Circle: Exercise 10.4

Q.1 Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.

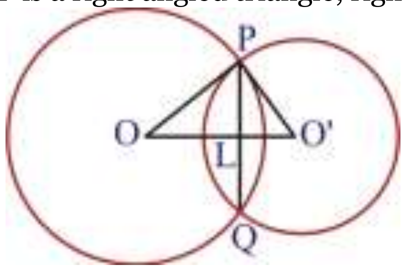
Sol. Suppose, O and O' are the centres of the circles of radii 5 cm and 3 cm respectively and PQ is their common chord.

Given: OP = 5 cm, O'P = 3 cm and OO' = 4 cm

Since, $OP^2 = PO'^2 + O'O^2$

$5^2 = 3^2 + 4^2$

⇒ Therefore, OO'P is a right angled triangle, right angled at P.



Now, Area of $\Delta OO'P = \frac{1}{2} \times O'P \times OO'$

$= \frac{1}{2} \times 3 \times 4$

= 6 sq. units... (i)

Also, Area of $\Delta OO'P = \frac{1}{2} \times OO' \times PL$

$= \frac{1}{2} \times 4 \times PL = 2PL$ ... (ii)

Since, eq. (i) and (ii) both are the area of ΔOO'P.

So, $2 \times PL = 6$

$\Rightarrow PL = 3$

Since, when two circles intersect at two points, then their centre lie on the perpendicular bisector of the common chord. So, OO' is the perpendicular bisector of PQ.

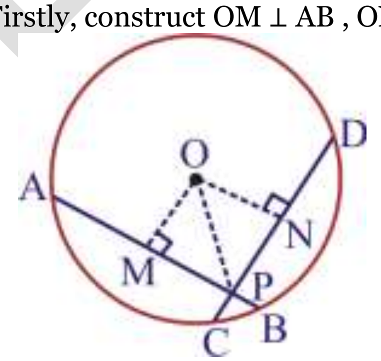
Thus, PQ = 2 × PL = (2 × 3) cm = 6 cm

Q.2 If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.

***Sol.* Given: Let** AB and CD be the chords of a circle with centre O. These chords AB and CD intersect at point P and AB = CD.

To prove: (i) AP = PD (ii) PB = CP.

Construction: Firstly, construct OM ⊥ AB , ON ⊥ CD and join OP.



Proof: AM = MB = $\frac{1}{2}$AB (Since, perpendicular from centre bisects the chord)

CN = ND = $\frac{1}{2}$CD (Since, perpendicular from centre bisects the chord)

So, AM = ND and MB = CN ......(i) (Since, bisectors of equal chords are equal)

Now, in ΔOMP and ΔONP,

OM = ON (Since, as we know that equal chords of a circle are equidistant from the centre)

∠OMP = ∠ONP (Since, OM ⊥ AB , ON ⊥ CD)

OP = OP (Common side)

So, from RHS' criterion of congruence,

ΔOMP≅ΔONP

⇒So, MP = PN ... (ii) (By C.P.C.T.)

By adding (i) and (ii),

AM + MP = ND + PN

⇒ AP = PD......Hence Proved (i)

By subtracting (ii) from (i),

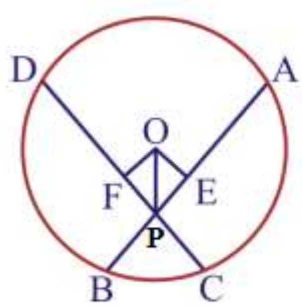
MB – MP = CN – PN

⇒ PB = CP............Hence Proved (ii)

Q.3 If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.

***Sol.* Given:** Let AB and CD be the chords of a circle with centre O. These chords AB and CD intersect at P and AB = CD.

To prove: ∠OPE = ∠OPF



Construction: Construct OE ⊥ AB and OF ⊥ CD. Now, join OP.

Proof: Firstly, in ΔOEP and ΔOFP,

∠OEP = ∠OFP (Since, OE ⊥ AB and OF ⊥ CD, each = 90°)

OP = OP (Common Side)

OE = OF (Since, as we know that equal chords of a circle are equidistant from the centre)

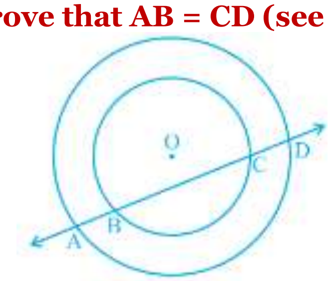
So, from RHS criterion of congruence,

ΔOEP ≅ ΔOFP

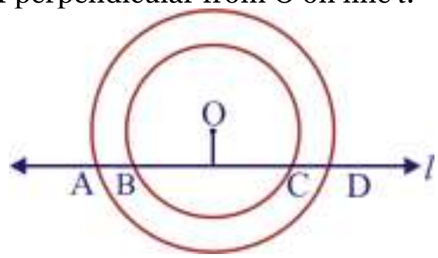
⇒ ∠OPE = ∠OPF (By C.P.C.T.)

