Coordinate Geometry: Exercise - 7.2

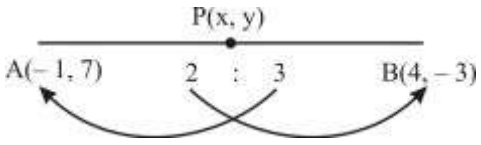
Q.1 Find the coordinates of the point which divides join of (−1, 7) and (4, −3) in the ratio 2:3

Sol. Suppose, P(x, y) is the required point. So,

From Section formula

$$P(x, y) = \left[\frac{mx_2 + nx_2}{m+n}, \frac{my_2 + ny}{m+n}\right]$$

Since, m = 2 and n = 3



$$P(x, y) = \left[\frac{2x4+3x(-1)}{2+3}, \frac{2x(-3)+3x7}{2+3}\right]$$

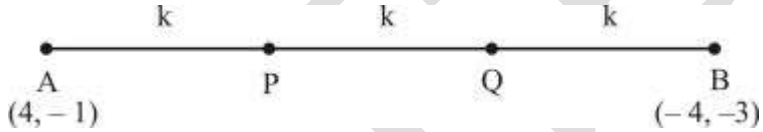
$$P(x, y) = \left[\frac{8-3}{5}, \frac{-6+21}{5}\right]$$

P(x, y) = (1, 3)

Thus, the coordinates of P are (1, 3).

Q.2 Find the coordinates of the points of trisection of the line segment joining (4,−1) and (−2,−3).

Sol. Let P and Q be the points of trisection of the line segment joining the points A (4,−1) and B (−2,−3).



Since, AP = PQ = QB = k (let).

Therefore, PB = PQ + QB = 2k

and, AQ = AP + PQ = 2k

So,

⇒ AP : PB = k : 2k = 1 : 2

So from the figure, P divides AB internally in the ratio of 1 : 2,

Thus, the coordinates of P $= \left[\frac{1x-2+2x4}{1+2}, \frac{1x(-3)+2x-1}{1+2}\right]$

$= \left[\frac{-2+8}{3}, \frac{-3-2}{3}\right]$

$= (2, -5/3)$

Now, AQ : QB = 2k : k = 2 : 1

So, Q divides AB internally in the ratio 2 : 1,

Thus the coordinates of Q $= \left[\frac{2x-2+1x4}{2+1}, \frac{2x(-3)+1x(-1)}{2+1}\right]$

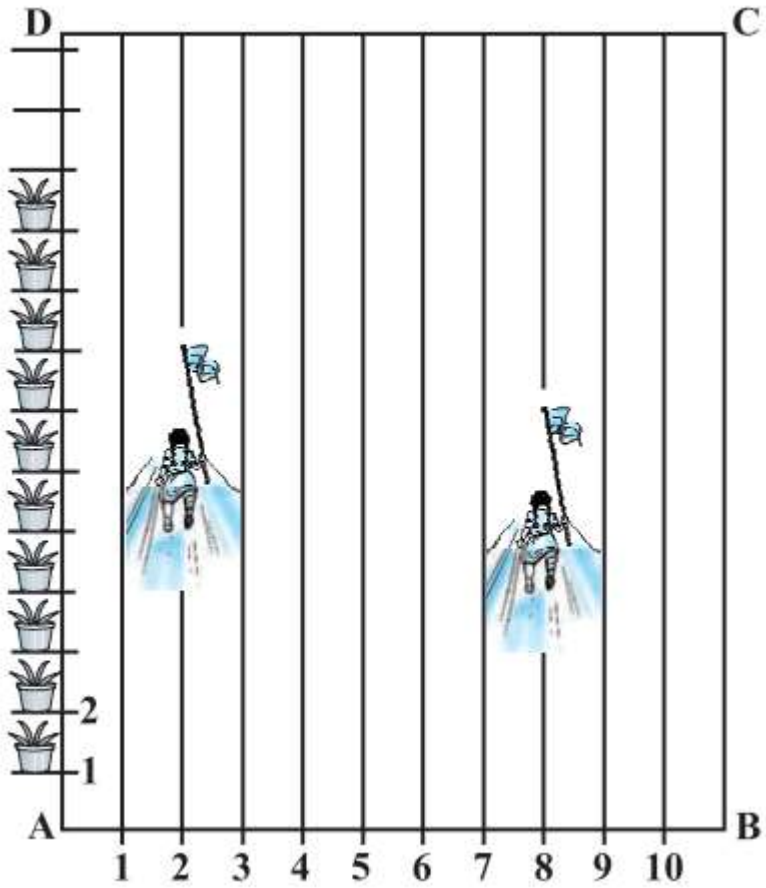
$= \left[\frac{-4+4}{3}, \frac{-6-1}{3}\right]$

$= (0, -7/3)$

Thus, the two points of trisection are P(2, −5/3) and Q(0, −7/3).

