Exponents and Power: Exercise 13.2

Q.1 Using laws of exponents, simplify and write the answer in exponential form:

(i) $3^2 \times 3^4 \times 3^8$ **(ii) $6^{15} \div 6^{10}$** **(iii) $a^3 \times a^2$**
(iv) $7^x \times 7^2$ **(v) $(5^2)^3 \div 5^3$** **(vi) $2^5 \times 5^5$**
(vii) $a^4 \times b^4$ **(viii) $(3^4)^3$** **(ix) $(2^{20} \div 2^{15}) \times 2^3$**
(x) $8^t \div 8^2$

Sol:

(i) Given: $3^2 \times 3^4 \times 3^8$
Since, $a^m \times a^n = a^{m+n}$
$3^2 \times 3^4 \times 3^8 = (3)^{2+4+8}$
$= 3^{14}$

(ii) Given: $6^{15} \div 6^{10}$
Since, $a^m \div a^n = a^{m-n}$
$6^{15} \div 6^{10} = (6)^{15-10}$
$= 6^5$

(iii) Given: $a^3 \times a^2$
Since, $a^m \times a^n = a^{m+n}$
$a^3 \times a^2 = (a)^{3+2}$
$= a^5$

(iv) Given: $7^x \times 7^2$

Since, $a^m \times a^n = a^{m+n}$
$7^x \times 7^2 = (7)^{x+2}$

(v) Given: $(5^2)^3 \div 5^3$
Since, $a^m \div a^n = a^{m-n}$
$(5^2)^3 \div 5^3 = (5)^{6-3}$
$= 5^3$

(vi) Given: $2^5 \times 5^5$
Since, $a^m \times b^m = (ab)^m$
$2^5 \times 5^5 = (2 \times 5)^5$
$= 10^5$

(vii) Given: $a^4 \times b^4$
Since, $a^m \times b^m = (ab)^m$
$a^4 \times b^4 = (a \times b)^4$
$= (ab)^4$

(viii) Given: $(3^4)^3$
Since, $(a^m)^n = a^{mn}$
$(3^4)^3 = (3)^{4 \times 3}$
$= 3^{12}$

(ix) Given: $(2^{20} \div 2^{15}) \times 2^3$
Since, $a^m \div a^n = a^{m-n}$ and $a^m \times a^n = a^{m+n}$
$(2^{20} \div 2^{15}) \times 2^3 = (2)^{20-15} \times 2^3$
$= 2^5 \times 2^3$
$= (2)^{5+3}$
$= 2^8$

(x) Given: $8^t \div 8^2$
Since, $a^m \div a^n = a^{m-n}$
$8^t \div 8^2 = (8)^{t-2}$

Q.2 Simplify and express each of the following in exponential form:

(i) $(2^3 \times 3^4 \times 4)/ (3 \times 32)$
(ii) $((5^2)^3 \times 5^4) \div 5^7$
(iii) $25^4 \div 5^3$
(iv) $(3 \times 7^2 \times 11^8)/ (21 \times 11^3)$
(v) $3^7/ (3^4 \times 3^3)$
(vi) $2^0 + 3^0 + 4^0$
(vii) $2^0 \times 3^0 \times 4^0$
(viii) $(3^0 + 2^0) \times 5^0$
(ix) $(2^8 \times a^5)/ (4^3 \times a^3)$
(x) $(a^5/a^3) \times a^8$
(xi) $(4^5 \times a^8b^3)/ (4^5 \times a^5b^2)$
(xii) $(2^3 \times 2)^2$

Sol:

(i) Given: $(2^3 \times 3^4 \times 4)/ (3 \times 32) = (2^3 \times 3^4 \times 2 \times 2)/ (3 \times 2 \times 2 \times 2 \times 2 \times 2)$
$= (2^3 \times 3^4 \times 2^2)/ (3 \times 2^5)$
Since, $a^m \times a^n = a^{m+n}$
$= (2^{3+2} \times 3^4) / (3 \times 2^5)$
$= (2^5 \times 3^4) / (3 \times 2^5)$
Since, $a^m \div a^n = a^{m-n}$
$= 2^{5-5} \times 3^{4-1}$
$= 2^0 \times 3^3$
Since, $a^0 = 1$
$= 1 \times 3^3$
$= 3^3$

(ii) Given: $((5^2)^3 \times 5^4) \div 5^7$
Since, $(a^m)^n = a^{mn}$
$((5^2)^3 \times 5^4) \div 5^7 = (5^6 \times 5^4) \div 5^7$
Since, $a^m \times a^n = a^{m+n}$
$= (5^{6+4}) \div 5^7$
$= 5^{10} \div 5^7$
Since, $a^m \div a^n = a^{m-n}$
$= 5^{10-7}$
$= 5^3$

