Exponent and power: Exercise 12.1

Q.1 Evaluate.

(i) 3^{-2} **(ii) $(-4)^{-2}$** **(iii) $\left(\frac{1}{2}\right)^{-5}$**

Sol. **(i) Given:** 3^{-2}

Since, $a^{-m} = \frac{1}{a^m}$

$= \frac{1}{3^2}$

$= \frac{1}{9}$

(ii) Given: $(-4)^{-2}$

Since, $a^{-m} = \frac{1}{a^m}$

$= \frac{1}{(-4)^2}$

$= \frac{1}{16}$

(iii) Given: $\left(\frac{1}{2}\right)^{-5}$

$= \frac{1}{(2)^{-5}}$

$= (2)^5$

$= 32$

Q.2 Simplify and express the result in power notation with positive exponent.

(i) $(-4)^5 \div (-4)^8$ **(ii) $\left(\frac{1}{2^3}\right)^2$** **(iii) $(-3)^4 \times \left(\frac{5}{3}\right)^4$**

(iv) $(3^{-7} \div 3^{-10}) \times 3^{-5}$ **(v) $2^{-3} \times (-7)^{-3}$**

Sol. **(i) Given:** $(-4)^5 \div (-4)^8$

Since, $a^m \div a^n = a^{m-n}$

$= (-4)^{5-8}$

$= (-4)^{-3}$

$= \frac{1}{(-4)^{-3}}$

(ii) Given: $\left(\frac{1}{2^3}\right)^2$

Since, $\left(\frac{a}{b}\right)^m = \frac{a^m}{a^n}$

$$= \frac{1^2}{(2^3)^2}$$

Since $(a^m)^n = a^{m \times n}$

$$= \frac{1}{2^{3\times2}}$$

$$= \frac{1}{2^6}$$

(iii) Given: $(-3)^4 \times \left(\frac{5}{3}\right)^4$

Since $\left(\frac{a}{b}\right)^m = \frac{a^m}{a^n}$

$$(-3)^4 \times \left(\frac{5}{3}\right)^4 = (-3)^4 \times \frac{5^4}{3^4}$$

$$= [(-1)^4 \times (3)^4] \times \frac{5^4}{3^4}$$

Since, $a^m \div a^n = a^{m-n}$

$= 3^{4-4} \times 5^4$

$= 3^0 \times 5^4$

Since, $a^0 = 1$

$= 5^4$

(iv) $(3^{-7} \div 3^{-10}) \times 3^{-5}$

Since, $a^m \div a^n = a^{m-n}$

$(3^{-7} \div 3^{-10}) \times 3^{-5} = 3^{-7-(-10)} \times 3^{-5}$

$= 3^{-7+10} \times 3^{-5}$

$= 3^3 \times 3^{-5}$

Since, $a^m \times a^n = a^{m+n}$

$= 3^{3+(-5)}$

$= 3^{-2}$

Since, $a^{-m} = \frac{1}{a^m}$

$$= \frac{1}{3^2}$$

(v) Given: $2^{-3} \times (-7)^{-3}$

Since, $a^{-m} = \frac{1}{a^m}$

$$= \frac{1}{2^3} \times \frac{1}{(-7)^3}$$

Since, $a^m b^m = (ab)^m$

$$= \frac{1}{[2\times(-7)]^3}$$

$$= \frac{1}{(-14)^3}$$

Q.3 Find the value of.

(i) $(3^0 + 4^{-1}) \times 2^2$ **(ii)** $(2^{-1} \times 4^{-1}) \div 2^{-2}$ **(iii)** $\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$

(iv) $(3^{-1} + 4^{-1} + 5^{-1})^0$ **(v)** $\left\{\left(\frac{-2}{3}\right)^{-2}\right\}^2$

