Arithmetic Progressions: Exercise 5.4

Q.1 Which term of the AP: 121, 117, 113….., is its first negative terms?

Sol. *Given:*121, 117, 113, ….

First term, $a = 121$, common difference, $d = 117 - 121 = -4$

Since, $a_n = a + (n-1)\,d$

$= 121 + (n - 1) \times - 4$

$= 121 - 4n + 4 = 125 - 4n$..............(i)

According to question, for the first negative term, $a_n < 0$

So, $125 - 4n < 0$

$\Rightarrow 125 < 4n$

$\Rightarrow 125/4 < n$

$\Rightarrow 31\frac{1}{4} < n$

n is an integer. So, n should be greater than $31\frac{1}{4}$.

Therefore, first negative term is 32nd term.

Q.2 The sum of the third and the seventh terms of an AP is 6 and their product is 8. Find the sum of first sixteen terms of the AP.

Sol. Given: $a_3 = a-2d$

$a_7 = a-6d$

And their sum: $a_3 + a_7 = a-2d + a-6d = 6$

$2a - 8d = 6$

$a - 4d = 3$

$a = 3-4d$.................(i)

And their product: $a_3 \text{ x } a_7 = 8$

$(a - 2d)\,(a + 2d) = 8$

$\Rightarrow (3-4d + 2d) \times (3-4d+6d) = 8$ from (i)

$\Rightarrow (3 - 2d)\times(3+2d) = 8$

$\Rightarrow 3^2 - 2d^2 = 8$

$\Rightarrow 9 - 4d^2 = 8$

$\Rightarrow 4d^2 = 1$

$\Rightarrow d = 1/2$ or $-1/2$

Taking $d = 1/2$

$S_{16} = 16/2\,[2\times(a - 4d) + (16-1)\times d]$

$= 8\left[2\text{x}(3 - 4\text{x}\frac{1}{2}) + 15\text{x}\frac{1}{2}\right]$

$= 8[2 + \frac{15}{2}] = 8\text{x}\frac{19}{2} = 76$

Now, taking $d = -1/2$

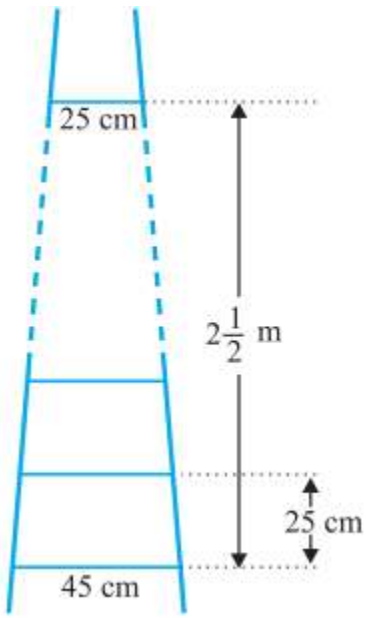
$S_{16} = 16/2[2 \times (a-4d) + (16-1)\times d]$

$= 8\left[2\text{x}(3 + 4\text{x}\frac{1}{2}) + 15\text{x-}\frac{1}{2}\right]$

$= 8[2\text{x}5 - \frac{15}{2}] = 8\text{x}\left[\frac{20-15}{2}\right] = 20$

Thus, the sum of first sixteen terms of the AP $S_{16} = 20, 76$

Q.3 A ladder has rungs 25 cm apart (see figure). The rungs decrease uniformly in length from 45 cm, at the bottom to 25 cm at the top. If the top and the bottom rungs are $2\frac{1}{2}$ m apart, what is the length of the wood required for the rungs?



Sol. ***Given:*** Number of rungs, $n = \dfrac{2\frac{1}{2}m}{25\ cm}$

$=250cm / 25cm = 10$

So, Number of rungs = 10 rungs

Therefore, the length of the wood required for rungs = Sum of 10 rungs

$S_{10} = 10/2\ [25+45]$

$= 5\times70 = 350cm$

So, the length of the wood is required for the rungs= 350 cm

Q.4 The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find this value of x.

Sol. Since, according to question, first term, a = 1, and common difference, d = 1.

$$So,\ S_{x-1} = \frac{x-1}{2}[2\times1+(x-1-1)\times1]$$

$$= \frac{x-1}{2}[2+x-2]$$

$$= \frac{x^2-x}{2}\ldots\ldots\ldots\ldots..(i)$$

$S_x = x/2\ [2\times1+(x-1)\times1] = (x/2)(x+1)$

$$= \frac{x^2+x}{2}\ldots\ldots\ldots\ldots..(ii)$$

and, $S_{49} = 49/2\ [2\times1+(49-1)\times1]$

$=49/2[2+48] = (49/2)\times50$

$=49\times25$

So, according to the question,

$S_{x-1} = S_{49} - S_x$

$$\frac{x^2 - x}{2} = 49x25 - \frac{x^2 + x}{2} \quad \text{From (i) \& } (ii)$$

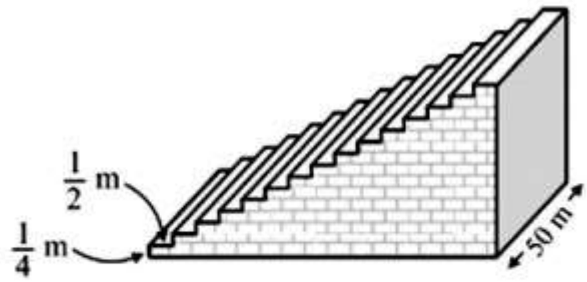
$$\frac{x^2 - x}{2} + \frac{x^2 + x}{2} = 49x25$$

$$x^2 = 49x25$$

$$x^2 = \pm 7x5$$

Since, value of x should be positive square root, x = 7 × 5 = 35.

Q.5 A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete.



Each step has a rise of 1/4m **and a tread of** 1/2m **(see figure). Calculate the total volume of concrete required to build the terrace.**

Sol. Volume of concrete required to build the first step $= \frac{1}{4} x \frac{1}{2} x50$

Second step $= \left(2x\frac{1}{4}\right) x \frac{1}{2} x50,$

Third step $= \left(3x\frac{1}{4}\right) x \frac{1}{2} x50 .... (in\ cm^3)$

And so on...

This make the sequence:

$$\frac{1}{4} x \frac{1}{2} x50,\ \left(2x\frac{1}{4}\right) x \frac{1}{2} x50,\ \left(3x\frac{1}{4}\right) x \frac{1}{2} x50, ...$$

$$i.e.\ \frac{50}{8},\ 2x\frac{50}{8},\ 3x\frac{50}{8}, ...$$

So, total volume of concrete required.

$$= \frac{50}{8} + 2x\frac{50}{8} + 3x\frac{50}{8} + ...$$

$$= \frac{50}{8}[1+2+3+......]$$

Since, S_n = n/2 [2a + (n-1)d]

$$= \frac{50}{8} x \frac{15}{2}[2x1 + [15-1]x1] \text{ [since, n=15]}$$

$$= \frac{50}{8} x \frac{15}{2} x16$$

$$= 750 \text{ cm}^3$$

So, total required volume: 750 cm^3