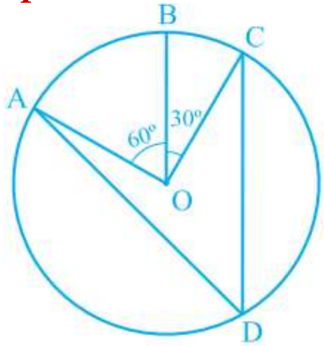
Circle: Exercise 10.5

Q. 1 In figure A, B and C are three points on a circle with centre O such that ∠BOC = 30⁰ and ∠AOB = 60⁰. If D is a point on the circle other than the arc ABC, find ∠ADC.



Sol. Since, in given figure, arc ABC makes ∠AOC = ∠AOB+∠BOC
= 60⁰ + 30⁰ = 90⁰

So, ∠AOC = 90⁰ at center
And ∠ADC at a point on the remaining part of the circle.

So, ∠ADC = $\frac{1}{2}$ (∠AOC)

= $\frac{1}{2}$ × 90°

= 45°

Q.2 A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major are.

Sol. Let PQ be chord of the circle with center O. Now, join OP and OQ.

Since, chord of circle is equal to the radius of the circle.
So, PQ = OP = OQ
Therefore, ΔOPQ is an equilateral.
⇒ ∠POQ = 60⁰
Since, arc PBQ makes reflex POQ = 360⁰ – 60⁰ = 300⁰ at centre of the circle and ∠PBQ at a point in the minor arc of the circle.

So, ∠PBQ = $\frac{1}{2}$ (reflex ∠POQ)

= $\frac{1}{2}$ × 300°

$= 150^0$

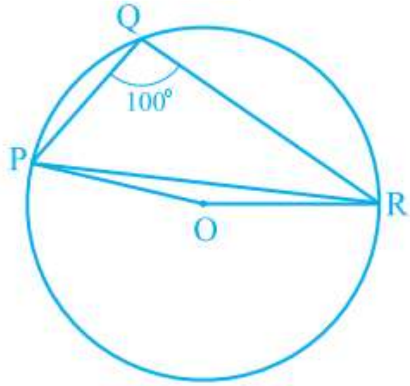
In the same way, $\angle PAQ = \frac{1}{2}(\angle POQ)$

$= \frac{1}{2}(60^0)$

$= 30^0$

Thus, angle subtended by the chord on the minor = 150^0 and on the major chord = 30^0.

Q.3 In figure $\angle PQR = 100^0$, there P, Q and R are points on a circle with centre O. Find $\angle OPR$.



Sol. Since, as we know that the angle subtended by an arc of a circle at its centre is twice the angle subtended by the same arc at any point on remaining part of the circumference.

So, Reflex $\angle POR = 2\angle PQR$

$\Rightarrow$ Reflex $\angle POR = 2\times100^\circ$

$= 200^\circ$

$\Rightarrow$So, $\angle POR = 360^0 - 200^0$

$= 160^0$

Now, in ΔOPR,

OP = OR (Radii of the same circle with center O)

$\Rightarrow \angle OPR = \angle ORP$ (Since, angles opp. to equal sides are equal)

and $\angle POR = 160^0$ .... (i)

And in ΔOPR,

$\angle OPR + \angle ORP + \angle PQR = 180^\circ$ (Angle sum property of a triangle)

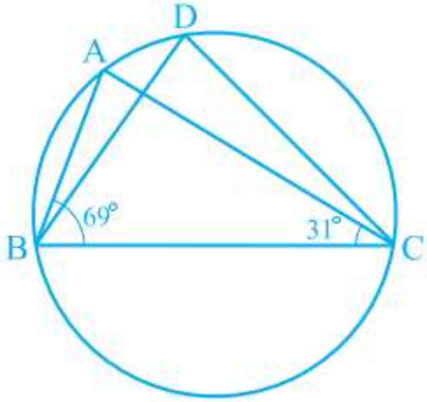
$\Rightarrow 160^\circ + 2\angle OPR = 180^\circ$