(iii) Given: $25^4 \div 5^3 = (5 \times 5)^4 \div 5^3$
$= (5^2)^4 \div 5^3$
Since, $(a^m)^n = a^{mn}$
$= 5^8 \div 5^3$
Since, $a^m \div a^n = a^{m-n}$
$= 5^{8-3}$
$= 5^5$

(iv) Given: $(3 \times 7^2 \times 11^8)/ (21 \times 11^3) = (3 \times 7^2 \times 11^8)/ (3 \times 7 \times 11^3)$
Since, $a^m \div a^n = a^{m-n}$
$= 3^{1-1} \times 7^{2-1} \times 11^{8-3}$
$= 3^0 \times 7 \times 11^5$
$= 1 \times 7 \times 11^5$
$= 7 \times 11^5$

(v) Given: $3^7/ (3^4 \times 3^3)$
Since, $a^m \times a^n = a^{m+n}$
$3^7/ (3^4 \times 3^3) = 3^7/ (3^{4+3})$
$= 3^7/ 3^7$
Since, $a^m \div a^n = a^{m-n}$
$= 3^{7-7}$
Since, $a^0 = 1$

$= 3^0$
$= 1$

(vi) Given: $2^0 + 3^0 + 4^0$
Since, $a^0 = 1$
$2^0 + 3^0 + 4^0 = 1 + 1 + 1$
$= 3$

(vii) Given: $2^0 \times 3^0 \times 4^0$
Since, $a^0 = 1$
$2^0 \times 3^0 \times 4^0 = 1 \times 1 \times 1$
$= 1$

(viii) Given: $(3^0 + 2^0) \times 5^0$
Since, $a^0 = 1$
$(3^0 + 2^0) \times 5^0 = (1 + 1) \times 1$
$= (2) \times 1$
$= 2$

(ix) Given: $(2^8 \times a^5)/ (4^3 \times a^3) = (2^8 \times a^5)/ [(2 \times 2)^3 \times a^3]$
$= (2^8 \times a^5)/ [(2^2)^3 \times a^3]$
Since, $(a^m)^n = a^{mn}$
$= (2^8 \times a^5)/ [2^6 \times a^3]$
Since, $a^m \div a^n = a^{m-n}$
$= 2^{8-6} \times a^{5-3}$
$= 2^2 \times a^2$
Since, $(a^m)^n = a^{mn}$
$= 2a^2$

(x) Given: $(a^5/a^3) \times a^8$
Since, $a^m \div a^n = a^{m-n}$
$(a^5/a^3) \times a^8 = (a^{5-3}) \times a^8$
$= (a^2) \times a^8$
Since, $a^m \times a^n = a^{m+n}$
$= a^{2+8}$
$= a^{10}$

(xi) Given: $(4^5 \times a^8b^3)/ (4^5 \times a^5b^2)$
Since, $a^m \div a^n = a^{m-n}$
$= 4^{5-5} \times (a^{8-5} \times b^{3-2})$
$= 4^0 \times (a^3b)$
Since, $a^0 = 1$
$= 1 \times a^3b$
$= a^3b$

(xii) Given: $(2^3 \times 2)^2$
Since, $a^m \times a^n = a^{m+n}$
$= (2^{3+1})^2$
$= (2^4)^2$
Since, $(a^m)^n = a^{mn}$
$= (2)^{4 \times 2}$
$= 2^8$

Q.3 Say true or false and justify your answer:
(i) $10 \times 10^{11} = 100^{11}$ **(ii) $2^3 > 5^2$**
(iii) $2^3 \times 3^2 = 6^5$ **(iv) $3^0 = (1000)^0$**

Sol:
(a) Given: $10 \times 10^{11} = 100^{11}$
By taking LHS,
Since, $a^m \times a^n = a^{m+n}$
$10 \times 10^{11} = 10^{1+11}$
$= 10^{12}$
Now, by taking RHS,
$100^{11} = (10 \times 10)^{11}$
Since, $a^m \times a^n = a^{m+n}$
$= (10^{1+1})^{11}$
$= (10^2)^{11}$
Since, $(a^m)^n = a^{mn}$
$= (10)^{2 \times 11}$
$= 10^{22}$
From above, $10^{12} < 10^{22}$
So, given statement is not true.

(b) Given: $2^3 > 5^2$
By taking LHS,
$2^3 = 2 \times 2 \times 2$
$= 8$
Now, by taking RHS,
$5^2 = 5 \times 5$
$= 25$
From above, $2^3 < 5^2$
So, given statement is not true.