Hence Proved.

Q.4 If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that AB = CD (see figure)



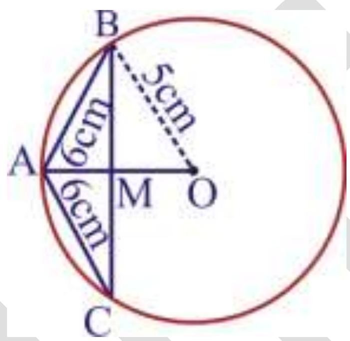
***Sol.* Given:** let l be the line which intersects two concentric circles with centre O at A, B, C and D.
To prove: AB = CD
Construct: Draw OM perpendicular from O on line l.



Proof: Since, perpendicular from the centre of a circle to a chord, bisects the chord.
Here, BC is a chord of the smaller circle and OM ⊥ BC
So, BM = CM ... (i)
And AD is a chord of the larger circle and OM ⊥ AD.
So, AM = DM ... (ii)
Now, subtracting (i) from (ii),
AM – BM = DM – CM
⇒ AB = CD............Hence Proved.

Q.5 Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5 m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6 m each, what is the distance between Reshma and Mandip?

***Sol*.** Let B, A and C be the points on circle of radius 5 cm, where three girls Reshma, Salma and Mandip are standing respectively.



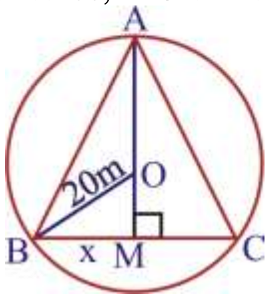
Since, AB = 6 cm and AC = 6 cm.
If AB and AC are two equal chords of a circle, then the centre of the circles lies on the bisector of ∠BAC. So, OA is the bisector of ∠BAC.
Since the internal bisector of an angle divides the opposite sides in the ratio of the sides containing the angle.
So, M divides BC in the ratio 6 : 6 = 1 : 1
Therefore, we can say M is the middle point of BC.
⇒ OM ⊥ BC.
Now, in right angled ΔABM,
$AB^2 = AM^2 + BM^2$
$\Rightarrow 36 = AM^2 + BM^2$
$\Rightarrow BM^2 = 36 - AM^2$ ... (i)

And in the right angled ΔOBM,
$OB^2 = OM^2 + BM^2$
$\Rightarrow 25 = (OA-AM)^2 + BM^2$
$\Rightarrow BM^2 = 25 - (OA-AM)^2$
$\Rightarrow BM^2 = 25-(5-AM)^2$ ... (ii)
So, from (i) and (ii),
$36 - AM^2 = 25 - (5-AM)^2$
$\Rightarrow 11 - AM^2 + (5-AM)^2 = 0$
$\Rightarrow 11 - AM^2 + 25 - 10AM + AM^2 = 0$
$\Rightarrow 10AM = 36$

⇒ AM = 3.6
Now, putting the value of AM = 3.6 in (i)
$BM^2 = 36 - (3.6)^2$
$= 36 - 12.96$
⇒ BM = $\sqrt{36 - 12.96}$
= $\sqrt{23.04}$ = 4.8cm
⇒ BC = 2BM
= 2 × 4.8 = 9.6cm
therefore, the distance between Reshma and Mandip is 9.6 cm

Q.6 A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.

Sol. Let A, B and C be the points on a circular path of radius 20 cm, where three boys Ankur, Sayed and David are sitting at equal distance. So, ABC is an equilateral triangle of side 2x metres.



BM = $\dfrac{BC}{2} = \dfrac{2x}{2}$ = x metres
Now, in right angled ΔABM,
$AM^2 = AB^2 - BM^2$
$= (2x)^2 - x^2$
$= 4x^2 - x^2$
$= 3x^2$
⇒AM = $\sqrt{3}$ x
Now, OM = AM – OA
= $\sqrt{3x}$ – 20 m
In right angled ΔOBM,
$OB^2 = BM^2 + OM^2$
⇒ $20^2 = x^2 + (\sqrt{3x} - 20)^2$
⇒ $400 = x^2 + 3x^2 - 40\sqrt{3x} + 400$
⇒ $4x^2 - 40\sqrt{3x} = 0$
⇒ $4x\,(x - 10\sqrt{3}) = 0$
Since, x ≠ 0; So, $x - 10\sqrt{3} = 0$
⇒ $x = 10\sqrt{3}$
Since, BC = 2BM
= 2x
$= 20\sqrt{3}$
Thus, the length of each string is $20\sqrt{3}$ m