Q.3 To conduct Sports Day activities in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1 m from each other along AD, as shown in figure. Niharika runs 1/4 th

the distance AD on the 2nd line and posts a green flag. Preet runs $1/5^{th}$ the distance AD on the eighth line and posts a red flag. What is the distance between both the flags? If Rashmi has to post a blue flag exactly halfway between the line (segment) joining the two flags, where should she post her flag?



Sol. *F*rom the figure, the position of green flag posted by Niharika is,

$P\left(2, \frac{1}{4}\times 100\right)$,

i.e., P (2, 25)

and red flag posted by Preet is, $Q\left(8, \frac{1}{5}\times 100\right)$

i.e. Q (8, 20)

So, distance between them,

By the distance formula = $\sqrt{(x_2-x_2)^2+(y_2-y_2)^2}$

Now, PQ= $\sqrt{(8-2)^2+(20-25)^2}$

$=\sqrt{(6)^2+(-5)^2}=\sqrt{61}$

Thus, the distance between the flags is $\sqrt{61}$ m

Let M be the position of the blue flag posted by Rashmi which is the halfway of line segment PQ.

So,, $M = \left[\frac{2+8}{2}, \frac{25+20}{2}\right]$

$= \left[\frac{10}{2}, \frac{45}{2}\right]$

= (5, 22.5)

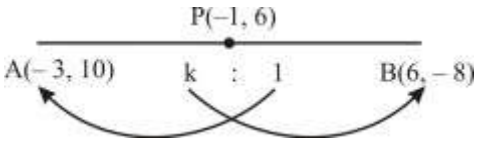
Therefore, the blue flag is on the fifth line at a distance 22.5 m above it.

Q.4 Find the ratio in which the line segment joining the points of (–3, 10) and (6, – 8) is divided by (–1, 6).

Sol. Suppose the point P (–1, 6) divide the line joining points A(–3, 10) and B(6, –8) in the ratio of k : 1.

So, coordinates of P: $\left(\frac{6k-3}{k+1}, \frac{-8k+10}{k+1}\right)$.

Since, the coordinates of P: (–1, 6).



So, $\frac{6k-3}{k+1} = -1$ and $\frac{-8k+10}{k+1} = 6$

$\Rightarrow 6k - 3 = -k - 1$ and $-8k + 10 = 6k + 6$

$\Rightarrow 6k + k = -1 + 3$ and $-8k - 6k = 6 - 10$

$\Rightarrow 7k = 2$ and $-14k = -4$

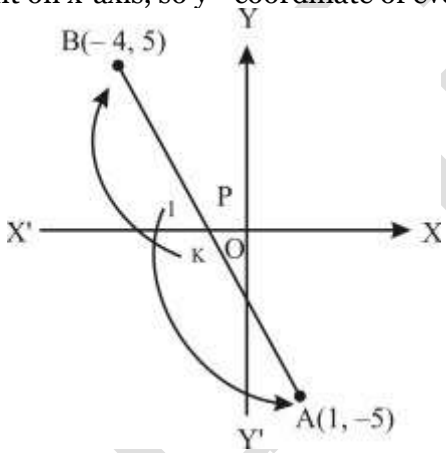
$\Rightarrow k = 2/7$

Thus, the point P divides line segment AB in the ratio 2:7.

Q.5 Find the ratio in which the line segment joining A(1, –5) and B(– 4, 5) is divided by the x-axis. Also find the coordinates of the point of division.

Sol. Let k: 1 be the required ratio. Then, the coordinates of the point P of division by the x-axis: $\left(\frac{-4k+1}{k+1} = \frac{5k-5}{k+1}\right)$.

Since this point on x-axis, so y - coordinate of every point is zero.



Therefore, $\frac{5k-5}{k+1} = 0$

$\Rightarrow 5k - 5 = 0$

$\Rightarrow 5k = 5$

$\Rightarrow k = 1$

Therefore, the required ratio is 1 : 1.

The point of division P: $\left(\frac{-4x1+1}{1+1}, 0\right)$,

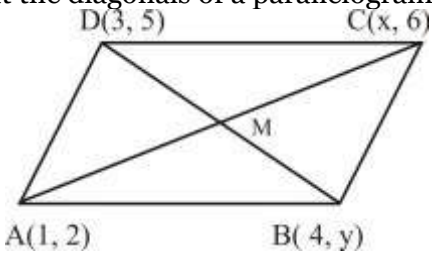
$\left(\frac{-4+1}{2}, 0\right)$

$(-3/2, 0)$

Thus the coordinates of the point of division: (–3/2, 0)

Q.6 If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

Sol. Suppose, points A(1, 2), B(4, y), C(x, 6) and D(3, 5) are the vertices of a parallelogram ABCD.
Since, as we know that the diagonals of a parallelogram bisect each other at point M,



Therefore, $\frac{x+1}{2} = \frac{3+4}{2}$

$\Rightarrow x + 1 = 7$

$\Rightarrow x = 6$

and, $\frac{5+y}{2} = \frac{6+2}{2}$

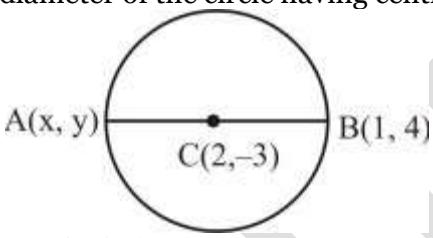
$\Rightarrow 5 + y = 8$

$\Rightarrow y = 3$

Thus, the values of x = 6 and y = 3.