(iii) Given: $2^3 \times 3^2 = 6^5$
By taking LHS,
$2^3 \times 3^2 = 2 \times 2 \times 2 \times 3 \times 3$
$= 72$
Now, by taking RHS,
$6^5 = 6 \times 6 \times 6 \times 6 \times 6$
$= 7776$
From above, $2^3 \times 3^2 < 6^5$
So, given statement is not true.

(iv) Given: $3^0 = (1000)^0$
By taking LHS,
$3^0 = 1$
Now, by taking RHS,
$(1000)^0 = 1$ (Since, $a^0=1$)
From above, $3^0 = (1000)^0$
So, given statement is true.

Q.4 Express each of the following as a product of prime factors only in exponential form:
(i) 108 × 192 (ii) 270 (iii) 729 × 64 (iv) 768
Sol:
(i) Given: 108×192
Prime factors of $108 = 2 \times 2 \times 3 \times 3 \times 3$
$= 2^2 \times 3^3$
And prime factors of $192 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3$
$= 2^6 \times 3$
So, $108 \times 192 = (2^2 \times 3^3) \times (2^6 \times 3)$
Since, $a^m \times a^n = a^{m+n}$
$= 2^{2+6} \times 3^{3+1}$
$= 2^8 \times 3^4$

(ii) Given: 270

Prime factors of 270 = $2 \times 3 \times 3 \times 3 \times 5$

Since, $a^m \times a^n = a^{m+n}$

$= 2 \times 3^3 \times 5$

(iii) Given: 729×64

Prime factors of 729 = $3 \times 3 \times 3 \times 3 \times 3 \times 3$

Since, $a^m \times a^n = a^{m+n}$

$= 3^6$

Prime factors of 64 = $2 \times 2 \times 2 \times 2 \times 2 \times 2$

Since, $a^m \times a^n = a^{m+n}$

$= 2^6$

So, $729 \times 64 = (3^6 \times 2^6)$

$= 3^6 \times 2^6$

(iv) Given: 768

Prime factors of 768 = $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3$

Since, $a^m \times a^n = a^{m+n}$

$= 2^8 \times 3$

Q.5 Simplify:

(i) $((2^5)^2 \times 7^3)/ (8^3 \times 7)$

(ii) $(25 \times 5^2 \times t^8)/ (10^3 \times t^4)$

(iii) $(3^5 \times 10^5 \times 25)/ (5^7 \times 6^5)$

Sol:

(i) Given: $((2^5)^2 \times 7^3)/ (8^3 \times 7)$ $= ((2^5)^2 \times 7^3)/ ((2^3)^3 \times 7)$

Since, $(a^m)^n = a^{mn}$

$= (2^{5 \times 2} \times 7^3)/ ((2^{3 \times 3} \times 7)$

$= (2^{10} \times 7^3)/ (2^9 \times 7)$

Since, $a^m \div a^n = a^{m-n}$

$= (2^{10-9} \times 7^{3-1})$

$= 2 \times 7^2$

$= 2 \times 7 \times 7$

$= 98$

(ii) Given: $(25 \times 5^2 \times t^8)/ (10^3 \times t^4) = (5^2 \times 5^2 \times t^8)/ (5^3 \times 2^3 \times t^4)$

Since, $a^m \times a^n = a^{m+n}$

$= (5^{2+2} \times t^8)/ (5^3 \times 2^3 \times t^4)$

$= (5^4 \times t^8)/ (5^3 \times 2^3 \times t^4)$

Since, $a^m \div a^n = a^{m-n}$

$= (5^{4-3} \times t^{8-4})/ 2^3$

$= (5 \times t^4)/ (2 \times 2 \times 2)$

$= (5t^4)/ 8$

(iii) Given: $(3^5 \times 10^5 \times 25)/ (5^7 \times 6^5) = (3^5 \times 5^5 \times 2^5 \times 5^2)/ (5^7 \times 2^5 \times 3^5)$

Since, $a^m \times a^n = a^{m+n}$

$= (3^5 \times 5^{5+2} \times 2^5)/ (5^7 \times 2^5 \times 3^5)$

$= (3^5 \times 5^7 \times 2^5)/ (5^7 \times 2^5 \times 3^5)$

Since, $a^m \div a^n = a^{m-n}$

$= (3^{5-5} \times 5^{7-7} \times 2^{5-5})$

$= (3^0 \times 5^0 \times 2^0)$

Since, $a^0=1$

$= 1 \times 1 \times 1$

$= 1$