Factorisation: Exercise 14.2

Q.1 Factorise the following expressions.
(i) $a^2 + 8a + 16$
(ii) $p^2 - 10p + 25$
(iii) $25m^2 + 30m + 9$
(iv) $49y^2 + 84yz + 36z^2$
(v) $4x^2 - 8x + 4$
(vi) $121b^2 - 88bc + 16c^2$
(vii) $(l + m)^2 - 4lm$ (Hint: Expand $(l + m)^2$first)
(viii) $a^4 + 2a^2b^2 + b^4$

Sol. Factorisation of the expressions:

(i) Given: $a^2 + 8a + 16$
$a^2 + 8a + 16 = (a)^2 + 2 \times a \times 4 + (4)^2$
By using identity, $(a + b)^2 = a^2 + 2ab + b^2$
$= (a + 4)^2$

(ii) Given: $p^2 - 10p + 25$
$p^2 - 10p + 25 = (p)^2 - 2 \times p \times 5 + (5)^2$
By using identity, $(a - b)^2 = a^2 - 2ab + b^2$
$= (p - 5)^2$

(iii) Given: $25m^2 + 30m + 9$
$25m^2 + 30m + 9 = (5m)^2 + 2 \times 5m \times 3 + (3)^2$
By using identity, $(a + b)^2 = a^2 + 2ab + b^2$
$= (5m + 3)^2$

(iv) Given: $49y^2 + 84yz + 36z^2$
$49y^2 + 84yz + 36z^2 = (7y)^2 + 2 \times 7y \times 6z + (6z)^2$
By using identity, $(a + b)^2 = a^2 + 2ab + b^2$
$= (7y + 6z)^2$

(v) Given: $4x^2 - 8x + 4$
$4x^2 - 8x + 4 = (2x)^2 - 2 \times 2x \times 2 + (2)^2$
By using identity, $(a - b)^2 = a^2 - 2ab + b^2$

$= (2x - 2)^2$
By taking common,
$= [2(x-1)]^2$
$= 4(x-1)^2$

(vi) Given: $121b^2 - 88bc + 16c^2$
$121b^2 - 88bc + 16c^2 = (11b)^2 - 2 \times 11b \times 4c + (4c)^2$
By using identity, $(a - b)^2 = a^2 - 2ab + b^2$
$= (11b - 4c)^2$

(vii) Given: $(l + m)^2 - 4lm$
$(l + m)^2 - 4lm = l^2 + 2 \times l \times m + m^2 - 4lm$
$= l^2 - 2lm + m^2$
By using identity, $(a - b)^2 = a^2 - 2ab + b^2$
$= (l - m)^2$

(viii) Given: $a^4 + 2a^2b^2 + b^4$
$a^4 + 2a^2b^2 + b^4 = (a^2)^2 + 2 \times a^2 \times b^2 + (b^2)^2$

By using identity, $(a + b)^2 = a^2 + 2ab + b^2$

$= (a^2 + b^2)^2$

Q.2 Factorise:

(i) $4p^2 - 9q^2$

(ii) $63a^2 - 112b^2$

(iii) $49x^2 - 36$

(iv) $16x^5 - 144x^3$

(v) $(l + m)^2 - (l - m)^2$

(vi) $9x^2y^2 - 16$

(vii) $(x^2 - 2xy + y^2) - z^2$

(viii) $25a^2 - 4b^2 + 28bc - 49c^2$

Sol. Factorisation of the expressions:

