Arithmetic Progressions: Exercise 5.3

Q.1 Find the sum of the following APs:

(i) 2, 7, 12, ... to 10 terms.

(ii) – 37, –33, – 29, ... to 12 terms.

(iii) 0.6, 1.7, 2.8...., to 100 terms.

(iv) 1/15, 1/12, 1/10,.... to 11 terms.

Sol. **(i) Given: AP : 2, 7, 12, ... to 10 terms.**

First term, a = 2 and common difference, d = 7 – 2 = 5

The sum of 10 terms of the given AP:

a = 2, d = 5, n = 10

Since, $S_n = n/2\,[2a + (n - 1)\,d]$

Putting values

$S_{10} = 10/2\,[\,2\times2 + (10 - 1)\,5]$

$=5(4 + 9\times5)$

$= 5(4 + 45) = 5 \times 49 = 245$

(ii) Given: AP: – 37, –33, – 29, ... to 12 terms.

First term, a = – 37 and common difference, d = – 33 – (– 37) = – 33 + 37 = 4

n = 12

Since, $S_n = n/2\,[2a + (n-1)\,d]$

$S_{12} = 12/2\,[2 \times -37 + (12-1)4]$

$= 6\,(- 74 + 11\times4)$

$= 6\,(- 74 + 44) = 6 \times (- 30) = - 180$

(iii) Given: AP: 0.6, 1.7, 2.8...., to 100 terms.

First term, a = 0.6 and Common difference, d = 1.7 – 0.6 = 1.1

n = 100

Since, $S_n = n/2\,[2a + (n-1)\,d]$

$S_{100} = 100/2\,[2\times0.6 + (100-1)1.1]$

$= 50(1.2 + 99 \times 1.1)$

$= 50\,(1.2 + 108.9)$

$= 50 \times 110.1 = 5505$

(iv) Given: AP: $\frac{1}{15},\frac{1}{12},\frac{1}{10}$,........ to 11 terms.

First term, a=1/15 and Common difference, $d = \frac{1}{12} - \frac{1}{15} = \frac{5-4}{60} = \frac{1}{60}$

n=11

Since, $S_n = n/2\,[2a + (n-1)\,d]$

$$S_{11} = \frac{11}{2}\left[2x\frac{1}{15} + (11-1)\frac{1}{60}\right]$$

$$S_{11} = \frac{11}{2}\left[\frac{2}{15} + (10)\frac{1}{60}\right]$$

$$S_{11} = \frac{11}{2}\left[\frac{2}{15} + \frac{1}{6}\right]$$

$$S_{11} = \frac{11}{2}\left[\frac{4+5}{30}\right]$$

$$S_{11} = \frac{11}{2}x\left[\frac{9}{30}\right]$$

$$S_{11} = \frac{33}{20}$$

Q.2 Find the sums given below:

(i) $7+10\frac{1}{2}+14+....+84$

(ii) $34 + 32 + 30 + ... + 10$

(iii) $-5 + (-8) + (-11) + ... + (-230)$

Sol. **(i)** Given: AP: $\mathbf{7+10\frac{1}{2}+14+....+84}$

First term , a = 7, Common difference, $d=10\frac{1}{2}-7=3\frac{1}{2}=\frac{7}{2}, l=a_n=84$

