Coordinate Geometry: Exercise - 7.1

Q.1 Find the distance between the following pairs of points:
(i) (2, 3), (4, 1) (ii) (−5, 7), (−1, 3) (iii) (a, b),(−a,−b)

Sol. **(i)** Given: Points (2, 3) and Q (4, 1)

Where, $x_1 = 2$, $y_1 = 3$ and $x_2 = 4$, $y_2 = 1$

Therefore distance formula, $d = \sqrt{(x_2 - x_2)^2 + (y_2 - y_2)^2}$

$\Rightarrow d = \sqrt{(4-2)^2 + (1-3)^2}$

$\Rightarrow d = \sqrt{(2)^2 + (-2)^2} = 2\sqrt{2}$

(ii) Given: Points (− 5, 7) and Q (−1, 3)

Where $x_1 = -5$, $y_1 = 7$ and $x_2 = -1$, $y_2 = 3$

Therefore distance formula, $d = \sqrt{(x_2 - x_2)^2 + (y_2 - y_2)^2}$

$\Rightarrow d = \sqrt{(-1+5)^2 + (3-7)^2}$

$\Rightarrow d = \sqrt{(4)^2 + (-4)^2} = 4\sqrt{2}$

(iii) Given: Points (a, b) and Q (−a, −b)

Where, $x_1 = a$, $y_1 = b$ and $x_2 = -a$, $y_2 = -b$

Therefore distance formula, $d = \sqrt{(x_2 - x_2)^2 + (y_2 - y_2)^2}$

$\Rightarrow d = \sqrt{(-a-a)^2 + (-b-b)^2}$

$\Rightarrow d = \sqrt{(-2a)^2 + (-2b)^2} = 2\sqrt{a^2 + b^2}$

Q.2 Find the distance between the points (0, 0) and (36, 15). Can you now find the distance between the two towns A and B discussed in Section 7.2?

Sol. Given: Points (0, 0) and Q (36, 15)

Where $x_1 = 0$, $y_1 = 0$ and $x_2 = 36$, $y_2 = 15$

Therefore distance formula, $d = \sqrt{(x_2 - x_2)^2 + (y_2 - y_2)^2}$

$\Rightarrow d = \sqrt{(36-0)^2 + (15-0)^2}$

$\Rightarrow d = \sqrt{1296 + 225} = \sqrt{1521} = 39$

Since, the positions of towns A and B are given by (0, 0) and (36, 15) respectively.
Therefore, the distance between towns = 39 km .

Q.3 Determine if the points (1, 5), (2, 3) and (−2, −11) are collinear.

Sol. Let P(1, 5), Q(2, 3) and R(−2, −11) be the given points.

Lets, find the distance between the points: PQ, QR and PR

Therefore distance formula, $d = \sqrt{(x_2 - x_2)^2 + (y_2 - y_2)^2}$

$\Rightarrow PQ = \sqrt{(2-1)^2 + (3-5)^2}$

$\Rightarrow PQ = \sqrt{1+4} = \sqrt{5}$ ..... (i)

$\Rightarrow QR = \sqrt{(-2-2)^2 + (-11-3)^2}$

$\Rightarrow QR = \sqrt{16+196} = 2\sqrt{53}$ ..... (ii)

and, $\Rightarrow PR = \sqrt{(-2-1)^2 + (-11-5)^2}$

$\Rightarrow PR = \sqrt{9+256} = \sqrt{256}$ ..... (iii)

From (i), (ii) & (iii),
BC ≠ AB + AC, AB ≠ BC + AC and AC ≠ BC
Thus, These points A,B and C are not collinear.

Q.4 Check whether (5, –2), (6, 4) and (7, – 2) are the vertices of an isosceles triangle.