Q.7 Find the coordinates of a point A, where AB is the diameter of a circle whose centre is (2, –3) and B is (1, 4).

Sol. Given: AB is the diameter of the circle having centre, C (2, –3) and the coordinates of B (1, 4).



Suppose the coordinates of A is (x, y).
As we know that centre, C is the mid - point of diameter AB,
SO,

$\frac{x+1}{2} = 2$

$\Rightarrow x+1 = 4$

$\Rightarrow x = 3$

and, $\frac{y+4}{2} = -3$

$\Rightarrow y + 4 = -6$

$\Rightarrow y = -10$

Thus, the coordinates of A: (3, –10).

Q.8 If A and B are (–2, –2) and (2, – 4) respectively, find the coordinates of P such that AP = $\frac{3}{7}$ AB and P lies on the line segment AB.

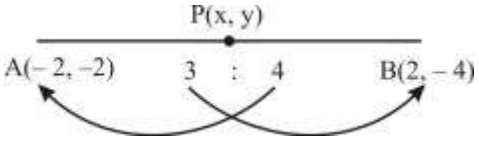
Sol. Given: Points A (–2, –2) and B (2, – 4) and Point p lies on the line segment AB such that AP= $\frac{3}{7}$ AB.

$\Rightarrow AB/AP = 7/3$

$\Rightarrow \frac{AP+PB}{AP} = \frac{3+4}{3}$

$\Rightarrow 1+\frac{PB}{AP} = 1+\frac{4}{3}$

⇒PB/AP = 4/3
⇒AP/PB = ¾
⇒AP : PB = 3 : 4



Suppose P(x, y) is the point which divides the line segment by joining the points A(–2, – 2) and B(2, – 4) in the ratio of 3 : 4.

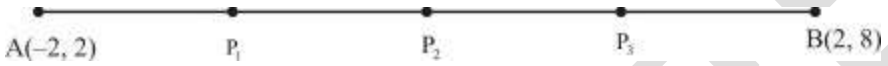
So, P: $\left[\frac{3x2+4x(-2)}{3+4}, \frac{3x(-4)+4x(-2)}{3+4}\right]$

P: $\left[\frac{6-8}{7}, \frac{-12-8}{7}\right]$

Thus, the coordinates of the point P are (–2/7, –20/7).

Q.9 Find the coordinates of the points which divide the line segment joining A(– 2, 2) and B(2, 8) into four equal parts.

Sol. Suppose, points P_1, P_2 and P_3 are the points which divide the line segment joining points A(–2, 2) and B(2, 8) into four equal parts.



As shown in figure, point P_2 divides the segment AB into two equal parts.

So, coordinates of P_2: $\left(\frac{-2+2}{2}, \frac{2+8}{2}\right)$,

P_2: (0, 5)

From figure, point P_1 divides the line segment AP_2 into two equal parts.

So, coordinates of P_1: $\left(\frac{-2+0}{2}, \frac{2+5}{2}\right)$,

: (–1, 7/2)

Now, Point P_3 is the mid-point of line segment P_2B.

Therefore, coordinates of P_3: $\left(\frac{0+2}{2}, \frac{5+8}{2}\right)$

: (1, 13/2).

Q.10 Find the area of a rhombus if its vertices are (3, 0), (4, 5), (– 1, 4) and (– 2,–1) taken in order.

Sol. Suppose, points A(3, 0), B(4, 5), C(–1, 4) and D(–2,– 1) are the vertices of the rhombus ABCD.

So, Diagonal AC, by the distance formula = $\sqrt{(x_2-x_2)^2+(y_2-y_2)^2}$

Now, AC = $\sqrt{(-1-3)^2+(4-0)^2}$

AC = $4\sqrt{2}$

And another diagonal BD,

BD = $\sqrt{(-1-3)^2+(4-0)^2}$

= $6\sqrt{2}$

Since, Area of the rhombus ABCD = (1/2) × (Product of lengths of diagonals)

= (1/2) × AC × BD

= (1/2) × $4\sqrt{2}$ x $6\sqrt{2}$ sq. units

= 24 sq. Units

Thus area of rhombus ABCD 24sq. Units.