(i) Given: $4p^2 - 9q^2$

$4p^2 - 9q^2 = (2p)^2 - (3q)^2$

By using identity, $a^2 - b^2 = (a + b)(a - b))$

$= (2p + 3q)(2p - 3q)$

(ii) Given: $63a^2 - 112b^2$

By taking common,

$63a^2 - 112b^2 = 7(9a^2 - 16b^2)$

$= 7[(3a)^2 - (4b)^2]$

By using identity, $a^2 - b^2 = (a + b)(a - b))$

$= 7(3a + 4b)(3a - 4b)$

(iii) Given: $49x^2 - 36$

$49x^2 - 36 = (7x)^2 - (6)^2$

By using identity, $a^2 - b^2 = (a + b)(a - b))$

$= (7x - 6)(7x + 6)$

(iv) Given: $16x^5 - 144x^3$

By taking common,

$16x^5 - 144x^3 = 16x^3[(x)^2 - 9]$

$= 16x^3[(x)^2 - (3)^2]$

By using identity, $a^2 - b^2 = (a + b)(a - b))$

$= 16x^3(x - 3)(x + 3)$

(v) Given: $(l + m)^2 - (l - m)^2$

By using identity, $a^2 - b^2 = (a + b)(a - b))$

$(l + m)^2 - (l - m)^2 = [(l + m) - (l - m)][(l + m) - (l - m)]$

$= (l + m - l + m)(l + m - l + m)$

$= 2m \times 2l$

$= 4lm$

(vi) Given: $9x^2y^2 - 16$

$9x^2y^2 - 16 = (3xy)^2 - (4)^2$

By using identity, $a^2 - b^2 = (a + b)(a - b))$

$= (3xy - 4)(3xy + 4)$

(vii) Given: $(x^2 - 2xy + y^2) - z^2$

By using identity, $a^2 - 2ab + b^2 = (a-b)^2$

$(x^2 - 2xy + y^2) - z^2 = (x - y)^2 - (z)^2$

By using identity, $a^2 - b^2 = (a + b)(a - b))$

$= (x - y - z)(x - y + z)$

(viii) Given: $25a^2 - 4b^2 + 28bc - 49c^2$

$25a^2 - 4b^2 + 28bc - 49c^2 = 25a^2 - (4b^2 - 28bc + 49c^2)$

$= (5a)^2 - [(2b)^2 - 2\times 2b\times 7c + (7c)^2)]$

By using identity, $a^2 - 2ab + b^2 = (a-b)^2$

$= (5a)^2 - (2b - 7c)^2$

By using identity, $a^2 - b^2 = (a + b)(a - b))$

$= (5a + (2b - 7c))(5a - (2b - 7c))$

$= (5a + 2b - 7c)(5a - 2b + 7c)$

Q.3 Factorise the expressions.

(i) $ax^2 + bx$

(ii) $7p^2 + 21q^2$

(iii) $2x^3 + 2xy^2 + 2xz^2$

(iv) $am^2 + bm^2 + bn^2 + an^2$

(v) $(lm + l) + m + 1$

(vi) $y(y + z) + 9(y + z)$

(vii) $5y^2 - 20y - 8z + 2yz$

(viii) $10ab + 4a + 5b + 2$

(ix) $6xy - 4y + 6 - 9x$

Sol. Factorisation of the expressions:

(i) Given: $ax^2 + bx$

$ax^2 + bx = a \times x \times x + b \times x$

By taking common,

$= x(ax + b)$

(ii) Given: $7p^2 + 21q^2$

$7p^2 + 21q^2 = 7 \times p \times p + 21 \times q \times q$

By taking common,

$= 7(p^2 + 3q^2)$

(iii) Given: $2x^3 + 2xy^2 + 2xz^2$

By taking common,

$2x^3 + 2xy^2 + 2xz^2 = 2x(x^2 + y^2 + z^2)$

(iv) Given: $am^2 + bm^2 + bn^2 + an^2$

By taking common,

$am^2 + bm^2 + bn^2 + an^2 = m^2(a + b) + n^2(a + b)$

$= (m^2 + n^2)(a + b)$

(v) Given: $(lm + l) + m + 1$

$(lm + l) + m + 1 = lm + m + l + 1$

By taking common,

$= m(l + 1) + 1(l + 1)$

$= (l + 1)(m + 1)$

(vi) Given: $y(y + z) + 9(y + z)$

$y(y + z) + 9(y + z) = (y + z)(y + 9)$

(vii) Given: $5y^2 - 20y - 8z + 2yz$

By taking common,

$5y^2 - 20y - 8z + 2yz = 5y(y - 4) + 2z(y - 4)$

$= (y - 4)(5y + 2z)$

(viii) Given: $10ab + 4a + 5b + 2$

By taking common,
$10ab + 4a + 5b + 2 = 5b\,(2a + 1) + 2(2a + 1)$
$= (2a + 1)\,(5b + 2)$

