Coordinate Geometry: Exercise - 7.3

Q.1 Find the area of the triangle whose vertices are:
(i) (2, 3), (−1, 0), (2, − 4) (ii) (− 5,−1), (3, − 5), (5, 2)

Sol. **(i) Given: Vertices of triangle, Let P = (x_1, y_1) = (2, 3),**
Q = (x_2, y_2) = (−1, 0) and R = (x_3, y_3) = (2, −4)

Thus, Area of ΔPQR: $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[2(0 + 4) + (-1)(-4-3) + 2(3-0)]$

$= \frac{1}{2}(8 + 7 + 6) = 21/2 = 10.5$ sq.units

Thus, the area of the triangle = 10.5 sq. Units

(ii) Given: Vertices of triangle, Let P = (x_1, y_1) = (−5, −1),
Q = (x_2, y_2) = (3, −5) and R = (x_3, y_3) = (5, 2)

Thus Area of ΔPQR: $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[-5(-5-2) + 3(2 + 1) + 5(-1 + 5)]$

$= \frac{1}{2}(35 + 9 + 20) = \frac{1}{2} \times 64$

= 32 sq. Units

Thus, the area of the triangle = 32 sq. Units

Q.2 In each of the following find the value of 'k' for which the points are collinear:
(i) (7, −2), (5, 1), (3, k) (ii) (8, 1), (k, − 4), (2, − 5)

Sol. **(i)** Let the points be P = (x_1, y_1) = (7, −2), Q = (x_2, y_2) = (5, 1)
and R = (x_3, y_3) = (3, k).
If these points lie on a line, then Area of triangle PQR should be zero.
Area (ΔPQR) = 0

$\Rightarrow \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

$\Rightarrow 7(1 - k) + 5(k + 2) + 3(-2 - 1) = 0$
$\Rightarrow 7 - 7k + 5k + 10 - 9 = 0$
$\Rightarrow 8 - 2k = 0$
$\Rightarrow 2k = 8$
$\Rightarrow k = 4$

Thus, the points are collinear for k = 4

(ii) Let the points be P = (x_1, y_1) = (8, 1), Q = (x_2, y_2) = (k, −4)
And R = (x_3, y_3) = (2, −5).
If these points lie on a line, then Area of triangle PQR should be zero.
Area (ΔPQR) = 0

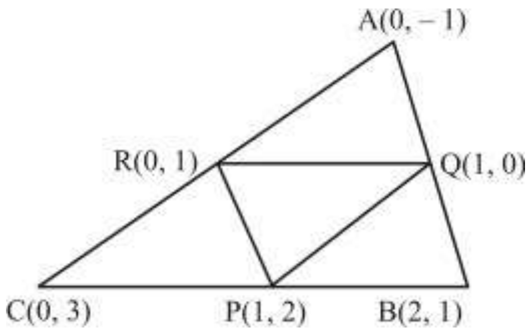
$\Rightarrow \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

$\Rightarrow 8(-4 + 5) + k(-5 - 1) + 2(1 + 4) = 0$
$\Rightarrow 8 - 6k + 10 = 0$
$\Rightarrow -6k = -18$
$\Rightarrow k = 3$

Thus, the points are collinear for k = 3.

Q.3 Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are (0, –1), (2, 1), and (0, 3). Find the ratio of this area to the area of the given triangle.

Sol. Suppose, the vertices of triangles ABC, A = (x_1, y_1) = (0, –1), B = (x_2, y_2) = (2, 1) and C = (x_3, y_3) = (0, 3).



Since, Area of the triangle: $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

So, Area (ΔABC) = $\frac{1}{2}[0(1-3) + 2(3 + 1) + 0(-1-1)]$

$= \frac{1}{2}(0 + 8 + 0) = 4$ sq. Units

Since, triangle PQR is formed by joining the mid-points of the sides of the triangle ABC. So, Vertices of triangle PQR are:

Let P $\left(\frac{0+2}{2}, \frac{3+1}{2}\right)$ = P (1, 2); Q $\left(\frac{2+0}{2}, \frac{1-1}{2}\right)$ = Q (1, 0) and R $\left(\frac{0+0}{2}, \frac{3-1}{2}\right)$=R (0, 1).

Now area of Triangle PQR: = $\frac{1}{2}[1(0 - 1) + 1(1 - 2) + 0(2 - 0)]$

$= \frac{1}{2}(-1 -1 + 0) = -1$

= 1 sq. unit (numerically)

Thus, the Ratio of the area (ΔPQR) to the area (ΔABC) = 1 : 4.

Q.4 Find the area of the quadrilateral whose vertices, taken in order, are (– 4, –2), (– 3, –5), (3, –2) and (2, 3).

Sol. Suppose, P (– 4, –2), Q (– 3, –5), R (3, –2) and S (2, 3) are the vertices of the quadrilateral PQRS. So, the area of quadrilateral PQRS = Area of Δ PQR + Area of Δ PRS

Since, Area of the triangle: $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[-4(-5 + 2) - 3(-2 + 2) + 3(-2 + 5)] + \frac{1}{2}[-4(-2 - 3) + 3(3 + 2) + 2(-2 + 2)]$

$= \frac{1}{2}(12 - 0 + 9) + \frac{1}{2}(20 + 15 + 0)$

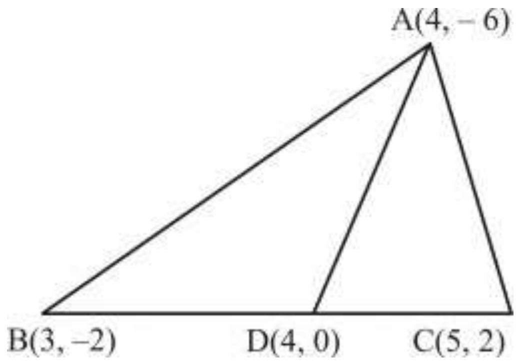
$= \frac{1}{2}(21 + 35) = \frac{1}{2} \times 56 = 28$ sq. Units

Thus, the area of quadrilateral PQRS = 28 sq. units

Q.5 You have studied in class IX (Chapter 9 Example 3) that a median of a triangle divides it into two triangles of equal areas. Verify this result for Δ ABC whose vertices are A (4, –6), B (3, –2) and C (5, 2).

Sol. Given: AD is the median of Δ ABC, therefore, D is the mid-point of BC as shown in fogure. So, coordinates of D are:

$$D\left(\frac{3+5}{2}, \frac{2+2}{2}\right) = D\ (4, 0).$$



Now, area of ΔADC by the formula: $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[4\ (0 - 2) + 4\ (2 + 6) + 5\ (-6 - 0)]$

$= \frac{1}{2}(-8 + 32 - 30)$

$= \frac{1}{2} \times -6 = -3$

= 3sq. units (Since area cannot be negative)

Now, area of ΔABD by the formula: $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[4\ (-2 - 0) + 3(0 + 6) + 4(-6 + 2)]$

$= \frac{1}{2}(-8+18-16)$

$= \frac{1}{2}(-6)$

$= -3$

= 3 sq. units (Since area cannot be negative)

from above, area (ΔADC) = area (ΔABD)

Therefore, It is proved that the median of the triangle divides it into two triangles of equal areas.