$\Rightarrow 2\angle OPR = 180^\circ - 160^\circ$

$\Rightarrow 2\angle OPR = 20^\circ$

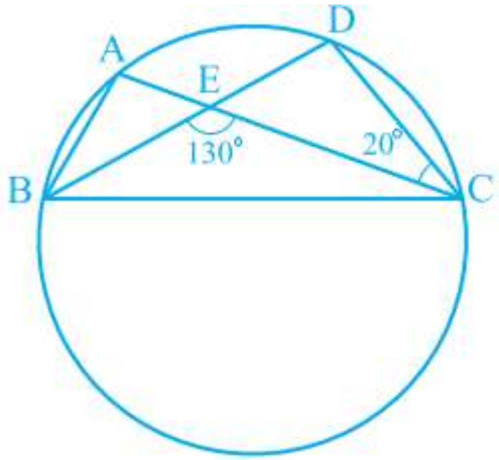
$\Rightarrow$ Thus, $\angle OPR = 10^\circ$

Q.4 In figure $\angle ABC = 69^0$, $\angle ACB = 31^0$, Find $\angle BDC$.



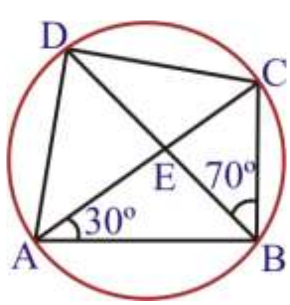
Sol. **Given:** $\angle ABC = 69^o$, $\angle ACB = 31^o$
Firstly, in ΔABC,
$\angle BAC + \angle ABC + \angle BCA = 180^o$ (Angle sum property of a triangle)
$\Rightarrow \angle BAC + 69^o + 31^o = 180^o$
$\Rightarrow \angle BAC = 180^o - (69^o + 31^o)$
$= 180^o - 100^o$
$= 80^o$
Since, as we know that angles in the same segment are equal
So, $\angle BDC = \angle BAC = 80^o$
Thus, $\angle BDC = 80^o$

Q.5 In figure A, B, C and D are four points on a circle. AC and BD intersect at a point E such that $\angle BEC = 130^o$ and $\angle ECD = 20^o$. Find $\angle BAC$.



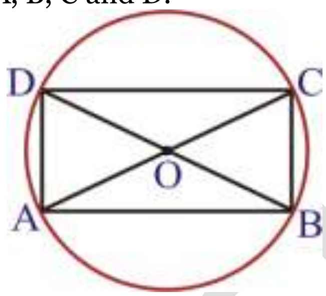
Sol. **Given:** Points A, B, C and D are the four points on a circle. AC and BD intersect at a point E such that $\angle BEC = 130^o$ and $\angle ECD = 20^o$.
$\angle CED + \angle CEB = 180^o$ (Linear pair angles)
$\Rightarrow \angle CED + 130^o = 180^o$
$\Rightarrow \angle CED = 180^o - 130^o$
$= 50^o$
Now, in ΔECD,
$\angle EDC + \angle CED + \angle ECD = 180^o$ (Angle sum property of a triangle)
$\Rightarrow \angle EDC + 50^o + 20^o = 180^o$ (Since, $\angle CED = 50^o$)
$\Rightarrow \angle EDC = 180^o - 50^o - 20^o$
$= 110^o$
$\Rightarrow \angle BDC = \angle EDC = 110^o$
Since, as we know that angles in the same segment are equal.
Thus, $\angle BAC = \angle BDC = 110^o$.