Sol. Let P (5, –2), Q (6, 4) and R (7, –2) are the given points of the triangle. Let's, find the distance between the points:

Therefore distance formula, $d = \sqrt{(x_2 - x_2)^2 + (y_2 - y_2)^2}$

$\Rightarrow PQ = \sqrt{(6-5)^2 + (4+2)^2}$

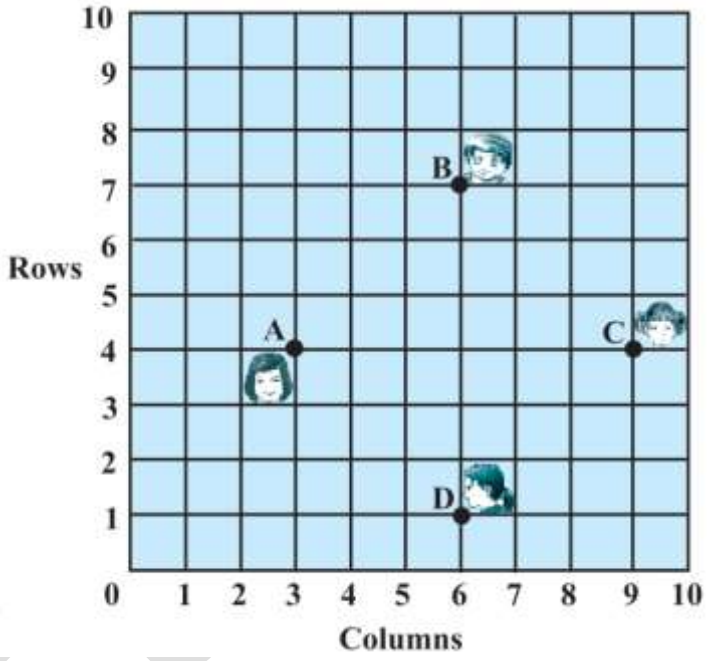
$\Rightarrow PQ = \sqrt{1+36} = \sqrt{37}$ ..... (i)

$\Rightarrow QR = \sqrt{(7-6)^2 + (-2-4)^2}$

$\Rightarrow QR = \sqrt{1+36} = \sqrt{37}$ ..... (ii)

From (i) & (ii), AB = BC
Thus, ΔABC is an isosceles triangle.

Q.5 In a classroom, four friends are seated at the points A, B, C and D as shown in figure. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, "Don't you think ABCD is a square?" Chameli disagrees. Using distance formula, find which of them is correct, and why?



Sol. From the figure, the coordinates of points A, B, C and D are (3, 4), (6, 7), (9, 4) and (6, 1) which tell the position of four students.
Now, find the distance the between the students.

By using distance formula, $d = \sqrt{(x_2 - x_2)^2 + (y_2 - y_2)^2}$

$AB = \sqrt{(6-3)^2 + (7-4)^2}$

$\Rightarrow AB = \sqrt{9+9} = 3\sqrt{2}$ ..... (i)

$BC = \sqrt{(9-6)^2 + (4-7)^2}$

$\Rightarrow BC = \sqrt{9+9} = 3\sqrt{2}$ ..... (ii)

$CD = \sqrt{(6-9)^2 + (1-4)^2}$

$\Rightarrow CD = \sqrt{9+9} = 3\sqrt{2}$ ..... (iii)

and, $DA = \sqrt{(3-6)^2 + (4-1)^2}$

$\Rightarrow DA = \sqrt{9+9} = 3\sqrt{2}$ ..... (iv)

⇒Since, from above AB = BC = CD = DA = $3\sqrt{2}$

And diagonals:

Also, AC = $\sqrt{(9-3)^2+(4-4)^2}$

$\Rightarrow$ AC = $\sqrt{36+0}$ = 6 .....(v)

and, BD = $\sqrt{(6-6)^2+(1-7)^2}$

$\Rightarrow$ BD = $\sqrt{0+36}$ = 6 .....(vi)

$\Rightarrow$AC = BD = 6

Since, the four sides and diagonals are equal. Therefore, ABCD is a square.

Thus, Champa is right.

Q.6 Name the type of quadrilateral formed, if any, the following points, and give reasons for your answer:

(i) (−1,−2), (1, 0), (−1, 2), (−3,0) (ii) (−3, 5), (3, 1), (0, 3), (−1,− 4) (iii) (4, 5), (7, 6), (4, 3), (1, 2)