Since, $a_n = a + (n-1)d$, Where $a_n = 84$

$\Rightarrow 84 = 7 + (n-1)\frac{7}{2}$

$\Rightarrow (n-1)\frac{7}{2}=84-7$

$\Rightarrow (n-1)\frac{7}{2}=77$

$\Rightarrow (n-1)=77\text{x}\frac{2}{7}$

$\Rightarrow (n-1)=22$

$\Rightarrow n=23$

Since, $S_n = n/2\,(a + \ell)$

$\Rightarrow S_{23} = 23/2\,(7 + 84) = 23/2 \times 91$

$==\frac{2093}{2}=1046\frac{1}{2}$

(ii) Given: AP: $34 + 32 + 30 + ... + 10$

First term, $a = 34$, common difference, $d = 32 - 34 = -2$, $\ell = a_n=10$

Since, $a_n = a + (n-1)d$, $a_n =10$

$\Rightarrow 10 = 34 + (n-1)(-2)$

$\Rightarrow (-2)(n-1) = 10 - 34$

$\Rightarrow (-2)(n-1) = -24$

$\Rightarrow n - 1 = 12$

$\Rightarrow n = 12 + 1 = 13$

Since, $S_n = n/2\,(a + \ell)$

$S_{13} =13/2\,(34 + 10) = 13/2 \times 44 = 13 \times 22 = 286$

(iii) Given: AP: $\mathbf{-5 + (-8) + (-11) + ... + (-230)}$

First term, $a = -5$, common difference, $d = -8 - (-5) = -8 + 5 = -3$,

$\ell = a_n = -230$

Since, $a_n = a + (n-1)d$

$\Rightarrow -230 = -5 + (n-1)(-3)$

$\Rightarrow (-3)(n-1) = -230 + 5$

$\Rightarrow (-3)(n-1) = -225$

$\Rightarrow n-1= -225/-3$

$\Rightarrow n - 1 = 75$

$\Rightarrow n = 75 + 1 = 76$

Since, $S_n = n/2\,(a + \ell)$

$S_{76} = 76/2\,(-5-230) = 38 \times -235 = -8930$

Q.3 In an AP:

(i) Given $a = 5$, $d = 3$, $an=50$, find n and S_n.

(ii) Given $a = 7$, $a_{13} = 35$,find d and S_{13}

(iii) Given $a_{12} = 37$, $d = 3$, find a and S_{12}

(iv) Given $a_3 =15$, $S_{10} = 125$, find d and a_{10}

(v) Given $d=5$, $S_9 = 75$, find a and a_9

(vi) Given $a = 2$, $d = 8$, $S_n = 90$, find n and a_n

(vii) Given $a = 8$, $a_n = 62$, $S_n = 210$, find n and d.

(viii) Given $a_n = 4$, $d = 2$, $S_n = -14$, find n and a.

(ix) Given $a = 3$, $n = 8$, $S = 192$, find d.
(x) Given $l = 28$, $S = 144$, and there are total 9 terms. Find a.

Sol. **(i)** Given: $a = 5$, $d = 3$ and $a_n=50$

$\Rightarrow a + (n - 1) d = 50$
$\Rightarrow 5 + (n - 1) 3 = 50$
$\Rightarrow 3(n - 1) = 50 - 5$
$\Rightarrow n-1 = 45/3 = 15$
$\Rightarrow n = 15 + 1 = 16$
$n = 16$, $a = 5$ and $\ell = a_n=50$
Since, $S_n = n/2 (a + \ell)$

$S_{16} = 16/2 (5 + 50) = 8\times 55 = 440$
Thus, $n = 16$ and $S_{16} = 440$

(ii) Given: $a = 7$ and $a_{13} = 35$
$\Rightarrow a_{13} = 35$ [Since, $a_n = a + (n-1) d$]
$\Rightarrow a + 12 d = 35$
$\Rightarrow 7 + 12 d = 35$
$\Rightarrow 12 d = 35 - 7 = 28$
$\Rightarrow d = 28/12 = 7/3$

$n = 13$, $a = 7$ and $\ell = a_{13} = 35$
$S_n = n/2 (a + \ell)$
$S_{13} = 13/2 (7 + 35)$
$=13/2 \times 42$
$= 13 \times 21 = 273$
Thus, $d=73$ and $S_{13} = 273$

(iii) Given: $a_{12} =37$, $d=3$
$a_{12} = 37$ [Since, $a_n = a + (n-1) d$]
$\Rightarrow a + 11d = 37$
$\Rightarrow a + 11(3) = 37$
$\Rightarrow a + 11(3) = 37$
$\Rightarrow a = 37 - 33 = 4$

$n = 12$, $a = 4$ and $\ell = a_{12} = 37$
$S_n = n/2 (a + \ell)$
$S_{12} = 12/2 (4 + 37) = 6 \times 41 = 246$
Thus, $a = 4$ and $S_{12} = 246$