Q.6 ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If $\angle DBC = 70^o$, $\angle BAC$ is 30^o, find $\angle BCD$. Further, if AB = BC, find $\angle ECD$.
Sol. **Given:** ABCD is a cyclic quadrilateral and its diagonals intersect at a point E. $\angle DBC = 70^o$, $\angle BAC = 30^o$ and AB = BC



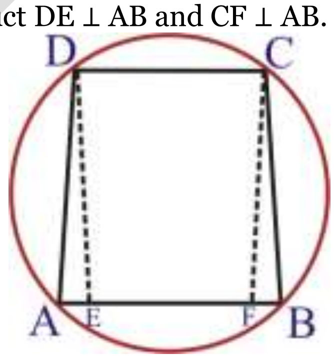
Since, ∠BDC = ∠BAC (Since, angles in the same segment)
⇒ So, ∠BDC = 30° (Since, given that ∠BAC = 30°)
Firstly, in ΔBCD,
∠BDC + ∠DBC + ∠BCD = 180° (Angle sum property of a triangle)
⇒ 30° + 70° + ∠BCD = 180° (Since, given that ∠DBC = 70° and ∠BDC = 30°)
⇒ ∠BCD = 180° – 30° – 70° = 80°
Now, if AB = BC,
Then, ∠BCA = ∠BAC = 30° (Since, angles opposite to equal sides)
∠BCA = ∠BCE = 30°
Now, ∠ECD =∠BCD – ∠BCE
= 80° – 30°
= 50°
Thus, ∠BCD= 80° and ∠ECD = 50°

Q.7 If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.
***Sol.* Given:** Let AC and BD be the diagonals of a cyclic quadrilateral ABCD which are diameters of the circle through the vertices A, B, C and D.



To prove: Quadrilateral ABCD is a rectangle.
Solution: Since, as we know that all the radii of the same circle are equal.
So, OA = OB = OC = OD
⇒ OA = OC = $\frac{1}{2}$ AC
and OB = OD = $\frac{1}{2}$ BD
⇒ AC = BD (Diameters of the same circle)
So, the diagonals of the quadrilateral ABCD are equal and bisect each other.
⇒Thus, quadrilateral ABCD is a rectangle.
Hence Proved.

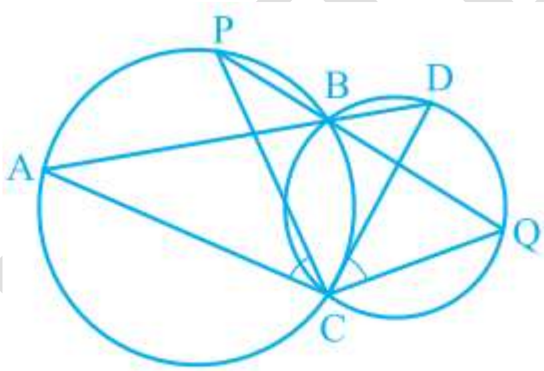
Q.8 If the non- parallel sides of a trapezium are equal prove that it is cyclic.
***Sol.* Given:** Let AD and BC are the non - parallel sides of a trapezium ABCD are equal. (i.e. AD = BC)
To prove: Trapezium ABCD is a cyclic.
Construction: construct DE ⊥ AB and CF ⊥ AB.



Proof: To prove that ABCD is a cyclic trapezium, we need to prove that

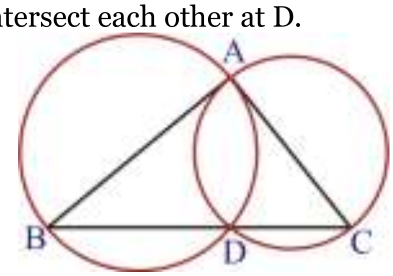
$\angle B + \angle D = 180^\circ$
Firstly, in ΔDEA and ΔCFB,
$AD = BC$ (Given)
$\angle DEA = \angle CFB$ (Since, $DE \perp AB$ and $CF \perp AB$ Each $= 90^\circ$)
and $DE = CF$ (Since, distance between two parallel lines is always equal.)
So, from RHS criterion of congruence,
$DEA \cong CFB$
$\Rightarrow \angle A = \angle B$ and $\angle ADE = \angle BCF$ (By C.P.C.T.)
Now, $\angle ADE = \angle BCF$
By adding 90° both the sides
$\Rightarrow 90^\circ + \angle ADE = 90^\circ + \angle BCF$
$\Rightarrow \angle EDC + \angle ADE = \angle FCD + \angle BCF$ (Since, $\angle EDC = 90^\circ$ & $\angle FCD = 90^\circ$)
$\Rightarrow \angle ADC = \angle BCD$
$\Rightarrow \angle D = \angle C$
Therefore, $\angle A = \angle B$ and $\angle C = \angle D$
Since, sum of the angles of a quadrilateral is 360°.
So, $\angle A + \angle B + \angle C + \angle D = 360^\circ$
$\Rightarrow 2\angle B + 2\angle D = 360^\circ$
$\Rightarrow \angle B + \angle D = 180^\circ$
Similarly, $\angle A + \angle C = 180^\circ$
Thus, ABCD is a cyclic trapezium.