Sol. **(i)** Let P(−1,−2), Q(1, 0), R(−1, 2) and S(−3, 0) be the given points. So, we need to find the distance between them

by the distance formula, d = = $\sqrt{(x_2-x_2)^2+(y_2-y_2)^2}$

PQ = $\sqrt{(1+1)^2+(0+2)^2}$

$\Rightarrow$ PQ = $\sqrt{4+4}$ = $\sqrt{8}$

QR = $\sqrt{(-1-1)^2+(2-0)^2}$

$\Rightarrow$ QR = $\sqrt{4+4}$ = $\sqrt{8}$

RS = $\sqrt{(-3+1)^2+(0-2)^2}$

$\Rightarrow$ RS = $\sqrt{4+4}$ = $\sqrt{8}$

SP = $\sqrt{(-1+3)^2+(-2-0)^2}$

$\Rightarrow$ SP = $\sqrt{4+4}$ = $\sqrt{8}$

And Diagonals, PR = $\sqrt{(-1+1)^2+(2+2)^2}$

$\Rightarrow$ PR = $\sqrt{0+16}$ = 4

and QS = $\sqrt{(-3-1)^2+(0-0)^2}$

$\Rightarrow$ QS = $\sqrt{16+0}$ = 4

From above, four sides PQ, QR, RS and SP are equal. Also, diagonals PR and QS are equal.

Thus, the quadrilateral PQRS is a square.

(ii) Let P(−3, 5), Q(3, 1), R(0, 3) and S(−1,− 4) are the given points.

we need to find the distance between them

by the distance formula, d = $\sqrt{(x_2-x_2)^2+(y_2-y_2)^2}$

PQ = $\sqrt{(-3-3)^2+(1-5)^2}$

$\Rightarrow$ PQ = $\sqrt{36+16}$ = $2\sqrt{13}$

QR = $\sqrt{(0-3)^2+(3-1)^2}$

$\Rightarrow$ QR = $\sqrt{9+4}$ = $\sqrt{13}$

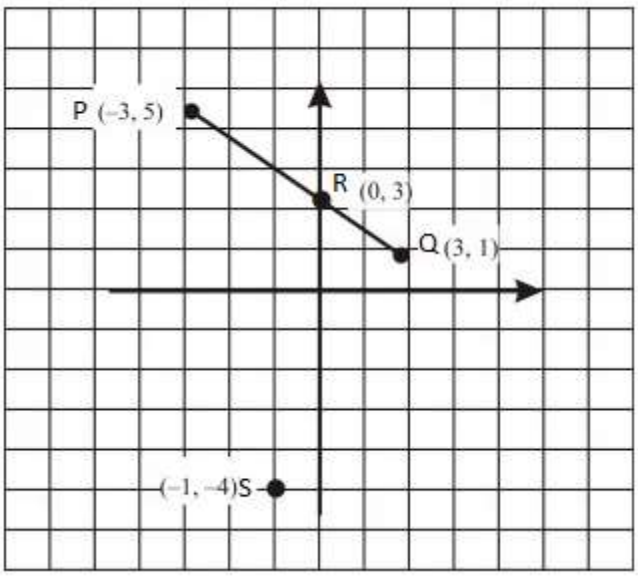
RS = $\sqrt{(-1-0)^2+(-4-3)^2}$

$\Rightarrow$ RS = $\sqrt{1+49}$ = $5\sqrt{2}$

SP = $\sqrt{(-1+3)^2+(-4-5)^2}$

$\Rightarrow$ SP = $\sqrt{4+81}$= $\sqrt{85}$

Now, plot these points as shown.



From the figure, points P, R and Q are collinear. So, no quadrilateral is formed by these points.

(iii) Let P(4, 5), Q(7, 6), R(4, 3) and S(1, 2) be the given points.
we need to find the distance between them

by the distance formula, $d = \sqrt{(x_2 - x_2)^2 + (y_2 - y_2)^2}$

$PQ = \sqrt{(7-4)^2 + (6-5)^2}$
$\Rightarrow PQ = \sqrt{9+1} = \sqrt{10}$

$QR = \sqrt{(4-7)^2 + (3-6)^2}$
$\Rightarrow QR = \sqrt{9+9} = \sqrt{18}$

$RS = \sqrt{(1-4)^2 + (2-3)^2}$
$\Rightarrow RS = \sqrt{9+1} = \sqrt{10}$

$SP = \sqrt{(1-4)^2 + (2-5)^2}$
$\Rightarrow SP = \sqrt{9+9} = \sqrt{18}$

And Diagonals, $PR = \sqrt{(4-4)^2 + (3-5)^2}$
$\Rightarrow PR = \sqrt{0+4} = 2$

and $QS = \sqrt{(1-7)^2 + (2-6)^2}$
$\Rightarrow QS = \sqrt{36+16} = 2\sqrt{13}$

From above, PQ = QR, RS = SP and but PR ≠ SP
Thus, the quadrilateral PQRS is a parallelogram.

