\sqrt{5}$ cm

Now, Area of shade III $= \frac{1}{2} \times EF \times AL$

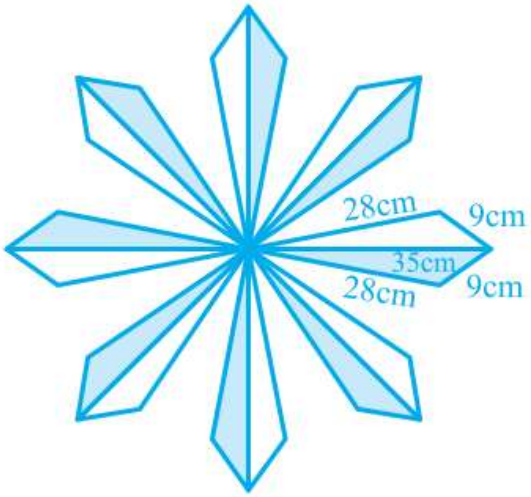
$= \frac{1}{2} \times 8 \times 2\sqrt{5}$ cm²

$= 8\sqrt{5}$ cm²

=17.92sqcm (approx)

Thus, area of region-I: 256 cm², region –II: 256 cm² and region-III: 17.92sqcm

Q.8 A floral design on floor is made up of 16 tiles which are triangular, the sides of the triangle being 9 cm, 28 cm and 35 cm (see figure). Find the cost of polishing the tiles at the rate of 50 p per cm².



Sol. For finding the cost of polishing the 16 tiles, we need to firstly find out the total area.
So, ***f***or one triangular tile: Let a = 9 cm, b = 28 cm and c = 35 cm be the side of the triangle.

Now, half of the perimeter, $s = \frac{1}{2}(a + b + c)$

$= \frac{1}{2}(9 + 28 + 35)$ cm

$= 36$cm

Then s–a = (36–9) cm = 27cm; s–b = (36–28) cm = 8cm
and s–c = (36–35) cm = 1cm

So, Area of one tile $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{36 \times 27 \times 8 \times 1}$ cm²

$= 36\sqrt{6}$ cm²

= 88.2 cm² approx.

Therefore, Area of 16 such tiles = (16×88.2) cm²

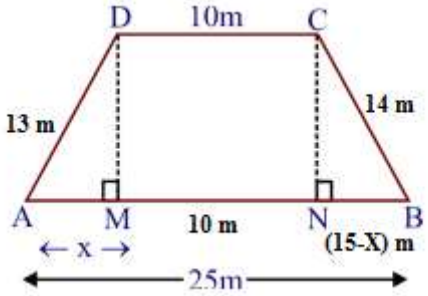
= 1411.2 cm² approx.

Cost of polishing is 50 P per cm² = Rs. $(1411.2 \times \frac{50}{100})$

= Rs. 705.60 approx.

Q.9 A field is in the shape of a trapezium whose parallel sides are 25 m and 10 m. The non-parallel sides are 14 m and 13 m. Find the area of the field.

Sol. **Given:** let ABCD be the field in shape of trapezium in which parallel sides AB = 25 cm and CD = 10 cm and non-parallel sides BC = 14m and AD = 13.



In figure, let AM = X, MN = 10 m and then BN = (15-X)

From figure, DM = CN (Distance between parallel sides are always equal.)

⇒ $DM^2 = CN^2$

So, in Right angled triangles AMD and CBN, from the Pythagoras theorem,

⇒ $(13)^2 - x^2 = 14^2 - (15-x)^2$

⇒ $169 - x^2 = 196 - (225 - 30x + x^2)$

⇒ $50x = 169 - 196 + 625$

⇒ $50x = 598$

⇒ $x = 598/50 = 11.96$

So, DM = $\sqrt{AD^2 - AM^2} = \sqrt{13^2 - (11.96)^2}$

$= \sqrt{(13+11.96)(13-11.96)}$

$= \sqrt{25.9584} = 5.09$

Now, Area of the trapezium = $\frac{1}{2} \times (AB + CD) \times DM$

$= \frac{1}{2} \times (25 + 10) \times 5.09$ m²

= 89.075 m²

Thus, area of the trapezium is 89.075 m².