(iv) Given: $a_3 =15$, $S_{10} = 125$
Since, $a_n = a + (n-1) d$
$\Rightarrow a + 2d = 15$ ... (i)
and $S_n = n/2 (2a + (n-1) d)$
$10/2 [2a + (10 - 1) d] = 125$
$\Rightarrow 5(2a + 9d) = 125$
$\Rightarrow 2a + 9d = 25$ ... (ii)
Multiply (i) by 2 and subtract (ii), we get

$2(a + 2d) - (2a + 9d) = 2 \times 15 - 25$
$\Rightarrow 4d - 9d = 30 - 25$
$\Rightarrow - 5d = 5$
$\Rightarrow d = -5/5 = -1$
Since, $a_{10} = a + 9d = (a+2d)+7d$
$a_{10} = 15 + 7 (- 1)$ by using (i)
$a_{10} = 15 - 7 = 8$
Thus, $d = - 1$ and $a_{10}=8$

(v) Given: $d = 5$, $S_9=75$
Since, $S_n = n/2 (2a + (n-1) d)$
$\Rightarrow 9/2 [2a + (9-1)5]=75$
$\Rightarrow 9/2 (2a + 40)=75$

$\Rightarrow 9a + 180 = 75$
$\Rightarrow 9a = 75 - 180$
$\Rightarrow 9a = -105$
$\Rightarrow a = -105/9 = -35/3$
Since, $a_9 = a + 8d = (-35/3) + 8\times5$
$= \frac{-35+120}{3} = \frac{85}{3}$
Thus, $a = -35/3$ and $a_9 = 85/3$

(vi) Given: $a = 2$, $d = 8$, $S_n=90$
Since, $S_n = n/2\,(2a + (n-1)\,d)$
$\Rightarrow n/2\,[2\times2 + (n-1)8] = 90$
$\Rightarrow n/2\,(4+8n-8) = 90$
$\Rightarrow n/2\,(8n-4) = 90$
$\Rightarrow n\,(4n-2)=90$
$\Rightarrow 4n^2 - 2n - 90 = 0$
So, $n = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \text{ x } 4 \text{ x } (-90)}}{2\text{x}4}$

$=(2\pm38)/8$
$=40/8, -36/8 = 5, -92$
Since, n cannot be negative
So, $n = 5$
Now $a_n = a + (n-1)d$
$\Rightarrow a_5 = 2+(5-1)8$
$=2+32=34$
Thus, $n = 5$ and $a_n=34$

(vii) Given: $a = 8$, $a_n = 62$, $S_n=210$

Since, $S_n = n/2\,(a + \ell)$
$\Rightarrow n/2\,(a + \ell) = 210$
$\Rightarrow n/2\,(8 + 62) = 210$
$\Rightarrow n/2 \times 70 = 210$
$\Rightarrow n=210\times(2/70) = 3\times2=6$
and $a_n = 62$
$\Rightarrow a_6 = 62$
$\Rightarrow a + 5d = 62$
$\Rightarrow 8 + 5d = 62$
$\Rightarrow 5d = 62 - 8 = 54$
$\Rightarrow d=54/5$
Thus, $d = 54/5$ and $n = 6$

(viii) Given: $a_n = 4$, $d = 2$, $S_n = -14$
Since, $a_n = a + (n-1)\,d$
$\Rightarrow a + (n - 1)2 = 4$
$\Rightarrow a = 4 - 2\,(n - 1)$ ... (i)
and $S_n = -14$
Since, $S_n = n/2\,(a + \ell)$
$\Rightarrow n/2(a + \ell) = -14$
$\Rightarrow n\,(a + 4) = -28$ from (i)
$\Rightarrow n[4 - 2\,(n - 1) + 4] = -28$
$\Rightarrow n\,(4 - 2n + 2 + 4) = -28$
$\Rightarrow n(-2n + 10) = -28$
$\Rightarrow n\,(-n + 5) = -14$
$\Rightarrow -n^2 + 5n = -14$
$\Rightarrow n^2 - 5n - 14 = 0$
$\Rightarrow (n - 7)\,(n + 2) = 0$