Q.9 Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively (see figure). Prove that $\angle ACP = \angle QCD$.

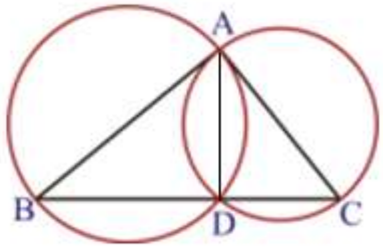


Sol. Since, as we know that angles in the same segment are equal.
So, $\angle ACP = \angle ABP$ ... (i)
and $\angle QCD = \angle QBD$ ... (ii)
Also $\angle ABP = \angle QBD$ ... (iii) (Vertically opposite angles)
So, from eq. (i), (ii) & (iii),
$\angle ACP = \angle QCD$..........Hence Proved.

Q.10 If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.
***Sol.* Given**: Let ABC be the triangle in which two circles are drawn with sides AB and AC as diameters. Both the circles intersect each other at D.

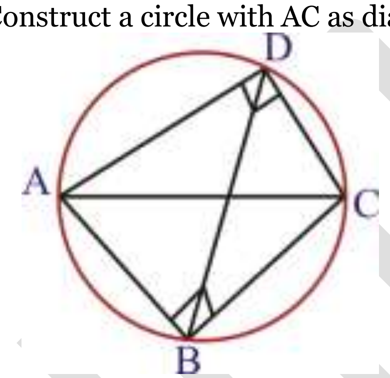


To prove: Point D lies on side BC.
Construction: Join A and D.



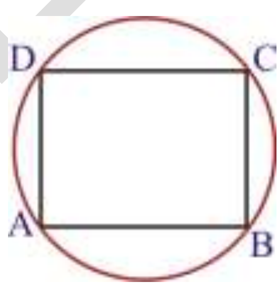
Proof: Since, AB and AC are the diameters of the two circles.
So, ∠ADB = 90⁰ (Since, angles in a semi- circle is 90º.)
and, ∠ADC = 90⁰ (Since, angles in a semi- circle is 90º.)
By adding,
∠ADB + ∠ADC = 90⁰ + 90⁰
= 180⁰
⇒ Thus, BDC is a straight line. Hence, D lies on BC.
Hence Proved.

Q.11 ABC and ADC are two right triangles with common hypotenuse AC. Prove that ∠CAD = ∠CBD.
***Sol.* Given:** ΔABC and ΔADC are right angled with common hypotenuse AC.
To Prove: ∠CAD = ∠CBD
Construction: Construct a circle with AC as diameter passing through the points B and D. Now, join BD.



Since, as we know that angles in the same segment are equal.
So, ∠CAD =∠CBD
Hence Proved.

Q.12 Prove that a cyclic parallelogram is a rectangle.
***Sol.* Given:** Let ABCD be a parallelogram inscribed in circle.
To prove: Cyclic parallelogram ABCD is a rectangle.
Proof: Since, given that ABCD is a cyclic parallelogram.



So, ∠A + ∠C = 180⁰ ... (i)
But ∠A = ∠C ... (ii) (Since Cyclic quadrilateral have same opposite angles.)
From (i) and (ii),

$\angle A + \angle A = 180^\circ$

$2\angle A = 180^\circ$

$\angle A = 90^\circ$

So, $\angle A = \angle C = 90^\circ$

In the same way, $\angle B = \angle D = 90^\circ$

So, each angle of cyclic parallelogram ABCD is 90°.

Thus, ABCD is a rectangle.

Hence Proved.