Heron's Formula: Exercise 12.1

Q.1 A traffic signal board, indicating 'SCHOOL AHEAD', is an equilateral triangle with side 'a'. Find the area of the signal board using Heron's formula. If its perimeter is 180 cm, what will be the area of the signal board?

Sol. Given: Perimeter of board = 180 cm Let a, b and c be the lengths of sides BC, CA and AB respectively of \triangle ABC and

$$S = \frac{1}{2} (a + b + c),$$

So, Area of $\triangle ABC = \sqrt{s(s-a)(s-b)(s-c)}$ Since, given that $\triangle ABC$ is equilateral of side = a So, here a = b = c (= a) and $s = \frac{1}{2}$ (a + a + a) $= \frac{3a}{2}$ Now, $s - a = \frac{3a}{2} - a = \frac{a}{2}$; $s - b = \frac{3a}{2} - a = \frac{a}{2}$ and $s - c = \frac{3a}{2} - a = \frac{a}{2}$ So, Area $= \sqrt{\frac{3a}{2}x(\frac{a}{2})x(\frac{a}{2})x(\frac{a}{2})}$ $= \frac{a}{2} \times \frac{a}{2}\sqrt{3} = \frac{\sqrt{3}a^2}{4} \dots$ (i) Since, its perimeter = 180 cm So, a + a + a = 180 cm $\Rightarrow a = 60$ cm By putting a = 60 cm in eq. (i) So, Required area $= \frac{\sqrt{3}(60)^2}{4}$ cm² $= \frac{\sqrt{3} \times 3600}{4}$ $= 900\sqrt{3}$ cm²

Thus required area of the board is $900\sqrt{3}$ cm².

Q.2 The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 122 m, 22 m and 120 m (see figure). The advertisements yield an earning of Rs 5000 per m² per year. A company hired one of its walls for 3 months. How much rent did it pay?



Sol. Given: Let a = 122 m, b = 22 m and c = 120 m be the sides of the triangular side walls of a flyover. So, firstly, we have to find the semi perimeter of triangular.

So, s = $\frac{1}{2}$ (a + b + c) = $\frac{1}{2}$ (122 + 22 + 120) m = $(\frac{1}{2} \times 264)$ m = 132m Now, s - a = (132 - 122) m = 10 m; s - b = (132 - 22) m = 110 m and s - c = (132 - 120) m = 12 m So, Area of triangular wall = $\sqrt{s(s-a)(s-b)(s-c)}$ (From Heron's formula) = $\sqrt{132 \times 10 \times 110 \times 12}$ m² = $(\sqrt{10 \times 10 \times 11 \times 11 \times 12 \times 12}$ m² = $(10 \times 11 \times 12)$ m² = 1320 m² Advertising rent charges = Rs 5000 per m² per year So, rent charged from a company for 3 month = Rs (5000 × 1320 × $\frac{3}{12}$) = Rs16,50,000 Thus, advertising rent charges is Rs. 16,50,000 Q.3 There is a slide in a park. One of its side walls has been painted in blue colour with a

message "KEEP THE PARK GREEN AND CLEAN" (See figure). If the sides of the wall are 15 m, 11 m and 6 m, find the area painted in colour.

KEEP THE PARK

GREEN AND CLEAN

15 m

Sol. Given: Let a = 15 m, b = 11 m and c = 6 m be the side of wall. So, firstly find the semi perimeter of side wall, s = $\frac{1}{2}$ (a + b + c) = $\frac{1}{2}$ (15 + 11 + 6)m =16m Now, s - a = (16 - 15) m = 1 m; s - b = (16 - 11) m = 5 m and s - c = (16 - 6) m = 10 m So, area of side wall = $\sqrt{s(s-a)(s-b)(s-c)}$ = $\sqrt{16 \times 1 \times 5 \times 10}$ m²

11 m

 $= 20\sqrt{2} \text{ m}^2$ Thus, the area painted in colour = Area of side wall $= 20\sqrt{2} \text{ m}^2$

Q.4 Find the area of a triangle two sides of which are 18 cm and 10 cm and the perimeter is 42 cm.

Sol. Given: Let a = 18 cm, b = 10 cm two sides of triangle and a + b + c = 42 cm Therefore, third side of triangle c = 42 - a - b \Rightarrow c = (42 - 18 - 10) cm \Rightarrow = 14 cm Now, s = $\frac{1}{2}$ (a + b + c) = $\frac{1}{2} \times 42$ cm = 21cm s-a = (21-18) cm = 3cm; s-b = (21-10) cm = 11cm and s-c = (21 - 14)cm = 7cm Now, area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$ $= \sqrt{21 \times 3 \times 11 \times 7}$ $= \sqrt{3 \times 7 \times 3 \times 11 \times 7}$ $= 21\sqrt{11}$ cm²

Thus, area of the triangle is $21\sqrt{11}$ cm²

Q.5 Sides of a triangle are in the ratio of 12 : 17 : 25 and its perimeter is 540 cm. Find its area. *Sol.* Given: ratio of the sides of triangle = 12 : 17 : 25 and perimeter = 540 cm Let a, b and c be the sides of the triangle.

So, a : b : c = 12 : 17 : 25

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 \Rightarrow

$$\Rightarrow \frac{a}{12} = \frac{b}{17} = \frac{c}{25} = k(say)$$

 \Rightarrow Therefore, a = 12 k, b = 17 k and c = 25 k

 \Rightarrow So, perimeter a + b + c = 540

 $\Rightarrow \qquad 12k + 17k + 25k = 540$

- $\Rightarrow 54 k = 540$ $\Rightarrow k = 540/54$
 - k = 540/54 k = 10

Thus, sides a = 12 × 10 = 120 cm, b = 17 × 10 cm = 170 and c = 25 × 10 = 250 cm

Now,
$$s = \frac{1}{2} \times 540 \text{ cm} = 270 \text{ cm};$$

 $s - a = (270 - 120) \text{ cm} = 150 \text{ cm}; \quad s - b = (270 - 170) \text{ cm} = 100 \text{ cm}$
and $s - c = (270 - 250) \text{ cm} = 20 \text{ cm}$
Thus, area of the triangle = $\sqrt{s(s - a)(s - b)(s - c)}$
 $= \sqrt{270 \times 150 \times 100 \times 20}$
 $= 9000 \text{ cm}^2$
Thus, area of triangle is 9000 cm^2

Q.6 An isosceles triangle has perimeter 30 cm and each of the equal sides is 12 cm. Find the area of the triangle.

Sol. Given: Let ABC be the isosceles triangle and one side of the equal side i.e. a = b = 12 cm and a + b + c = perimeter = 30 cm