$\Rightarrow n = 7$ or -2
Since, n cannot be negative
So, $n = 7$
Now put $n = 7$ in (i),
$a = 4 - 2(7 - 1) = 4 - 2 \times 6$
$= 4 - 12 = -8$
Thus, $n = 7$ and $a = -8$

(ix) Given: $a = 3, n = 8, S = 192$
Since, $S_n = n/2\,[2a + (n-1)d]$
$\Rightarrow 192 = 8/2\,[2\times3+(8-1)d]$
$\Rightarrow 192 = 4(6 + 7d)$
$\Rightarrow 48 = 6 + 7d$
$\Rightarrow 7d = 48 - 6$
$\Rightarrow 7d = 42$
$\Rightarrow d = 42/7 = 6$
Thus, $d = 6$

(x) Given: $\ell = 28, S = 144, n = 9$
Since, $S_n = n/2\,(a + \ell)$
$\Rightarrow n/2\,(a + \ell) = 144$
$\Rightarrow 9/2\,(a + 28) = 144$
$\Rightarrow a+28=144 \times (2/9)$
$\Rightarrow a + 28 = 32$
$\Rightarrow a = 32 - 28 = 4$
Thus, $a = 4$

Q.4 How many terms of the AP : 9 , 17, 25, … must be taken to give a sum of 636 ?
Sol. **Given: AP: 9, 17, 25, …**
First term, $a = 9$ and common difference, $d = 17 - 9 = 8$.
Let n be the number of terms. Then,
$S_n = 636$
Since, $S_n = n/2\,[2a + (n-1)d]$
$\Rightarrow n/2\,[2a + (n-1)d] = 636$
$\Rightarrow n/2\,[2\times9+(n-1)8] = 636$
$\Rightarrow n/2\,(18+8n-8) = 636$
$\Rightarrow n/2\,(8n+10) = 636$
$\Rightarrow n(4n + 5) = 636$
$\Rightarrow 4n^2 + 5n - 636 = 0$
Therefore, $n = \dfrac{-(5) \pm \sqrt{25 - 4x4x(-636)}}{2x4}$
$= (-5\pm101)/8 = 96/8, -106/8$
$12, -53/4$
Since, n cannot be negative.
So, $n = 12$
Therefore, the sum of 12 terms = 636.

Q.5 The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.
Sol. **Given: First term, a = 5, last term ℓ or a_n= 45 and S = 400**
Since, $S_n = n/2\,(a + \ell)$
$\Rightarrow n/2\,(a + \ell) = 400$
$\Rightarrow n(5 + 45) = 400\times2$
$\Rightarrow n(50) = 400 \times 2$
$\Rightarrow n = (400\times2) / 50 = 8\times2 = 16$
Since, $\ell = 45$

$\Rightarrow a + (n - 1)\,d = 45$
$\Rightarrow 5 + (16 - 1)d = 45$
$\Rightarrow 15d = 45 - 5 = 40$
$\Rightarrow a = 40/15 = 8/3$
Thus, the number of term, $n = 16$
and the common difference, $d = 8/3$.

Q.6 The first and the last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Sol. **Given: First term, $a = 17$, last term, ℓ or $a_n = 350$, $d = 9$**

Since, $a_n = a + (n-1)\,d$
$\Rightarrow a + (n - 1)\,d = 350$
$\Rightarrow 17 + (n - 1)9 = 350$
$\Rightarrow 9(n - 1) = 350 - 17$
$\Rightarrow n-1 = 333/9 = 37$
$\Rightarrow n = 37 + 1 = 38$
Since, $S_n = n/2\,(a + \ell)$,
Now put $a = 17$, $\ell = 350$, $n = 38$
$S_{38} = 38/2\,(17 + 350)$
$= 19 \times 367 = 6973$
Thus, Number of term = 38 terms and their sum = 6973.

Q.7 Find the sum of first 22 terms of an AP in which $d = 7$ and 22nd term is 149.