Sol. **(i) Given:** $(3^0 + 4^{-1}) \times 2^2$

Since, $a^{-m} = \frac{1}{a^m}$

$= \left(1+\frac{1}{4}\right)\times 2^2$

$= \left(\frac{4+1}{4}\right)\times 2^2 = \frac{5}{4}\times 2^2$

$= \frac{5}{2^2}\times 2^2$

Since, $a^m \div a^n = a^{m-n}$

$= 5 \times 2^{2-2}$

$= 5 \times 2^0$

Since, $a^0 = 1$

$= 5 \times 1$

$= 5$

(ii) Given: $(2^{-1} \times 4^{-1}) \div 2^{-2}$

Since, $a^{-m} = \frac{1}{a^m}$

$= \left(\frac{1}{2^1}\times\frac{1}{4^1}\right)\div 2^{-2}$

Since, $a^m \times a^n = a^{m+n}$

$= \left(\frac{1}{2}\times\frac{1}{2^2}\right)\div 2^{-2}$

$= \left(\frac{1}{2^3}\right)\div 2^{-2}$

Since, $a^m \div a^n = a^{m-n}$

$= 2^{-3} \div 2^{-2}$

$= 2^{-3-(-2)}$

$= 2^{-1}$

$= \left(\frac{1}{2}\right)$

(iii) Given: $\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$

Since, $a^{-m} = \frac{1}{a^m}$

$= (2^{-1})^{-2} + (3^{-1})^{-2} + (4^{-1})^{-2}$

Since, $a^m \times a^n = a^{m+n}$

$= 2^{-1 \times (-2)} + 3^{-1 \times (-2)} + 4^{-1 \times (-2)}$

$= 2^2 + 3^2 + 4^2$
$= 4 + 9 + 16$
$= 29$

(iv) Given: $(3^{-1} + 4^{-1} + 5^{-1})^0$

Since, $a^{-m} = \frac{1}{a^m}$

$$= \left(\frac{1}{3}+\frac{1}{4}+\frac{1}{5}\right)^0$$

$$= \left(\frac{20+15+12}{60}\right)^0$$

$$= \left(\frac{47}{60}\right)^0$$

(Since, $a^0 = 1$)
$= 1$

(v) Given: $\left\{\left(\frac{-2}{3}\right)^{-2}\right\}^2$

Since, $(a^m)^n = a^{m \times n}$

$$= \left(\frac{-2}{3}\right)^{-2\times2}$$

Since, $a^{-m} = \frac{1}{a^m}$

$$= \left(\frac{-3}{2}\right)^4$$

$$= \frac{81}{16}$$

Q.4 Evaluate

(i) $\frac{8^{-1}\times5^3}{2^{-4}}$ **(ii) $(5^{-1} \times 2^{-1}) \times 6^{-1}$**

Sol. **(i) Given:** $\frac{8^{-1}\times5^3}{2^{-4}}$

We can write as $= \frac{(2^3)^{-1}\times5^3}{2^{-4}}$

Since, $(a^m)^n = a^{m \times n}$

$$= \frac{2^{-3} \times 5^3}{2^{-4}}$$

Since, $a^m \div a^n = a^{m-n}$
$= 2^{-3-(-4)} \times 5^3$
$= 2 \times 125$
$= 250$

(ii) Given: $(5^{-1} \times 2^{-1}) \times 6^{-1}$

Since, $a^{-m} = \frac{1}{a^m}$

$$= \left(\frac{1}{5} \times \frac{1}{2}\right) \times \frac{1}{6}$$

$$= \frac{1}{10} \times \frac{1}{6}$$

$$= \frac{1}{60}$$

Q.5 Find the value of *m* for which $5^m \div 5^{-3} = 5^5$.

Sol. Given: $5^m \div 5^{-3} = 5^5$

Since, $a^m \div a^n = a^{m-n}$

$$5^{m-(-3)} = 5^5$$

$$5^{m+3} = 5^5$$

By equating exponent of both sides,

$$m + 3 = 5$$

$$m = 5-3$$

$$m = 2$$

Q.6 Evaluate (i) $\left\{\left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right\}^{-1}$ (ii) $\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-4}$

***Sol.* (i) Given:** $\left\{\left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right\}^{-1}$

Since, $\left(\frac{a}{b}\right)^m = \frac{a^m}{a^n}$

$$= \left\{\left(\frac{3}{1}\right)^1 - \left(\frac{4}{1}\right)^1\right\}$$

$$= 3 - 4$$

$$= -1$$

(ii) Given: $\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-4}$

Since, $\left(\frac{a}{b}\right)^m = \frac{a^m}{a^n}$

$$= \frac{5^{-7}}{8^{-7}} \times \frac{8^{-4}}{5^{-4}}$$

Since, $\left(\frac{a}{b}\right)^m = \frac{a^m}{a^n}$

$= 5^{-7-(-4)} \text{ x } 8^{-4-(-7)}$

$= 5^{-7+4} \text{ x } 8^{-4+7}$

$= 5^{-3} \text{ x } 8^{3}$

$= \dfrac{8^3}{5^3}$

$= \dfrac{512}{125}$

Q.7 Simplify.

(i) $\dfrac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}} (t \neq 0)$ **(ii)** $\dfrac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}}$

Sol. **(i) Given:** $\dfrac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}}$

We can write as,

$= \dfrac{5^2 \times t^{-4}}{5^{-3} \times 5 \times 2 \times t^{-8}}$

Since, $a^m \div a^n = a^{m-n}$

$= \dfrac{5^{2-(-3)-1} \times t^{-4-(-8)}}{2}$

$= \dfrac{5^{2+3-1} \times t^{-4+8}}{2}$

$= \dfrac{5^4 \times t^4}{2}$

$= \dfrac{625}{2} t^4$

(ii) Given: $\dfrac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}}$

We can write as,

$= \dfrac{3^{-5} \times (2 \times 5)^{-5} \times 5^3}{5^{-7} \times (2 \times 3)^{-5}}$

Since, $(ab)^m = a^m b^m$

$= \dfrac{3^{-5} \times 2^{-5} \times 5^{-5} \times 5^3}{5^{-7} \times 2^{-5} \times 3^{-5}}$

Since, $a^m \text{ x } a^n = a^{m+n}$

$= \dfrac{3^{-5} \times 2^{-5} \times 5^{-5+3}}{5^{-7} \times 2^{-5} \times 3^{-5}}$

$= \dfrac{3^{-3} \times 2^{-5} \times 5^{-2}}{5^{-7} \times 2^{-5} \times 3^{-5}}$

Since, $a^m \div a^n = a^{m-n}$

$= 3^{-5-(-5)} \text{ x } 2^{-2-(-5)} \text{ x } 5^{-2-(-7)}$

$= 3^{-5+5} \text{ x } 2^{-5+5} \text{ x } 5^{-2+7}$

$= 3^0 \text{ x } 2^0 \text{ x } 5^5$

Since $a^0 = 1$

$= 1 \times 1 \times 3125$

$= 3125$