(ix) Given: $6xy - 4y + 6 - 9x$
By taking common,
$6xy - 4y + 6 - 9x = 3x\,(2y - 3) - 2(2y - 3)$
$= (2y - 3)\,(3x - 2)$

Q.4 Factorise.
(i) $a^4 - b^4$
(ii) $p^4 - 81$
(iii) $x^4 - (y + z)^4$
(iv) $x^4 - (x - z)4$
(v) $a^4 - 2a^2b^2 + b^4$
***Sol.* Factorization:**
(i) Given: $a^4 - b^4$
$a^4 - b^4 = (a^2)^2 - (b^2)^2$
By using identity, $a^2 - b^2 = (a + b)\,(a - b))$
$= (a^2 - b^2)\,(a^2 + b^2)$
Again by using identity, $a^2 - b^2 = (a + b)\,(a - b))$
$= (a - b)\,(a + b)\,(a^2 + b^2)$

(ii) Given: $p^4 - 81$
$p^4 - 81 = (p^2)^2 - (9)^2$
By using identity, $a^2 - b^2 = (a + b)\,(a - b))$
$= (p^2 - 9)\,(p^2 + 9)$
By using identity, $a^2 - b^2 = (a + b)\,(a - b))$
$= (p^2 - 3^2)\,(p^2 + 9)$
$= (p - 3)\,(p + 3)\,(a^2 + b^2)$

(iii) Given: $x^4 - (y + z)^4$
$x^4 - (y + z)^4 = (x^2)^2 - [(y + z)^2]^2$
By using identity, $a^2 - b^2 = (a + b)\,(a - b))$
$= (x^2 - (y + z)^2)\,(x^2 + (y + z)^2)$
Again by using identity, $a^2 - b^2 = (a + b)\,(a - b))$
$= [\{x - (y + z)\}\,\{x + (y + z)\}]\,(x^2 + (y + z)^2)$
$= (x - y + z)\,(x + y + z)\,[x^2 + (y + z)^2]$

(iv) Given: $x^4 - (x - z)^4$
$x^4 - (x - z)^4 = (x^2)^2 - [(x-z)^2]^2$
By using identity, $a^2 - b^2 = (a + b)\,(a - b))$
$= (x^2 - (x-z)^2)\,(x^2 + (x-z)^2)$
Again by using identity, $a^2 - b^2 = (a + b)\,(a - b))$
$= (x - (x-z))\,(x + (x + z))\,(x^2 + (x-z)^2)$
By using identity, $(a-b)^2 = a^2 - 2ab + b^2$
$= (x - x + z)\,(x + x + z)\,(x^2 + x^2 - 2xz + z^2)$
$= x\,(2x + z)\,(2x^2 - 2xz + z^2)$

(v) Given: $a^4 - 2a^2b^2 + b^4$
$a^4 - 2a^2b^2 + b^4 = (a^2)^2 - 2a^2b^2 + (b^2)^2$
By using identity, $a^2 - 2ab + b^2 = (a-b)^2$
$= (a^2 - b^2)^2$
$= [(a - b)\,(a + b)]^2$
$= (a - b)^2\,(a + b)^2$

Q.5 Factorise the following expressions.
(i) $p^2 + 6p + 8$
(ii) $q^2 - 10q + 21$
(iii) $p^2 + 6p - 16$

Sol. **Factorization:**

(i) Given: $p^2 + 6p + 8$
By splitting the middle term,
$p^2 + 6p + 8 = p^2 + (4 + 2)\,p + 4 \times 2$
$= p^2 + 4p + 2p + 4 \times 2$
By taking common,
$= p\,(p + 4) + 2(p + 4)$
$= (p + 4)\,(p + 2)$

(ii) Given: $q^2 - 10q + 21$
By splitting the middle term,
$q^2 - 10q + 21 = q^2 - (7 + 3)q + 7\times3$
$= q^2 - 7q - 3q + 7\times3$
By taking common,
$= q\,(q - 7) - 3(q - 7)$
$= (q - 7)\,(q - 3)$

(iii) Given: $p^2 + 6p - 16$
By splitting the middle term,
$p^2 + 6p - 16 = p^2 + p\,(8-2) - 16$
$= p^2 + 8p - 2p - 16$
By taking common,
$= p\,(p + 8) - 2(p + 8)$
$= (p - 2)\,(p + 8)$