Sol. Suppose a is the first term and d is the common difference of the given AP.
Given: $d = 7$ and $a_{22} = 149$
Since, $a_n = a + (n-1)\,d$
$\Rightarrow a + (22 - 1)\,7 = 149$
$\Rightarrow a + 21 \times 7 = 149$
$\Rightarrow a = 149 - 147 = 2$
Since, $S_n = n/2\,[2a + (n-1)d]$
Now put $n = 22$, $a = 2$ and $d = 7$
$S_{22} = 22/2\,[2\times2 + (22-1)7]$
$= 11(4 + 21 \times 7)$
$= 11(4 + 147)$
$= 11 \times 151 = 1661$
Thus, the sum of first 22 terms = 1661.

Q.8 Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Sol. Given: $a_2 = 14$ and $a_3 = 18$
Since, $a_n = a + (n-1)\,d$
So, $\Rightarrow a + d = 14$ ....(i)
$a + 2d = 18$..................(ii)
Solve these equations, we get
$d = 4$ and $a = 10$
Since, $S_n = n/2\,[2a + (n-1)d]$
Now put $a = 10$, $d = 4$ and $n = 51$
$S_{51} = 51/2\,[2\times10 + (51-1) \times 4]$
$= 51/2\,[20 + 50\times4]$
$= 51/2\,(20 + 200) = (51/2) \times 220$
$= 51 \times 110 = 5610$

Q.9 If the sum of 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of n terms.

***Sol.* Given: $S_7 = 49$ and $S_{17} = 289$**

Since, $S_n = n/2\,[2a + (n-1)d]$

$\Rightarrow 7/2[2a+(7-1)d]=49$

$\Rightarrow 7/2(2a + 6d) = 49$

$\Rightarrow a + 3d = 7$ .... (i)

$17/2\,[2a + (17-1)d] = 289$

$\Rightarrow 17/2\,(2a + 16d) = 289$

$\Rightarrow a + 8d = 17$ .... (ii)

Solve these two equations

$\Rightarrow 5d = 10$, $d = 2$ and $a = 1$

Since, $S_n = n/2\,[2a + (n-1)d]$

$= n/2\,[2\times1 + (n-1)2]$

$=n/2\,(2 + 2n - 2) = (n/2) \times 2n = n^2$

So, $S_n = n^2$

Q.10 Show that $a_1, a_2 \ldots a_n \ldots$ form an AP where a_n is defined as below:

(i) $a_n = 3 + 4n$ (ii) $a_n = 9 - 5n$

Also find the sum of the first 15 term in each case.

***Sol.* (i)** Given: $a_n = 3 + 4n$

By putting n = 1, 2, 3, 4, ... , n , then we get

The sequence 7, 11, 15, 19, ... $(3 + 4n)$

This sequence forms an AP with first term, a = 7, common difference, d = 4 and n = 15

So, the sum of the first 15 term

Since, $S_n = n/2\,[2a + (n-1)d]$

$S_{15} = 15/2\,[2 \times 7 + (15-1)4]$

$=15/2\,(14 + 14\times4)$

$=15/2\,(14 + 56)$

$= (15/2) \times 70 = 15\times35$

$= 525$

(ii) Given: $a_n = 9 - 5n$

By putting n = 1, 2, 3, 4, ... n, then we get

The sequence 4, – 1, – 6, – 11, ... $(9 - 5n)$.

This forms an AP with first term, a = 4, common difference, d = – 5 and n = 15.

Since, $S_n = n/2\,[2a + (n-1)d]$

$S_{15} =15/2\,[2\times4 + (15-1)(-5)]$

$=15/2\,(8 + 14\times-5)$

$=15/2\,(8-70) = (15/2) \times -62$

$= 15 \times -31 = -465$

Q.11 If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly, find the 3rd the 10th and the nth terms.