Q.7 Find the point on the x-axis which is equidistant from (2, – 5) and (– 2, 9).

Sol. As we know that the point on x - axis have its ordinate = 0, so point P(x, 0) is any point on the x – axis which is equidistant from A (2, –5) and B (–2, 9)

So, PA = PB
$\Rightarrow PA^2 = PB^2$
$\Rightarrow (x-2)^2 + (0+5)^2 = (x+2)^2 + (0-9)^2$
$\Rightarrow x^2 - 4x + 4 + 25 = x^2 + 4x + 4 + 81$
$\Rightarrow -4x - 4x = 81 - 25$
$\Rightarrow -8x = 56$
$\Rightarrow x = 56/-8$
$= -7$

Thus, the point equidistant from given points on the axis is P (–7, 0)

Q.8 Find the value of y for which the distance between the points P(2, –3) and Q (10, y) is 10 units.

Sol. Given: Points, P (2, –3) and Q (10, y) such that PQ = 10 units.

From the distance formula, $d = \sqrt{(x_2 - x_2)^2 + (y_2 - y_2)^2}$

But, $PQ = \sqrt{(10-2)^2 + (y+3)^2}$

$\Rightarrow 10 = \sqrt{64 + y^2 + 6y + 9}$

Squaring both the sides

$\Rightarrow 100 = 73 + y^2 + 6y$

$\Rightarrow y^2 + 6y - 27 = 0$

$\Rightarrow (y+9)(y-3) = 0$

$\Rightarrow y = -9$ or 3

Therefore, the values of y are – 9 or 3.

Q.9 If Q (0, 1) is equidistant from P (5, –3) and R(x, 6), find the values of x. Also find the distances QR and PR.

Sol. Given: The point Q (0, 1) is equidistant from P (5, –3) and R(x, 6),

$QP = QR$

$\Rightarrow QP^2 = QR^2$

$\Rightarrow (5-0)^2 + (-3-1)^2 = (x-0)^2 + (6-1)^2$

$\Rightarrow 25 + 16 = x^2 + 25$

$\Rightarrow x^2 = 16$

$\Rightarrow x = \pm 4$

Therefore point, R is (4, 6) or (– 4, 6).

Now the distances QR and PR:

QR = Distance between Q (0, 1) and R (4, 6)

From the distance formula, $d = \sqrt{(x_2 - x_2)^2 + (y_2 - y_2)^2}$

$QR = \sqrt{(4-0)^2 + (6-1)^2}$

$\Rightarrow QR = \sqrt{16 + 25} = \sqrt{41}$

Also, QR = Distance between Q(0, 1) and R(– 4, 6)

$QR = \sqrt{(-4-0)^2 + (6-1)^2}$

$\Rightarrow QR = \sqrt{16 + 25} = \sqrt{41}$

and, PR = Distance between P(5, –3) and R(4, 6)

$PR = \sqrt{(4-5)^2 + (6+3)^2}$

$\Rightarrow PR = \sqrt{1 + 81} = 9\sqrt{2}$

Also, PR = Distance between P(5, –3) and R(– 4, 6)

$PR = \sqrt{(4-5)^2 + (6+3)^2}$

$\Rightarrow PR = \sqrt{1 + 81} = 9\sqrt{2}$

Q.10 Find a relation between x and y such that the point (x, y) is equidistant from the points (3, 6) and (–3, 4).

Sol. Given: Point P(x, y) be equidistant from the points C(3, 6) and D(–3, 4)

i.e., PC = PD

$\Rightarrow PC^2 = PD^2$

$\Rightarrow (x-3)^2 + (y-6)^2 = (x+3)^2 + (y-4)^2$

$\Rightarrow x^2 - 6x + 9 + y^2 - 12y + 36 = x^2 + 6x + 9 + y^2 - 8y + 16$

$\Rightarrow -6x - 6x - 12y + 8y + 36 - 16 = 0$

$\Rightarrow -12x - 4y + 20 = 0$

$\Rightarrow 3x + y - 5 = 0$

Hence, this is the relation between x and y.