***Sol.* *Given:* the sum of the first n terms of an AP is:**

$S_n = 4n - n^2$ ....(i)

$S_1 = 4\times1-1^2$

$= 4 - 1 = 3$

So, First term $S_1 = 3$

Now, sum of first two terms is by putting n=2 in (i)

$S_2 = 4\times2-2^2$

$=8-4=4$

So, Second term will be $= S_2 - S_1$

$= 4 - 3 = 1$

Similarly ,

$S_3 = 4\times3 - 3^2$

$= 12 - 9 = 3$

So, Third term will be $= S_3 - S_2$

$= 3 - 4 = -1$

Similarly, Tenth term $= S10 - S9$

$= [4\times10 - 10^2] - [4\times9 - 9^2]$

$= [36 - 81] - [40 - 100]$

$= -60 - (-45)$

$= -60 + 45 = -15$

Now n^{th} term will be: $S_n - S_{n-1}$

$= [4n - n^2] - [4(n-1) - (n-1)^2]$

$= 4n - n^2 - (-n^2 + 6n - 5)$

$= 4n - n^2 + n^2 - 6n + 5$

$= 5 - 2n$

Q.12 Find the sum of the first 40 positive integers divisible by 6.

Sol. Since, first positive integers divisible by 6: 6, 12, 18, ...

We can see that this sequence forms an AP with first term, a = 6 and common difference, d = 6.

So, the sum of the first 40 positive integers terms are:

Since, $S_n = n/2\,[2a + (n-1)d]$

$S_{40} = 40/2\,[2\times6 + (40-1)\,6]$

$= 20\,(12 + 39 \times 6)$

$= 20(12 + 234)$

$= 20 \times 246$

$= 4920$

Thus, the sum of the first 40 positive integers divisible by 6 =4920

Q.13 Find the sum of the first 15 multiples of 8.

Sol. Since, first 15 multiples of 8 are $8 \times 1, 8 \times 2, 8 \times 3, \ldots 8 \times 15$

Sequence: 8, 16, 24 120,

We can see that this sequence forms an AP with first term, a =8 and common difference, d =8 and last term, ℓ= 120.

So, sum of 1st 15 multiples of 8:

Since, $S_n = n/2\,(a + \ell)$

$S_{15} = 15/2\,(8+120)$

$= 15/2 \times 128$

$= 15 \times 64 = 960$

Q.14 Find the sum of the odd numbers between 0 and 50.

Sol. Since, odd numbers between 0 and 50 are:

Sequence: 1, 3, 5, ...49.

We can see that this sequence forms an AP with first term, a = 1 and the last term, ℓ = 49 and there are 25 terms.

Since, $S_n = n/2\,(a + \ell)$

$S_{25} = 25/2\,(1 + 49)$

$= 25/2 \times 50$

$= 25 \times 25 = 625$

Q.15 A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: Rs 200 for the first day, Rs 250 for the second day, Rs 300 for the third day, etc., the penalty for each succeeding day being Rs 50 more than for the preceding day. how much money the contractor has to pay as penalty, if he has delayed the work by 30 days?

Sol. As the question, penalties form the sequence: 200, 250, 300.........

Where, first term, a = 200 , common difference, d = 50 and n = 30

So, the money contractor has to pay as penalty, if he has delayed the work by 30 days:

Since, $S_n = n/2\,[2a + (n-1)d]$

So, $S_{30} = 30/2\,[2\times200 + (30-1)50]$

$= 15(400 + 29 \times 50)$

$= 15(400 + 1450)$

$= 15 \times 1850$

$= 27750$

Thus, a delay of 30 days costs the contractor Rs 27750.

Q.16 A sum of Rs 700 is to be used to give seven each prizes to students of a school for their overall academic performance. If each prize is Rs 20 less than its preceding prize, find the value of each of the prizes.

Sol: Suppose the respective prizes are:

Sequence: a + 60, a + 40, a + 20, a, a – 20, a – 40, a – 60

So, The sum of the prizes:

$a + 60 + a + 40 + a + 20 + a + a - 20 + a - 40 + a - 60 = 700$

$\Rightarrow 7a = 700$

$\Rightarrow a = 700/7 = 100$

Thus, the seven prizes will be:

=100 + 60, 100 + 40, 100 + 20, 100, 100 – 20, 100 – 40, 100 – 60

=160, 140, 120, 100, 80, 60, 40 (in Rs)

Q.17 In a school, students thought of planting trees in an around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying e.g., a section of Class I will plant 1 tree, a section of Class II will plant 2 trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?

Sol. According to question, there are three sections of each class, so the number of trees planted by class I, class II, class III,... class XII are 1 × 3, 2 × 3, 3 × 3, ... 12 × 3 respectively

Then, the sequence: 3, 6, 9, ... 36.

From above we can see that sequence forms an AP with first term, a=3 and last term, ℓ =36.

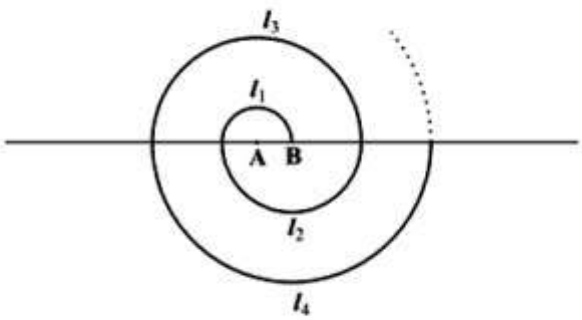
The sum of the number of the trees planted by each class:

Since, $S_n = n/2\,(a + \ell)$

$= 12/2\,(3 + 36)$

$= 6 \times 39 = 234$

Q.18 A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, ... as shown in fig.



What is the total length of such a spiral made up of thirteen consecutive semicircles?
(Take π = 22/7)

Sol. According to question, length of spiral made up of thirteen consecutive semicircles.

Since, length of a semi-circumference = πr

where r is the radius of the circle.
So, length of the thirteen consecutive semicircles:

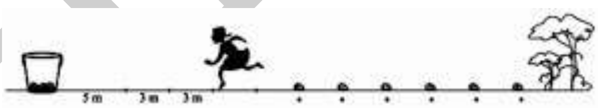
$= (\pi\times0.5 + \pi\times1.0 + \pi x1.5 + \pi\times2.0 + \ldots\ldots\ldots + \pi\times6.5)$cm
$= \pi\times0.5\ (1 + 2 + 3 + \ldots. + 13)$cm
$= \pi\times0.5\times132(2\times1+[13-1)\times1]$cm
$= (22/7) \times (5/10) \times (13/2) \times 14$cm
$=143$cm

Q.19 200 logs are stacked in the following manner. 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on (see figure). In how many rows are the 200 logs placed and how many logs are in the top row?

Sol. Since, logs stacked in each row form a sequence:
$20 + 19 + 18 + 17 + \ldots.$
So, we can see that this sequence forms an an AP with first term, $a = 20$, common difference, $d = 19 - 20 = -1$.

Let $S_n=200$
Since, $S_n = n/2\ [2a + (n-1)d]$
$\Rightarrow n\ (40 - n + 1) = 400$
$\Rightarrow n^2 - 41n + 400 = 0$
$\Rightarrow (n-16)\ (n-25)=0$
$\Rightarrow n = 16$ or 25
Since, the common difference is negative. So, the terms go on diminishing and 25^{st} term is negative. Thus $n = 25$ is not valid for this problem.
So we need to take $n = 16$. Therefore, 200 logs are placed in 16 rows.
Number of logs in the 16^{th} row are:
$a_{16} = a + (n-1)d$
$= a + 15d$
$= 20 + 15(-1)$
$= 20 - 15 = 5$

Q.20 In a potato race, a bucket is placed at the starting point, which is 5 cm from the first potato, and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line (see figure).



A competitor starts from the bucket, picks up the earest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?

Sol. Since, runner picks up the potato, runs back with it and drops it in the bucket. To pick up the first potato second potato, third potato, fourth potato, ...up to ten potato.
So, the distance travelled by the competitor are
$2 \times 5\ ;\ 2 \times (5 + 3),\ 2 \times (5 + 3 + 3),\ 2 \times (5 + 3 + 3 + 3), \ldots.$
Then sequence: 10, 16, 22, 28, ...
This sequence forms an AP with first term, $a = 10$, common difference, $d = 16 - 10 = 6$
So, The sum of first ten terms,
Since, $S_n = n/2\ [2a + (n-1)d]$
$S_{10} = 10/2\ [(2\times10 + (10-1)\times6)]$
$= 5(20 + 54)$
$= 5 \times 74 = 370$
Thus, The total distance the competitor has to run is 370 m.